

The Art and Logic of Ramon Llull

A USER'S GUIDE



BY

ANTHONY BONNER

The Art and Logic of Ramon Llull

Studien und Texte zur Geistesgeschichte des Mittelalters

Begründet von

Josef Koch

Weitergeführt von

Paul Wilpert, Albert Zimmermann und
Jan A. Aertsen

Herausgegeben von

Andreas Speer

In Zusammenarbeit mit

Tzotcho Boiadjiev, Kent Emery, Jr.
und Wouter Goris

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*To the memory of
Robert Pring-Mill
and David Rosenblatt*

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PREFACE

This is a book about method, not content; about the how, not the what. This does not mean that the one is independent of the other, that the form of Ramon Llull's message did not deeply influence its matter, or, more importantly, that the matter and, above all, its goals, were not primary in determining its form. We will, in fact, frequently touch on such subjects, but only as they come up in connection with the form. It is therefore a question of priorities; as Llull would have put it, the first intention of this book is the method, and the second intention the content.

Even though, in the course of developing his method, Llull made frequent and important programmatic statements presenting its foundations, these should not be confused with the method itself, any more than we should confuse the foundations of a building with the finished edifice. In both cases they can only provide a beginning for a future exploration of the structure. Moreover, Llull was not a speculative theologian, philosopher, or logician. His study of these subjects is never an end in itself, but only the means to an end. For the purposes of this book, it means that his interest in logic—and this applies to the Art insofar as it constitutes a logical system—is not theoretical but practical. His is not a *logica docens, speculativa, or theorica*, but rather what the Middle Ages called *logica utens*, one to be judged by its usefulness. Since his logic was directed to producing what he called 'necessary reasons', it is these arguments to which we must turn our attention, to see what it was about them that he felt justified in considering 'necessary'. This is particularly urgent because in the past a great deal of research has gone into these 'necessary reasons', but mostly from the perspective of what they imply concerning the question of faith vs. reason. Even more research has gone into the bases of the Art—the Dignities, the components of figures such as S and T, the Nine Subjects, etc. Singularly little effort, however, has gone into trying to bridge the gap between the theoretical foundations of Llull's Art and logic, or between its bases and their final expression in patterns of argument, and perhaps even less in the actual study of these patterns, which in fact constituted his 'necessary reasons', the backbone of his entire endeavor.

Since it seemed to me that the best—if not the only—way to study these argument patterns is to display them along with appropriate comments, I have structured this book as a kind of *explication de textes*, always quoting in full (and in translation) those Lullian *textes* on which my *explications* are based, instead of just referring the reader to the place where he might find the originals. That way the reader can see for himself the tenor of Llull's arguments, and judge more closely the correctness or error of my assessments. Perhaps the principal purpose in writing thus is to let Llull speak for himself, and to try to avoid explaining what I (or others) thought he might have meant instead of what he actually said. In the few places where I have offered non-Lullian models, it has been for purely hermeneutic purposes, as an aid in trying to understand and explain what he is doing.

Too often it has been assumed that when Llull used a particular word or discussed a particular philosophical or logical doctrine, it must be like that of contemporary usage, and if it wasn't, that was because he hadn't quite grasped what others meant. The latter option has, I think, now been discarded for a view of Llull as quite consciously setting up a system (or systems) alternate to that (or those) of his contemporaries. And because this alternate system of his was self-referential, it developed its own network of meanings and doctrinal interpretations. Llull seemed aware enough of this problem to offer many definitions and explanations of what he was doing, and if we can manage to listen carefully to his voice, we might better capture his intentions. Only when we finally see the structure as a whole can we begin to study its relation to outside structures. We must begin inside, and then, if we wish, begin to proceed outward.

After a first general chapter giving an outline of Llull's life and a brief introduction to the Art, come four specialized chapters explaining the nuts and bolts of his system, or systems. Chapters 2 and 4 are more or less straightforward presentations of the two phases of the Art, the quaternary and the ternary. Chapter 3 presents the transition between the two, with its important cosmological and systematic changes (along with some methodological experiments). Chapter 5 presents Llull's incursion into logic during the final stage of his career. I have thought it necessary to venture into this domain because it starts as a natural prolongation of the Art, and because it represents Llull's final endeavor in providing 'necessary reasons'. Chapter 6 is a conclusion, where I have tried to tie together some of the many threads woven throughout the book and arrive at some generalizations as to Llull's methods. These

chapters are to a certain extent independent of one another—in spite of many cross-references—and thus can, to the same extent, be read independently. For those more interested in the literary side of Llull, Chapter 2 deals with the period during which he produced his best-known literary works. Chapter 4 deals with the version of the Art for which Llull was best known in the sixteenth and seventeenth centuries. Chapter 5 might be for those interested in Llull's reshaping of classical logic. For a final overview there is Chapter 6.

As to things omitted from the book, I must first of all say that to try an *explication de textes* with a man who wrote 260 texts means that I have had to severely limit where I took my samples. I have, for instance, not discussed the immense *Book of Contemplation*, partly because it precedes the Art, and partly because its relation to the Art of the first quaternary phase has been so admirably discussed by Josep Rubio. I have said little about the earlier major works of the ternary Art such as the *Ars inventiva veritatis* or the *Art amativa*. I have instead chosen to concentrate on the works of the Art which seemed to best sum up its two main stages: the *Ars demonstrativa* for the first phase, and the *Ars generalis ultima* with its small satellite, the *Ars brevis*, for the second. Such choices were made on the basis of what seemed objectively most central to Llull's different formulations of his system, and what texts might serve best to exemplify them.

My other important omission has been to include almost nothing on the question of influences, both of predecessors and contemporaries on Llull, and of Llull on succeeding generations of thinkers. This is an area made delicate by two factors. On the one hand Llull practically never cites his sources. In a world so essentially intertextual as that of medieval thought, Llull's writings are unusual in lacking almost any references to other texts or to any outside justification. This originated as a tactical device, so as to remove interreligious dialogue from the no-win area of the interpretation of authorities, but soon it took on a life of its own. Even when he does write things that look like contemporary textbooks, on logic, theology, preaching, etc., Llull's purpose, as I said before, is not speculative but practical, which vitiates the apparent similarity, and complicates enormously the task of comparison.

On the other hand, Llull is indeed interested in contemporary methods and doctrines, not as justifications, but as springboards to give his reader or listener the impression that he is starting on familiar ground, from which Llull begins a recycling process to lead them down his particular path. This involved not only the cosmology and elemental

theory which all medievals inherited from Greek science, but also many of the techniques, vocabulary, and conceptual framework of Aristotle. Following recent innovative studies, we will show, on the one hand, how he tried to remodel the dialectical methods of the *Topics* so that they could produce scientific demonstrations of a sort comparable to the *Posterior Analytics*, ending up openly presenting an alternative to the latter, and, on the other hand, how his presentation of the Divine Dignities could be used to argue with Jews and Muslims, because their similarity to their *sefirot* and *ḥadrās* would allow him to start off on something that looked familiar. What is interesting about the differences in Lull's reworking of all this material, is—I would say—the conscious use he makes of these very differences, and the resulting structures he moulds from them, as well as the resonances he hoped to establish. Lull is thus original, not only because of what he creates *ab ovo*—because I don't think it can be denied that many aspects of his system have little in the way of predecessors—, but also because of the adaptive use he makes of contemporary material, with the resulting interplay between his original creation and this adaptation. Nor should one forget the purpose to which he subjects it: to persuade rather than to discuss, to convince rather than to speculate. He is not a schoolman, but a missionary, polemicist, and preacher, yet one who uses (and remodels) many of the weapons of the schoolmen.

I have not gone into the influence of Lull's Art on posterity, in spite of the enormous importance of this subject. This is partly because we know so little about the followers who in the later Middle Ages copied (or had copied) the thousand or so manuscripts in which the Art was faithfully handed down, and partly because the Lullists who made the tradition so well-known in the Renaissance did so by adapting it to contemporary tastes, which meant watering it down to compete with the encyclopedic-rhetorical systems of Ramus and others. One need only look at the two pseudo-Lullian works, the *Logica brevis* and *De audito cabbalistico*, along with Lavinheta's *Explanatio compendiosaque applicatio Artis Raymundi Lulli* and Agrippa von Nettesheim's commentary on the *Ars brevis*, all works of enormous influence throughout the sixteenth and seventeenth centuries, to see how many of the fundamental notions of Lull's Art, such as the dynamic ontology—reflected in his definitions and in the correlatives—, his intensional logic which dispenses with the classical square of opposition, his addition (to the traditional five) of a sixth sense—that of communication—which he called *affatus*, have for the most part quite simply disappeared, in an attempt to 'normalize'

Llull by eliminating all the more ‘idiosyncratic’ elements. In the seventeenth century Llull was chiefly read in the famous Zetzner anthology published in Strasbourg (1598, and reedited 1609, 1617, and 1651), in which genuine works such as the *Ars generalis ultima* and the *Ars brevis* were accompanied not only by three of the works just mentioned—the spurious *Logica brevis* and *De audito*, together with Agrippa’s commentary—, but also by the *In rhetoricam isagogem*, which for a time also passed as an authentic work, and in which an extreme of absurdity is reached. It is as if a stewpot full of bits and pieces of Llull’s works along with contemporary rhetorical formulations had exploded in the study of an author who then tried to catalogue the splatterings stuck to the walls, ceiling, and floor of his study. Since there is little doubt that it is this anthology in which Descartes read Llull, one can sympathize with his characterization of the Art as something that would allow one to speak on many subjects without knowing any of them.

These remarks are not intended to deny the importance of the Lullist Renaissance tradition, on which I myself have spent a certain amount of scholarly energy. It was a phenomenon of considerable cultural and historical importance, and served to generate a counter-proposition to the system of Petrus Ramus. It was, of course, based on Llull’s formulations, but what it omitted, added, or changed, as a result of differences in intent or goal, make it a separate field of study from that of the Art itself.

In modern times a different set of misapprehensions about the Art has been in circulation: that it functions essentially (the adverb is important) by a system of revolving combinatorial disks, that it presents the basic building blocks of human thought (like Leibniz’s *Mathesis universalis*), that it is a method for discovering all possible predicates to all possible subjects (equally like Leibniz’s *Mathesis universalis*), or that it is only capable of dealing with topics included in its own premises. Llull has also sometimes been seen as a simple soul quite out of his depth in philosophical or scientific discourse, and whose logic could not really be valid. The present book, in fact, started as an attempt to correct the many misapprehensions about Llull that have been in circulation for centuries now, an attempt which was finally abandoned for two reasons. The first was that I found that some of the misapprehensions which needed correction were my own. If the reader thinks this is a piece of false modesty as a kind of *captatio benevolentiae*, he has only to compare the presentation of the beginning of the *Ars brevis* in Chapter 4 here with that in my two anthologies. The second was that, after the

first three chapters, which more or less summarize material already proposed by others and by myself, I found myself venturing more and more into unexplored territory. Following the threads on which Llull actually articulated his arguments, coordinating this articulation with his own methodological presentations, and tracing the leads of his use of certain concepts (from the *Ars demonstrativa* on, his vocabulary becomes more and more consciously precise and consistent), has taken me down paths unforeseen when I began this book, to the point where I often had the feeling that the book was writing itself—or rather that Llull was writing it—and I was more in the position of a listener at a medieval lecture trying to give a reasoned *reportatio*. This process took me so far from my original notions about Llull's Art, that when one of the people who read an earlier version of this book exclaimed, "This isn't the Llull I know", I could only answer that it wasn't the Llull I knew either.

My aim, moreover, has not been to defend Llull, but rather to remain neutral in the matter of possible judgments upon his Art and logic. In the past people have been too quick to criticize (or praise) Llull, without fully understanding why he was doing things the way he did. Here I have tried to supply that why. My one conviction is that he built a structure—or rather several structures—of extraordinary consistency and interest, and that it would be wrong to judge them until we fully understand how their parts fit together and how his systems were meant to function. Until then, we will be like a person complaining that a piece of machinery or a computer program doesn't work, when he hasn't bothered to read the instruction manual. Instruction manuals do not necessarily make for easy, rapid reading, and especially if the machine or a computer program has many and important differences from any we know. In Llull's case, the Art itself was a kind of user's manual, telling us how to produce what he called 'necessary reasons'. It is this manual and its resulting arguments we will try to explain in this book.

ACKNOWLEDGMENTS

The first person to read and comment on this book, while it was still in a very primitive and as yet disorganized form, was Charles Lohr, who, among other things, saved me from some important errors in my understanding of contemporary scholastic discourse. In an already much more finished form, both Lola Badia and Robert Hughes read it, making many important observations and suggestions, in addition to long lists of corrections of detail. The most extensive list of suggestions and corrections came from Josep Maria Ruiz Simon, who saved me not only from some serious errors, but also from many imprecisions of language, as well as from possible confusions as a result of inadequate explanations on my part. My wife, Eve, read the book in a first draft, when she helped get it into a more finished, coherent form, and then a second time for the final draft, to clear up errors of content, meaning, and presentation, as well as many errors of detail. To all my deepest thanks; this book might still have flaws, but without them it would have had many more.

As for people or institutions that let me use material, Yanis Damberg kindly let me cite the translations from his website <http://lullianarts.net/>; if I have in some cases altered them (as I have occasionally altered my own previous translations), this in no way implies a criticism, but only a difference of setting and purpose in which they appear. The Badische Landesbibliothek in Karlsruhe kindly allowed me to reproduce the eleventh miniature from the well-known gem of their collection, the *Breviculum* of Thomas Le Myésier (St. Peter perg. 92).

Finally there is my debt to the two people to whose memory this book is dedicated. Robert Pring-Mill, as one of the first to write on the Art and to do so in a most articulate way, was chief among those whose writings started my interest in the subject. David Rosenblatt, as a mathematician convinced that Lull had not yet found his proper place in the history of logic, for many years acted as my guide on that side of the question. I am very sorry that, whatever their opinion might have been, neither of them lived to see this book in print.

ABBREVIATIONS

Works

<i>AA</i>	=	<i>Art amativa (Ars amativa boni)</i>
<i>AB</i>	=	<i>Ars brevis</i>
<i>ACIV</i>	=	<i>Ars compendiosa inveniendi veritatem</i>
<i>AD</i>	=	<i>Art demonstrativa (Ars demonstrativa)</i>
<i>AGU</i>	=	<i>Ars generalis ultima</i>
<i>AIPU</i>	=	<i>Ars inveniendi particularia in universalibus</i>
<i>AIV</i>	=	<i>Ars inventiva veritatis</i>
<i>LFAD</i>	=	<i>Lectura super figuras Artis demonstrativae</i>
<i>LN</i>	=	<i>Logica nova</i>
<i>PropAD</i>	=	<i>Liber propositionum secundum Artem demonstrativam compilatus</i>
<i>TG</i>	=	<i>Taula general (Tabula generalis)</i>

Manuscripts

Breviculum = Karlsruhe, Badische Landesbibliothek, St. Peter perg. 92. Reproduced in its entirety (not only the miniatures) with a reconstruction of its missing parts in Le Myésier 1990, which on p. xxv has a bibliography of previous editions. For a reproduction of the miniatures along with translations of the speeches and explanations in several languages (including English), see <http://lulle.free.fr/>.

Electorium = Paris, Bibliothèque Nationale, lat. 15450. For a detailed description see Hillgarth 1971, 348ff., reproduced in Catalan translation at <http://orbita.bib.ub.es/llull/docs/hillgarth.doc>.

These and almost all the other Lullian Latin manuscripts are consultable at <http://freimore.uni-freiburg.de/llullus/index.html> directly, or through the links on the Lull DB listed below.

Publications

- ATCA* = *Arxiu de Textos Catalans Antics* (Barcelona, 1982–).
- DI* = *Doctor Illuminatus. A Ramon Llull Reader*, ed. Anthony Bonner and Eve Bonner (Princeton, 1993).
- EL* = *Estudios Lulianos* (Palma, 1957–1990. See *SL*)
- Llull DB = The bibliographical database online at <http://orbita.bib.ub.es/llull/>.
- MOG* = *Beati Raymundi Lulli Opera*, ed. Ivo Salzinger, 8 vols. (Mainz: Häffner, 1721–1742; reprint ed. F. Stegmüller, Frankfurt, 1965).¹
- NEORL* = *Nova Edició de les Obres de Ramon Llull* (Palma: Patronat Ramon Llull, 1990–).
- OED* = *Oxford English Dictionary*
- ORL* = *Obres de Ramon Llull*, ed. Salvador Galmés et al., 21 vols. (Palma, 1906–1950).
- ROL* = *Raimundi Lulli Opera Latina*, I–V (Palma, 1959–1967) and VI– (Turnholt, Belgium: Brepols, 1975–).
- SL* = *Studia Lulliana* (Palma, 1991–. Continuation of *EL*)
- SW* = *Selected Works of Ramon Llull (1232–1316)*, ed. Anthony Bonner, 2 vols. (Princeton, 1985).

¹ Citations will be given in the form “*MOG* I, vii, 44: 476”, with volume number, number of internal division, and page of this last, followed (after the colon) by the continuous pagination of the 1965 reprint.

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CHAPTER ONE

INTRODUCTION

A Brief Life

Ramon Llull was born in 1232, or possibly 1233, only two or three years after the Catalans under the Aragonese crown had conquered the island of Majorca from its Muslim rulers. This meant that he grew up in a place where Muslims (enslaved by the conquerors) occupied perhaps a third of the population, and where Jews, although numerically far inferior, were economically, politically, and even culturally important.¹

At the age of thirty, after a dissolute, typically aristocratic upbringing, during which he married and had two children, Ramon Llull had visions of the crucified Christ which he took as a sign that he should dedicate his life to his service. This he decided to do in three ways: to accept the possibility of martyrdom in an attempt to missionize; to try “to write a book, the best in the world, against the errors of the unbelievers”; and “to go to the pope, to kings, and to Christian princes to incite them” to establish language monasteries for missionaries.² All three were aims clearly directed towards the conversion of the Muslims and Jews he knew so well from his own multicultural society. He therefore abandoned his former way of life, left provisions for his family, and, after a pilgrimage to Rocamadour in Southern France and Santiago de Compostela, settled down to nine years of study, during which he

¹ Near the beginning of his career, at the beginning of the *Book of the Gentile and the Three Wise Men*, Llull says “Since for a long time we have had dealings with unbelievers...” (*SW* I, 110; *DI*, 85). For the extraordinary cultural importance of Jews, see Hillgarth 1991, 45, 50, and Tables I and II on pp. 63–64.

² The biographical passages in quotes in this Introduction, if not otherwise attributed, come from the *Vita coetania* Llull dictated when he was almost eighty to some monks of the Chartreuse of Vauvert in Paris. The standard edition is that of *ROL* VIII, 259–309. See *SW* I, 13–48 (or *DI*, 11–40) for the English version used here, and for an explanation of the relation between the Latin and Catalan originals. The two passages quoted here are from *SW* I 15–16 or *DI* 13.

bought himself an Arab slave so as to learn his language and culture in preparation for the task ahead.

This information is gleaned from a sort of autobiography, as told to the monks of the Chartreuse of Vauvert in Paris when he was almost eighty, who wrote down what he told them. This has become known as the *Vita coetanea*.³ It is an extraordinary document, probably unmatched in the annals of medieval philosophy since Abelard's *History of my Calamities*, but has frustrating gaps, as for instance what he actually studied beside the Arabic language during his nine years of formation. All we can do is glean bits of information from some of his early works.

We can see, for instance, from the *Book of the Gentile and the Three Wise Men*, that in addition to learning the language, he also learnt something of Islam—as well as Judaism. In the *Doctrina pueril* he cites ten works of Aristotle, showing that he must have become conversant with the basic philosophical texts of his time. The first work he wrote, as a kind of “school exercise”—in the words of one scholar—is a compendium of al-Ghazali's logic, with large sections taken from the *Summulae logicales* of Peter of Spain.⁴ In his first medical work he cites Avicenna, Platearius and Constantine the African. Some of the material of the *Book of the Beasts* comes from the Arabic *Kalila wa Dimna*, and the *Book of the Lover and the Beloved* he says is based on a Sufi model. But that is about all we can infer about Llull's preparation during these years.

Towards the end of his nine years of study, aside from the already mentioned adaptation of al-Ghazali's logic, he wrote the vast *Book of Contemplation*, which contains the seeds of much of his later thought. The only major work he wrote in the first person, it is a hymn of thanks to God for guiding his understanding in its ascent to the divine. At the same time, one can feel him searching, especially in the last section of the work, for some method to embody all his discoveries, to explain them, and to be able to forge them into a missionary tool.

At the end of these years of study, as the culmination of this search, on Mount Randa, a solitary hillock rising out of the plain some 15

³ It first appeared in a huge compilation of Llull's writings known as the *Electorium* put together by his Parisian disciple, Thomas Le Myésier. A shortened version, called the *Breviculum* was prepared for the queen of France, with twelve magnificent miniatures illustrating Llull's life.

⁴ The phrase is from Cruz Hernández 1977, 67. See Lohr 1967 for the sources of the *Compendium logicae Algazelis*.

miles east of Palma, he experienced another divine intervention, but this time it was not a vision, but rather what we might now call a methodological illumination, or as he puts it, he was given the “form and method [*forma et modus*]” for writing the book (or books) he felt he had to write against the errors of the unbelievers. If he had already unfolded the foundations of his philosophy and theology in the *Book of Contemplation*, what was new in this illumination was its structuralization—the bringing together of all the various bits found throughout that work, and especially its latter portion, into a single organic whole which he called his ‘Art’.⁵ *Ars* was the standard Scholastic translation of the Greek *technē*. Lull’s Art was therefore a technique; it was not a body of doctrine, but, like medieval medicine, it was a practical art.⁶ Lull’s innovation was how he systematized his own Art. Moreover, it was not embodied in any single work, but presented in many different works, sometimes, as we shall see, with notable changes.⁷

After this illumination, he abandoned his mountain hermitage for a Cistercian monastery just outside Palma where he wrote his first work embodying this “form and method”, and hence the first work of the Art, the *Ars compendiosa inveniendi veritatem*. It has been calculated that this took place in about 1274, and with that began what has since been termed the quaternary phase of the Art.⁸ To this work he wrote four satellite works showing how to apply it to theology, philosophy, jurisprudence and medicine.⁹ It was also during this first period or cycle of the quaternary phase that Lull wrote his most important apologetic work, the *Book of the Gentile and the Three Wise Men*, as well as the first of his two remarkable novels, *Blaquerna*, one of the last portions of which is occupied by his great mystical work, the *Book of the Lover and*

⁵ For a detailed study of the transition from the *Book of Contemplation* to the first version of the Art, explaining what elements of the former appeared in the latter, and hence giving an important exposition of the beginnings of the Art, see Rubio 1997.

⁶ See p. 293 below for more information on the term *ars*.

⁷ As a result, when Lull refers to his *Ars generalis*, normally—and especially after the *Tabula generalis*—he is not referring to any specific work, but rather to his system as such.

⁸ Because many of the principal components of his system appeared in multiples of four.

⁹ The *Ars compendiosa inveniendi veritatem* with its four satellite works were edited in *MOG I*, and the latter were reedited with an introduction by Pring-Mill in Lullus 1969. As this book went to press, the *Quattuor libri principiorum*, as the four satellite works have become known, have received a critical edition in a new *ROL* volume. The only one of these five works extant in Catalan is the *Principles of Medicine*, with two editions (Lull 1989 II and *NEORL V*), and translated into English in *SW II*.

the *Beloved*. During this period, moreover, he wrote a pedagogical work, the *Doctrina pueril*, as well as a tract on knighthood, the *Book of the Order of Chivalry*, which was widely read in northern Europe in the fifteenth century.¹⁰ It was also during this period that he saw fulfilled the third of his three wishes upon his conversion, the founding in 1276 of a language monastery for missionaries on the north coast of Majorca.¹¹

In about 1283 Lull remodeled his system with the *Ars demonstrativa*, using much of the same material as the *Ars compendiosa inveniendi veritatem*, but now placed within a much clearer and better organized format. Within the quaternary Art, this started off a new cycle, with its own set of satellite works and many important commentaries. It was also now that he began his travels, but to explain that we must give another bit of background.

When James the Conqueror died in 1276, his realm was divided between his two sons: the elder, Peter III (II of Catalonia), received Aragon, Catalonia, and Valencia, while the younger, James II, received three scattered bits of real estate: the Balearic Islands, the Roussillon with its capital of Perpignan, and the city of Montpellier. This last had a university that included the leading medical school in Europe and an important law faculty. Probably for that reason, in addition to the fact that it was more centrally located for his eventual travels further north into France and across into Italy, Lull used it more and more as his home base. Moreover, it was probably there that he learned what he knew about medicine and jurisprudence.¹²

The *Vita coetanea*, after another frustrating gap of eleven years (1276–1287), now recounts that in that last year Lull, after going to Rome to have an audience with the pope only to find he had recently died, went on to Paris where he tried to teach his Art. It was there that he wrote his second great novel, *Felix or the Book of Wonders*, which contains the remarkable political parable, the *Book of the Beasts*. But the trip was not a success. As he says in the *Vita coetanea*:

¹⁰ It was translated then into English (by Caxton), and even into Scottish (see Lull 1988, 152, and the versions listed under that work in the Lull DB).

¹¹ See Garcías Palou, 1977, and Hillgarth 2001, 39–40.

¹² For medicine and law at Montpellier in Lull's time see Gayà 1977, as well as the same author's introduction to *ROL XX*. Some of the leading counsellors of Philip the Fair had studied law in Montpellier, and it is not impossible that Lull might have known some of them who could have helped him get a hearing with the king.

Having lectured on this *Commentary* [on the *Ars demonstrativa*] in Paris, and having observed the attitude of the students there, he returned to Montpellier, where he once again wrote and lectured on a book, this one entitled the *Ars inventiva veritatis*. In this book, as well as in all others he wrote from then on, he used only four figures, eliminating—or rather disguising, because of the weakness of human intellect which he had witnessed in Paris—twelve of the sixteen figures that had formerly appeared in his Art. (*SW* I, 29; *DI* 23–24)

I have quoted this passage in full because it presents the justification for a major reorientation of the Art. The negative reception in Paris was undoubtedly brought on by the radicalism of Lull's system, seen as too alien to the accepted ways of discussing theology and philosophy, which caused a head-on collision with masters and students of the University. Since Paris was important for him—if he was going to persuade the Christian establishment of the benefits of his methods, he couldn't do without the backing of the University there—he saw that he had to adapt his system and make it more acceptable. This change from sixteen to four figures signals the change from the quaternary to the ternary phase of the Art,¹³ which begins with the *Ars inventiva veritatis* written upon his return to Montpellier in 1290.

But notice how he says that any simplification of the Art was more apparent (“disguised”) than real. Indeed, as we will see, the ternary phase by no means represents a reduction of his system. The above biographical excerpt also glosses over the fact that the change—as we will see in Chapter 3—had in fact been brewing for some time, and therefore how much of it was really due to intellectual pressure from Paris is difficult to determine. But he did find there two disciples who were to be of great significance for his career, and who perhaps advised him on tactics and acceptable ways of going about his mission. The first was Pierre de Limoges, doctor and astronomer, who left his library, containing some of the earliest Lull manuscripts, to the Sorbonne. The second was Thomas Le Myésier, doctor to members of the royal family and also connected with the Sorbonne, who might have had a leading role in redirecting Lull's efforts to persuade the Parisian intellectual community, who was to draw up important anthologies of Lull's works after his master's death, and who thus was to exert a strong influence on

¹³ So named because the components are now in multiples of three.

subsequent French Lullism.¹⁴ The first may have been more important on this visit, whereas on subsequent visits to Paris the second seems to have become the center of Llull's attentions.¹⁵

Another problem with this visit has recently come to light in a note in a manuscript Llull himself sent to the doge of Venice in that same year of 1290. There he complains rather bitterly about the scandal caused by the title of *Ars demonstrativa*. At the end of the chapter on the ternary Art we will see what this meant, and how it may have affected the restructuring of the Art.

In any case, soon after writing the *Ars inventiva veritatis*, he wrote the *Art amativa* (*Ars amativa boni*), in which he says that just as the former work deals with *scientia*, and is thus directed towards the intellect, the second deals with *amantia* and is directed towards the will.¹⁶ After another unsuccessful attempt to have an audience with the pope, he went to Genoa, where he experienced a spiritual and psychological crisis. When he had recovered in 1293, he set out on his first mission to Africa, this time to Tunis. Just when it seemed as if some of the Muslim doctors of the law were becoming convinced by some his propositions, he was jailed as a threat to Islam and expelled.¹⁷

In Naples, where he arrived in 1294, he finished the *Tabula generalis* which he had begun in Tunis,¹⁸ and with it he started on a major reorientation of his strategies. In the first place, this is the first work in which the Art is described as "general". When we come to study the ternary Art, we will see what this implies. It is now too that he begins to sign his works (as opposed to earlier works where he said he was unworthy of having his name affixed to anything he had written), in addition to

¹⁴ For Pierre de Limoges see Soler 1993 and Soler 1992–3. The classic work on Le Myésier and Lullism in France is Hillgarth 1971.

¹⁵ It was, for instance, during his second visit to Paris when he answered the *Quaestiones* mentioned in n. 21 below. As for Llull's third visit, Pierre de Limoges had already died by then; see Soler 1993, 99.

¹⁶ *ORL* XVII, 4; *ROL* XXIX, 120. See Ruiz Simon 1986, 88–90, for an important passage from the *Art amativa* indicative of Llull's changed epistemological and cosmological orientation.

¹⁷ The crisis in Genoa and the subsequent trip to Tunis are recounted in dramatic detail in the *Vita coetanea*—see *SW* I, 30–37; *DI* 24–31.

¹⁸ Shortly afterwards (1294–5) he wrote an immense work applying the Art (principally to theology), the *Art of Proposing and Solving Questions* (traditionally called *Lectura super Artem inventivam et Tabulam generalem*), as well as a short commentary on this new version of the Art, the *Lectura compendiosa Tabulae generalis*.

usually giving the place and date of composition.¹⁹ There can be little doubt that this was part of a need to project himself into the public arena, or rather to project an image of himself which could explain why people should take his works seriously, and treat them, since they were not based on any previous authorities, as authoritative in their own right. In a poem called *Desconhort*, written a year later, structured as a dialogue with a hermit, he gives a dramatized explanation for the need for this reorientation, and at the same time—as if to reinforce the presentation of himself—gives some of the details of his life and conversion, things about which until then he had hardly said a word.

Right after this poem, in 1295–6, he wrote the immense *Tree of Science*, and followed it with the *Book of the Articles of Faith*, or *Apostrophe*, from the title of an introductory poem addressed to the pope. Whereas the first of these three works talks about the need of achieving credibility with the papacy and the second hopes that the pope will find it pleasing and help diffuse it because of the benefits it could bring, the third is openly dedicated to the pope, to show how Lull's system can effectively but unobjectionably prove the Articles of Faith.²⁰ Lull's new public image would thus seem to be part of his most concerted campaign to date to get a hearing with and ultimately backing from the papacy.

In 1297–9 we find Lull once again in Paris, but this time in a much more confident frame of mind, ready to show how his newly “general” Art can answer the principal questions occupying the theologians and philosophers of Paris. In one work he answers a series of questions Le Myésier had put to him, obviously to test how his Art can handle them.²¹ In another work he takes on no less than the *Sentences* of Peter Lombard, a text upon which every theology student had to comment in order to complete his university course.²² And finally, he writes a work to show how the 219 articles condemned twenty years before by the bishop of Paris, Étienne Tempier, were indeed wrong.²³ Furthermore,

¹⁹ The situation is actually more complicated; see Bonner 1998, 42–43 for details. See that article, as well as Badia 1995, on which it was largely based, for the following matter of the projection of a public image.

²⁰ See Bonner 2002a for the details of this Roman campaign. See also *Vita coetanea*, par. 31 (*SW* I, 37–38; *DI* 31–32), for Lull's insistence on his efforts to get a hearing from the pope.

²¹ *Quaestiones Attrebatenses*.

²² *Disputatio eremitaie et Raimundi super aliquibus dubiis quaestionibus Sententiarum Magistri Lombardi*.

²³ *Declaratio Raimundi per modum dialogi edita contra aliquorum philosophorum et eorum sequacium opiniones*. The condemnation of 1277 was of a list of doctrines attributable

he produces a shortened version of the Art, the *Ars compendiosa*, a kind of precursor of the *Ars brevis*, and which might also have been written as a text on which to lecture.

After an absence of many years, he returned to his native Majorca, where the news that Ghazan, the Mongol il-khan of Persia, was conquering Syria made him set sail for Cyprus, only to find out that, after a brilliant campaign, Ghazan had been forced to withdraw.²⁴ This was in 1301, a year in which before leaving Majorca he wrote the *Aplicació de l'Art general*, and in Cyprus, the *Rhetorica nova*, works in which he made his first hesitant moves into two new fields, logic and preaching.²⁵

His return to Europe was followed by a feverish two years (1303–5) of literary activity carried out back and forth between Genoa and Montpellier. Among the many works he wrote, there are two on the Art: an important commentary, the *Lectura Artis quae intitulat Brevis practica Tabulae generalis*, and in 1305 the first part of the *Ars generalis ultima*.²⁶ His ventures into new fields are now consolidated with the *Logica nova* of 1303 and the *Liber de praedicatione* of 1304, works in which he shows how his Art can be applied both to logic and to preaching respectively.

It was probably in 1307 that he set out on his second mission to North Africa, this one to Bejaya (Bougie) in present-day Algeria. It was the first and only time he deliberately tried to provoke the authorities by proclaiming that the Christian religion was true and Islam false. The crowds tried to lynch him, and he was only saved by the local authorities, who, after deciding against a death sentence, put him in jail, where he spent six months and held a long disputation with an emissary of the *cadi*, an account of which he then drew up.²⁷

to the more extreme Aristotelians of the Arts Faculty of the University of Paris (the so-called Parisian 'Averroists', but some of Aquinas' doctrines also turn up in the list). See Ch. 4, n. 133 below for bibliography.

²⁴ The Mongols were of interest not only to Llull for questions of conversion, but to the Christian west in general as possible allies in driving the Egyptian Mamelukes out of the Holy Land.

²⁵ We don't count the earlier *Compendium logicae Algazelis*, because of its nature as a largely derivative school exercise (see n. 4 above).

²⁶ The colophon of the work says it was begun in Lyon in November of 1305 and finished in Pisa in March of 1308. At the first date, Llull seems to have had an interview with the newly elected pope, Clement V, who was then in Lyon, and in which he tried—unsuccessfully—to interest him in the founding of language schools for missionaries.

²⁷ This is the *Disputatio Raimundi christiani et Homeri saraceni*.

Finally he was expelled, but on the return voyage to Genoa his ship sank in a storm. Among those who managed to make it ashore to the nearby Port of Pisa, were Llull and a companion. He had lost all his books, but in Pisa (we are now in 1308) he managed to rewrite his account of the disputation in Bejaya, as well finishing the *Ars generalis ultima* and writing its shorter companion, the *Ars brevis*, both of which he finished in 1308. The longer work he called *ultima* because, as he said, he intended to write no more works of the Art.²⁸ In that same year he produced one of the most important applications of the Art to theology, the *Ars compendiosa Dei*.

Parallel to these last presentations of the Art, thus producing an overlap similar to that at the end of the quaternary phase, Llull was simultaneously developing and perfecting his use of syllogistic logic, which would become preponderant during the last period of his production, the post-Art phase, (1309–1315), so-called because as he had promised, he produced no reformulations of his system. This does not mean that he abandoned the Art, but simply that it played a diminishing role in the works of this period, replaced as it was by an increasingly all-pervading logic and syllogistics. It was also a period of remarkable activity as an author: in these seven years he wrote 115 works, nearly half of his entire production. To be sure, many of them were short, but all the same it is astounding in a man who was then aged between 77–83 at a time when life expectancy was perhaps half that.

Near the beginning of this stage (1309–1311) took place Llull's third and last stay in Paris,²⁹ one which he could finally claim a success, with many students attending his lectures, and with his receiving letters of approval from the king, the chancellor of the university, and from forty masters and bachelors in Arts and Medicine. Llull—undoubtedly, as we have said, with the help of friends like Thomas Le Myésier—was someone who had clearly been able to learn from his intellectual surroundings, and thus to develop from his fascinating but rather home-grown beginnings into someone able to confront scholastic methods. It was also during this last stay in Paris that he conducted his so-called

²⁸ He in fact did write other works using the Art, but no new formulations thereof.

²⁹ The *Vita coetania* adds a mysterious—because otherwise undocumented and hard to fit in with his itinerary of those years—fourth trip some four or five years earlier. See *SW I*, 40 n. 154.

anti-Averroist campaign, against the more extreme Aristotelians who tried to separate faith from knowledge, theology from philosophy.

After dictating the *Vita coetanea* at the end of his stay in Paris, he left for the Council of Vienne, south of Lyon,³⁰ where he had his second success, with the Church agreeing to found language monasteries for missionaries. From there he went once again to Majorca, where he spent a year (1312–1313), and where he wrote a large collection of model sermons.³¹ As tireless as ever, even though he was now over eighty, he set out for Sicily, where he stayed in Messina for a year, until he set out for his third and last voyage to North Africa, to Tunis again. His last works are dated there in December, 1315, and we can only gather that he must have died sometime between then and March 1316, in Tunis, on the boat back, or upon arriving in Majorca.³²

The Centrality of the Art

Throughout Llull's enormous production of some 260 works on every subject imaginable, he never tires of reminding the reader that the work they are reading is based on the Art, or recommending it for a deeper comprehension of the subject under discussion, and this even in works that might strike one as purely literary. For example in his first novel, *Blaquerna*, he cites the *Ars compendiosa inveniendi veritatem* four times, and in his other novel, *Felix*, he similarly cites the *Ars demonstrativa* four times. Even in a supposedly purely mystical work such as the *Book of the Lover and Beloved*, although not cited explicitly, the Art is present more and more as the work progresses, and it is followed by an *Art of contemplation*, as a kind of how-to-do-it manual.³³ In short, it forms the backbone of all his other works.

³⁰ He wrote a delightful work, the *Disputatio Petri clerici et Raimundi phantastici*, recounting a dialogue with a worldly priest he supposedly met on the way to Vienne, who laughs at Llull, saying he's already heard of him, and has always held him to be a *phantasticus*, someone hopelessly impractical and idealistic, as opposed to himself, who has been most successful in feathering his own and his family's nest. It shows Llull's remarkable ability to see himself as others must have seen him, while at the same time staunchly defending his position.

³¹ Numbers IV.60–67 in the catalog of *SW* II or on the Llull DB. They have been edited in *ROL* XV and XVIII.

³² For what we know about Llull's death, as well the origin of the legend of his martyrdom, see the last three notes on *SW* I, 51–52.

³³ See Pring-Mill 1962–1967, reprinted in Pring-Mill 1991, 279–306.

After his death, his Parisian disciple, Thomas le Myésier, put together a vast anthology based on the Art, and a shorter one prepared for the queen of France based mainly on the *Ars brevis* (this is the anthology with the justly celebrated miniatures illustrating the *Vita coetanea*). Neither the problems of understanding, nor the difficulties and cost of copying the complicated manuscripts of the Art with their multiple figures, seemed to dissuade his many followers from tackling these works. Of the thousand or so manuscripts preserved of Lull's works, the great majority are dedicated to the Art.³⁴ This means that the Art was widely circulated in the fourteenth and fifteenth centuries, in far greater quantity than his works of a more literary nature, so much better known in our days.

With the introduction of printing and with the sixteenth-century search for a general method of science and learning, interest in Lull's Art increased even more. Agrippa von Nettesheim's commentary on the *Ars brevis* was printed fifteen or sixteen times between 1531 and 1651. Towards the end of the century, Giordano Bruno wrote five commentaries on Lull, four of them on the Art—more attention than he gave to any other thinker. Then in 1598 the Strasbourg publisher, Lazarus Zetzner, put out the equivalent of a modern paperback anthology of some of Lull's works—mainly of the Art—together with the commentaries of Agrippa and Bruno. This anthology was to become a minor best-seller, going through three reprintings in the seventeenth century, and with copies still extant in many European libraries.³⁵ Finally it was in this edition that Leibniz read the Art,³⁶ which inspired his first published work, *De arte combinatoria*, and which started him off on a project which would develop into his *mathesis universalis*.³⁷

The Art, therefore, while never underlying any main current of Western thought, provided a beacon for important secondary currents and has attracted the attention of major thinkers. In spite of this, however, as Frances Yates said almost half a century ago, “the Lullian

³⁴ See Bonner 2003b.

³⁵ See the introduction to Lullus 1996 for more information on this edition.

³⁶ There was also a copy in Newton's library, and two centuries later, in that of Charles Peirce.

³⁷ Still the best and most complete history of Lullism is that found in Vol. II of Carreras y Artau 1939–43. Nowadays this must be supplemented by Hillgarth 1971, Rossi 1960 and 2000, and Schmidt-Biggemann 1983.

Art still looms in mystery like some huge unclimbed mountain”.³⁸ With certain exceptions,³⁹ most modern studies offer descriptions of the basic components of the Art (its fundamental concepts along with the figures presenting them), and of its purpose (among other things, offering proofs or answering questions), without saying almost anything about how Llull went about achieving this purpose, which has always reminded me of those exploded diagrams of automobiles, showing all the parts in precise detail, without giving any clue as to how they actually function, much less how one could actually drive the car down a road.

Before beginning to study the Art, the reader should try to put aside preconceived notions of what to expect from a medieval text of theology, philosophy, or logic. If not, he will almost immediately become bogged down with the idea—an idea that could probably be traced back to the fifteenth century—that Llull’s lack of resemblance to standard authors such as Albert the Great, Thomas Aquinas, or Peter of Spain is due to his innocence and ignorance of contemporary intellectual styles. Llull, especially after his first visit to Paris, was acutely aware of the problems caused by the unusual nature, or one could even say oddity, of his presentation and language, but he would always insist that it was essential for a proper explanation and understanding of his message, one in which form and goal were inseparable.

It is this unusual nature which makes studying the Art like exploring uncharted territory, filled with unfamiliar signposts. But if we want to get past vague generalities which will never give us even a minimal feeling for what Llull is doing, then we must be prepared to take the plunge of assimilating many of the details of this new landscape, of studying the manual.

The Nature of the Art

We have seen that it was conceived as a missionary tool. In fact, in 1263, very likely the same year as his conversion, there took place in

³⁸ Yates 1954, 166, and Yates 1982, 66. In the following paragraph she says that “Lullism is no unimportant side-issue in the history of Western civilization. Its influence over five centuries was incalculably great.”

³⁹ The exceptions are Vol. I of Carreras y Artau 1939–43, the first work to explain that the Art was at the center of Llull’s production and thought, and the first to try to tackle its complexities. Recently we have had two important studies whose principal focus has been the quaternary Art: Ruiz Simon 1999 and Rubio 1997.

Barcelona a celebrated disputation in the presence of the king, between the leading rabbi of the Jewish community, Moses ben Nahman, or Nahmanides as he is usually known, and a Jewish convert, the Dominican, Pau Cristià. The innovation in this disputation was the use of post-biblical sources in addition to the common ground of the Old Testament, but, as Harvey Hames has said, “The mendicants... rarely, if at all, took into account the practices and beliefs of their Jewish contemporaries. In a way, they were still arguing with books, rather than with real and living people... Because the [Barcelona] disputation was based on the authority of the sources, each side could retreat into its own hermeneutics and interpretation of the sources without really having to confront head-on the actual issue being discussed. Friar Paul and Nahmanides did not argue about whether the Messiah had already come, they argued—academically as far as Nahmanides was concerned—about how to interpret key passages in the Jewish sources. Paul did not attempt to undermine or understand Nahmanides’ personal beliefs and opinions, and hence, Nahmanides was never really in any danger. If really pushed, his last resort could have been to remark that Christian and Jew differ on the interpretation of the passage in question and that, therefore, nothing had been proven either way.”⁴⁰

Some five years later another Dominican, identified with Ramon Martí, failed in a mission to Tunis, where he was expelled and his life even threatened after, as Lull tells it—in an *exemplum* he uncharacteristically repeats seven times in his writings—Martí had successfully proven to the sultan the falsity of Islam, and then been unable in return to prove the truth of Christianity, lamely showing the monarch

⁴⁰ Hames 1997, 134. See that article for the principle bibliography on the subject. See Chazan 1992, 243–4, for a listing of the many editions of the Christian and Hebrew versions of the debate. A great deal of argument has gone into trying to assess the relative accuracy of the differences between these two accounts of the debate, arguments that are to a certain extent futile, in that each side could genuinely feel that it had won (since it had successfully defended its own hermeneutics), and that it was therefore free to reshuffle the evidence to show the truth of this feeling. In the *Book of the Gentile and the Three Wise Men* (*SW* I, 170; *DI*, 110–111) Lull presents the problem clearly. The Jew, after explaining how, in spite of a common origin in the Mosaic Law, the three religions have different beliefs, says: “Moreover, we and the Christians agree on the text of the Law, but we disagree in interpretation and commentaries, where we reach contrary conclusions. Therefore, we cannot reach agreement based on authorities and must seek necessary arguments by which we can agree.” For more Lullian statements on the subject, see ch. 6. n. 97 below.

the articles of faith, saying they were things that could not be proved, only believed.⁴¹

These two events seem to have profoundly marked Llull's course. The first showing that he would have to avoid texts and their attendant hermeneutics, and instead find some way to address people's beliefs; the second that he would have to devise a method for proving the Articles of Faith, one which would be able to side-step the Church's distaste for such an undertaking.

The first had several direct effects on his Art. For one thing, it led him to develop an abstract system without exterior references, and hence put him into the extraordinary situation of a Christian polemicist and missionary who, except for three or four stock phrases he repeats frequently, and except for some themata of sermons employed in a most unusual way, almost never mentions the Bible, the Church Fathers or contemporary theologians.⁴² Secondly, it led him to base his system on fundamental notions acceptable to all three faiths. All could agree, for instance, on the existence of one God, and on his having a series of attributes, such as goodness, greatness, eternity, etc.⁴³ They could broadly agree on the ethical values embodied in a series of virtues and vices. In addition they shared a common inheritance of Greek science, logic, and philosophy, on which Llull would draw for his cosmology and much of his conceptual framework and vocabulary. These fundamental notions without exterior references or appeal to authorities would form the basis for a self-sufficient system, one which was almost totally inward-looking for its justification, one which could be called endo-referential. Finally, if such a closed system was to investigate and demonstrate things outside its own restricted domain, it would have to be generative.

As for proving the Articles of Faith, and especially those of the Trinity and Incarnation, most central to Christianity and most controversial for Muslims and Jews, Llull knew that he would have to find some non-standard (that is, non-Aristotelian) method of proof, if the attempt was

⁴¹ For the seven places in which he tells this story, see *SWI*, 58 n. 21 and 96 n. 12. The original identification with Ramon Martí, made in Longpré 1933, has been challenged in Ruiz Simon 1999, 362 n. 501, who favors another Dominican, André de Longjumeau, who is known to have accompanied Saint Louis on his fatal final expedition to Tunis in 1270.

⁴² See Bonner 1993. As for the themata of sermons, see pp. 266 and 280–1 below.

⁴³ For the Christian Divine Names, the Muslim *ḥadrās*, and the Jewish *sefirot*, see ch. 6. n. 76 below.

not to be condemned by the Church. After all, Ramon Martí's position was the correct one: how can you prove something that is a matter of faith?⁴⁴ For this Llull developed the Art, based first on comparisons within a closed set of terms, and later on what one might call dynamic definitions of an even smaller set of (general) terms, which could offer non-causal, non-Aristotelian proofs.

His development of new techniques of proof, however, does not mean the rejection of traditional methods. As he explains, they are as valid as ever where applicable, that is to say in the created world. Since for Aristotle demonstrating meant finding a cause, his techniques could not be applied to God, who cannot be the object of any kind of cause. His Trinity, for instance, would therefore have to be proved by other means. If these new techniques, however, are valid in the highest realm, they must also be so in the lesser realms of the created world, hence the greater generality of the Art. It must be remembered too that Llull's chief aim was to try to convince others, hence, with some important exceptions, he avoided confrontational attitudes.⁴⁵ In many realms the relation between the Art and traditional methods is not one of simple opposition, but usually more one of trying to adapt them to the Art, or to make the Art a tool for their re-use, all of which I hope will become clearer in the course of this book.⁴⁶

The non-oppositional nature had both an important cause and an important result. People who had tried to carry out reforms, or even reformulations, of a spiritual or theological nature, and had done it confrontationally, usually ended badly, with their endeavors cast by the wayside.⁴⁷ If Llull was serious about introducing a new kind of

⁴⁴ Gregory the Great had declared that "Fides non habet meritum, cui humana ratio praebebat experimentum," a phrase repeated by Pope Gregory IX in a letter of warning to the University of Paris written in 1228. The phrase was well known and clearly represented Church doctrine. Llull himself repeats it in many works to show that he is well aware of the problem, and he even wrote one to show how his system could side-step the problem, as proclaimed in the title, *Liber in quo declaratur quod fides sancta catholica est magis probabilis quam improbabilis* (*Book in which it is declared that the Catholic faith is more provable than not provable*), and where he quotes the phrase twice in the prologue (*ROL VI*, 328–9, 338).

⁴⁵ With the possible exceptions of his campaign against the Parisian 'Averroists', and of his provocations on his second trip to North Africa.

⁴⁶ Many of Llull's techniques are additive rather than oppositional with respect to previous formulations. See pp. 206 and 296 below.

⁴⁷ Or their propounders cast into prison. Llull lived in a time of what one might call a hardening of ecclesiastic arteries. Things which might have been permitted at the beginning of the 13th century, such as the Waldensians, at first more or less

missionizing and in getting his contemporaries to lead more genuinely Christian lives, he would have to do everything possible to get the Church on his side. This meant not only buttonholing, as it were, princes and prelates, but it meant trying to persuade the core of the Church's intellectual establishment, which in Llull's time meant the University of Paris. We will see how in the course of his development he seems to have become increasingly sensitive to the topics and methods of the scholastic world, and how he absorbs and uses—or rather re-uses and recycles—them.

Even from the beginning, however, there was a strong relationship between the Art and the tradition of the Aristotelian topics as inherited by the Middle Ages.⁴⁸ It is not so much a question of the 'origins' of the Art, but more one of how it shaped what Llull was doing and how it permitted him to create a framework and address problems of actuality to these university audiences.

Aristotle's *Topics*, it will be remembered, discussed dialectics, that is, techniques for arguing (or debating), starting from generally accepted premises and arriving at probable conclusions. This was in opposition to his *Prior* and *Posterior Analytics*, the second of which discussed demonstrations—usually by using the formal methods of the first—to arrive at necessary conclusions. The *Topics* dealt with *inventio*, a method for 'finding' or 'discovering' arguments or counter-arguments on almost any subject that could arise in a debate, and the enumeration of sets of *loci* or 'places' for guiding the user through this apparatus of arguments. The *Analytics* dealt with what Cicero, Boethius, and later writers called *judicium*, or the ability, based on a much narrower range of methods (usually purely syllogistic), to reach a valid judgment as to the truth or falsehood of an argument. The former was an art of persuasion; the latter a science of logical proof. The division between the two, however, was not always quite so sharp, either with Aristotle or with Llull's contemporaries. The most widely used logical text of the thirteenth century, the *Summulae logicales* of Peter of Spain, begins by saying that "Dialectics is...", as if logic *were* dialectics (see n. 51 below).

tolerated, or the Franciscans, quickly brought into the fold, along with a proclamation prohibiting the formation of new orders, by the end of the century were beginning to be more relentlessly persecuted

⁴⁸ The demonstration of this fact is one of the ground-breaking themes of Ruiz Simon 1999.

With this in mind, we can follow Ruiz Simon, in listing Llull's four repeatedly declared descriptions of the Art.⁴⁹

1. It is 'inventive', like Aristotelian dialectic, but here combinatorial mechanisms are used to 'multiply', as Llull says, his investigative techniques.
2. It is 'demonstrative', because it provides the means to arrive at necessary conclusions.
3. It is 'compendious', because a limited, finite set of principles permit the 'finding' of an unlimited number and variety of arguments.
4. It is 'general', like metaphysics and dialectic itself, because it is applicable to any subject and its principles are of all classes of being.

If the reader will look at any list of works of the Art, he will see how these four terms reappear again and again in Llull's titles.⁵⁰ The first clearly points to a method based on the topics, and we will see how certain aspects of his system address certain techniques of *inventio* as understood by his contemporaries. The last clearly mirrors a statement of both Lambert of Auxerre and Peter of Spain: "Dialectics is the art of arts, science of sciences, the road leading to all other methods."⁵¹ The third refers to an essential aspect of the Art, its very innovative generative techniques, whose workings will become clear as we unfold the methods of Llull's system. What will also become clear is the crucial second point, where Llull intentionally blurs the distinction between *inventio* and *judicium*, or rather fashions the Art so that its conclusions are not only probable but necessary, and thus capable of demonstrating all manner of things, even the Articles of the Faith.⁵²

For all these similarities, however, the reader must keep in mind what we have said before, that what Llull does, not only with the Topics, but with all of logic, with metaphysics, rhetoric, astronomy, literature,

⁴⁹ Ruiz Simon 1993, 77–78.

⁵⁰ Although with varying orders of importance in different periods, and with the term 'general' not applied to the Art until the *Tabula generalis* of 1293–4.

⁵¹ For Lambert of Auxerre, see Gilson 1947, 554, and for the latter, Peter of Spain 1972, 1. The quotations are substantially similar in both authors (but see the textual variants cited in the second), except that Lambert omits the "sciences of sciences". Cf. Ruiz Simon 1993, 92 and n. 44, where it is also pointed out that, according to Lohr 1967, Llull knew the tradition of Peter of Spain.

⁵² The opening chapters of Ruiz Simon, 1999, are particularly instructive on the relationship between these different aspects of the Art.

etc., is to recycle whatever models he uses. Any subject he touches is refashioned in the light of his needs and of his Art, and this all the way down to sermons, proverbs, and poetic forms.⁵³ This is why the word ‘new’ appears in so many of his titles: *Tractatus novus de astronomia*, *Liber de geometria nova*, *Logica nova*, *Metaphysica nova*, *Liber novus physicorum*, *Rhetorica nova*, etc. So with the Topics we must not expect to see something that looks very much like contemporary treatises. But if we keep in mind the general framework within which he is working, it will help us see not only how he has re-used or recycled it, but why he has chosen certain strategies or terms along the way.

Is the Art a Kind of Logic?

One might be induced to think it is by the fact that Llull announces it as ‘demonstrative’, or by the semi-algebraic look of some of Llull’s discourse: “The goal of E is A V Y, and thus E is the origin of . . .” (*SW* I, 367). These letters, however, represent constants and not variables as they have in formal logic since Aristotle’s time. The Art as a system, in fact, has nothing to do with formal logic. One could perhaps consider it a kind of material logic, although it would probably be best to treat it as another species entirely. Llull himself was quite clear about it, differentiating it clearly from both logic and metaphysics, in a passage worth quoting at length.

It should be clear that this Art and logic and metaphysics in a certain sense deal with the same thing, since the intention of each is to encompass all things; nevertheless it differs from the other two in two ways: in its manner of considering its subject matter, and in the manner in which it is based on principles. Metaphysics considers things exterior to the mind insofar as they concern their reason for being (*prout conveniunt in ratione entis*); logic, however, considers things according to their existence in the mind, since it deals with those intentions which follow on their being intelligible things, that is to say, genus, species, etc., and also about those things involved in the act of reason, such as syllogisms, consequences, etc.; but this Art, as the highest of all human sciences, considers being

⁵³ For the sermons, see the introduction by Fernando Domínguez to *ROL* XV. Llull’s proverbs, instead of collections of popular sayings, are more like philosophical or theological maxims, and often nothing more than thinly disguised definitions of the concepts under study. His longest poem, *Lo desconhort*, reshapes the dialogue of the Provençal *partimen* or *sirventesc* into a philosophical dialogue; see Höfle 1996.

indifferently according to one mode or another. Whence it follows that they differ in their manner of considering the subject matter (*ex parte subjecti*). (*Introductoria Artis demonstrativae* [MOG III, ii, 1: 55])

In Chapter 5 below we will see how the Art is applied to logic, how Llull consciously tries to rework many parts of Aristotle's *Organon*, and finally how Llull's logic takes on a more or less independent existence.

The Two Phases of the Art

The reader should know that there is a curious and very important difference in historical projection of these two phases. Judging from the number of manuscripts in which they were copied, all the principal works of the Art seem to have enjoyed a roughly similar, and not inconsiderable, popularity in the Middle Ages.⁵⁴ With the advent of the Renaissance, however, the only works of the Art which saw their way into print were those of the ternary phase. This is clear from what we said above about the commentaries on the *Ars brevis* by Agrippa von Nettesheim and Giordano Bruno, by the printing by Zetzner in 1598 of both the *Ars brevis* and the *Ars generalis ultima* together with these commentaries, and its being this anthology, reprinted three times in the seventeenth century, in which Leibniz read Llull. The complete absence of printed editions of works of the quaternary phase was not remedied until Salzinger's great anthology of the early eighteenth century, in which almost all of the works of that phase were printed for the first time.

Llull and his Parisian disciple, Thomas Le Myésier, also probably helped to bury the quaternary phase. In the *Breviculum*, the beautiful manuscript presented to the queen of France by Thomas Le Myésier, the eleventh miniature reproduced here shows Llull's disciple asking his master's permission to make an anthology of his works. In it he lists (along the left and upper edges)⁵⁵ the components of the two main phases of the Art, and then referring to the first (in the bubble labeled "a" rising above him), complains about "the confusion caused by the

⁵⁴ See Bonner 2003b. The exception is the *Ars brevis*, which has always outdone all Llull's other works in popularity.

⁵⁵ The one along the left side is that of the *Ars demonstrativa* reproduced on p. 64, and the one along the top edge is an earlier version (from the *Ars inventiva veritatis*) of that from the *Ars brevis* reproduced on p. 123 below.

meanings of the alphabet of the *Ars demonstrativa* and its sixteen figures, which confound the mind” (Hillgarth 1971, 179). We have seen how in the *Vita coetanea* Lull also explains how he was ‘forced’ by the attitude of the students in Paris to simplify the Art, leaving aside the complications of the quaternary phase. The net result was to present the quaternary phase as a sort of more primitive and difficult predecessor, whose main ‘faults’ were corrected in the ternary phase.



miniature from *Breviculum*

However daunting master and disciple might have felt these early versions of the Art appeared to the Parisian scholastic world, they don't seem to have been as daunting to other readers of the time. The two principal works of the quaternary phase, the *Ars compendiosa inveniendi veritatem* and the *Ars demonstrativa*, are preserved in almost twenty-five manuscripts each, most of them medieval, a very large number for works that didn't fit into contemporary academic or literary canons.⁵⁶ So they must have found readers with enough interest to pay copyists to reproduce not only the texts, but also the elaborate chain of colored figures.

While the ternary phase does perhaps represent a certain retreat from the methodologically (or logically) more experimental techniques of the quaternary phase, it offers a very real metaphysical and epistemological deepening. At the same time one should remember that it is the ternary phase which, because of the Renaissance and Baroque projection we have just mentioned, was for long considered as *the* Art. We will therefore present both here, not only to show Lull's methodological development, but also because any presentation of the Art with just one of them would be manifestly incomplete.

Finally, it should be added that Lull's logic, his use of syllogistics and the developments of the post-Art phase, can be seen as a continuation of his endeavors with the Art, redirected towards a new method of demonstration based on a reworking of the Aristotelian definition of a "syllogism which produces scientific knowledge" (see below pp. 194 and 259).

A Short Preview of the Mechanisms of the Art

In approaching the Art it is important that the reader keep separate in his mind five different types of components, which can be placed in two broad categories: three that present its foundations, that is the basic building-blocks of Lull's system; and two concerned with its working out, that is, how these foundations are used in the development of arguments or demonstrations. They are:

⁵⁶ See again Bonner 2003b.

The foundations of the Art:

1. Terms—the basic concepts of the Art, such as those of Figure A (goodness, greatness, eternity, etc.), of Figure T (difference, concordance, contrariety, etc.), the virtues and vices, etc.
2. Figures, or graphical devices, which show different groupings and interrelationships of terms.
3. Alphabet, the letters assigned to terms, or to figures (i.e., groups of terms).

The functioning of the Art:

4. The conditions
5. Systems for constructing demonstrations and answering questions

Of the first three, it is the figures which have aroused the most curiosity and caused most misunderstandings. Partly because of their novelty (no other medieval author seems to have used the same kind of figures as organizing principles of philosophical thought), partly because of the way this novelty is boldly set out at the beginning of every work, and partly because of their later appropriation by sixteenth- and seventeenth-century writers, many critics have assumed that, having described them, they have described the lion's share of the Art. Because of this, as well as other confusions about them, we must try to make clear their types, functions and role.

As to types of figures, there are five:

- (a) The circular, non-rotating, figures, often with interior lines connecting concepts written on the circumference.
- (b) The triangular figures usually made up of compartments containing binary combinations of the terms of a corresponding circular figure.
- (c) The revolving figures or volvelles, of which there are never more than one per work, used to generate ternary, and very occasionally higher order combinations.
- (d) The *tabula* of the ternary phase, which is not really a figure, but, as the name implies, a tabular display of all the ternary combinations (without repetitions) of the revolving Fourth Figure of that phase.

- (e) The elemental figure, which shows the possible combinations of the four elements, earth, air, fire, and water.⁵⁷

As will be shown in the next chapter, type (a) uses, *avant la lettre*, the graphs of modern mathematics. Type (b), which displays all the binary combinations implied by (a), uses what, in relation to a graph, is called an adjacency matrix (or half-matrix in Llull's case). Since graphs can only be used with binary relations, type (c), comes into play when Llull needs to deal with ternary (or higher order) relations.⁵⁸ Similarly the adjacency matrices are replaced with (d) which displays all possible ternary relations in tabular form. Type (e), which is quite different from the other figures, has certain similarities to what mathematicians call a 'lattice'.

The figures have four purposes in the Art. The first is as what we would nowadays call a visual aid. Llull says that "it is only natural for the intellect to understand better by means of a visual and auditory demonstration than by one that is purely auditory",⁵⁹ and in another work he says that "the sense can help the imagination, and the imagination the intellect."⁶⁰

The second function of the figures is mnemonic, or as Llull puts it, "so that the memory can more easily recall the universal principles" of the Art.⁶¹ This is what made them attractive to Renaissance thinkers, who saw in them mnemonic devices, which, by acting as instruments

⁵⁷ I omit the tree figures, because Llull does not use them in works of the Art. For a discussion, however, of the various types he uses and for their relation to graph theory, see pp. 30–31 below.

⁵⁸ In the quaternary phase it is the Demonstrative Figure which displays these possible multiple combinations. Volvelles were traditionally used to display astronomical or astrological configurations, as Llull himself does in his *Tractatus novus de astronomia*. To use them as an 'inventive' device, however, for generating arguments is, as far as I have been able to ascertain, an innovation of Llull's.

⁵⁹ *Principles of Medicine* (SW II, 1120). By "auditory" Llull is referring to the written text, which in the Middle Ages was still often intended to be read aloud rather than silently. Cf. SW I, 64.

⁶⁰ *Compendium seu commentum Artis demonstrativae* (MOG iii, VI, 1: 293). See also the *Book of Contemplation*, 330:6 and 331:2; *Ars inventiva veritatis* (MOG V, 2); *Art amativa* (ORL XVII, 9; ROL XXIX, 124).

⁶¹ *Compendium seu commentum Artis demonstrativae*, loc. cit. Since such figures are usually thought of as mnemonic, it should be said that in this passage the memory plays a secondary role, as something for which figures can also be helpful, while in the other passages cited in the previous note it is not mentioned at all.

for showing the structure of the world, made it both more intelligible and more easily memorable.

The third purpose of the figures is to display the different sets into which the fundamental concepts of the Art are divided. This could be seen as the undeclared function of the circle inside which the graphs are inscribed: a kind of symbol of set inclusion. Thus under Figure A we find—taking an example from the quaternary Art—the divine attributes or ‘dignities’ as Lull calls them, under Figure V the virtues and vices, etc. This sometimes makes it convenient (although, strictly speaking, not correct) to talk, for instance, of ‘Figure A’ when what we mean is the ‘dignities’ or ‘the concepts grouped in Figure A.’

The fourth function of the figures is to show the possible relationships between the components of each set. In the quaternary phase this is done graphically, by interconnecting lines showing concordance. In the ternary phase, the mere circularity, for instance of Figure A (from which the lines have disappeared), shows the mutual predicability of its terms. To this one must add a second layer of what one might call color-coding, when Lull mentions, for instance, ‘the green triangle of Figure T’ (referring to the triad of difference, concordance, and contrariety), or ‘red V’ (referring to the vices).⁶²

As for the Alphabet, as we said before, one must not be misled by the algebraic appearance of the letters in the discourse of the Art. They are not variables, but shorthand symbols for concepts or groups of concepts of the Art. To give just two examples, as Lull explains at the beginning of the *Ars demonstrativa*, “B stands for memory remembering”, and “T [stands] for principles”, i.e. for all the “principles” of Figure T. So it is just a matter of having at hand a crib as one reads the Art so that one can make the necessary substitutions. Why then did Lull do this, instead of just writing out the concepts for which the letters stood? I would say that it was basically to keep the reader constantly aware of the foundations of the system, and how these foundations were sufficient for setting in motion all the rest of the mechanisms of the Art. In other words, making clear the generative system in which the reader is immersed. It was also doubtless for the reader to keep in mind fundamental relationships, as in Figure S for instance, where a single concept is less basic than its coupling with its act, and in turn

⁶² In the ternary phase these colors are residual: they still appear in the drawings of Figure T, but are not used in the actual texts of the Art.

with the various combinations with the other two powers of the soul and *their* acts, all this in a game of shifting mirrors according to the attitudes of the inquirer, which, as we will see in the next chapter, could take on a complexity that would be much more difficult with a purely verbal presentation.⁶³

Finally we should add that the Alphabet has a very different use in the ternary phase, where it is reduced to 9 letters, with multiple possible meanings for each, and where algebraic notation disappears from the actual discourse of the Art. What I mean by that is that we will no longer encounter texts such as:

E I N are the subjects in which is carried out the transformation of Y into Z contrary to the red triangle, there being a difference between S and B C D, as well as between B C D and E I N, to which B C D are subject, beneath which B C D lies S. (see below p. 87)

In the ternary phase he only presents letters as spring-boards for a discourse from which they disappear. As an example, with the Table of the *Ars generalis ultima*, he says that C T B D (which we have notated C b d) means “What is great difference and contrariety?”, which he then explains:

To the fifteenth question, “What is great difference and contrariety?”, one must answer that it is the cause causing one contrary in another, like an elemented thing in which are mixed fire and water... (see below p. 150)

As for the conditions, in the quaternary Art they sometimes are presented purely as comparisons or as a hierarchization of its terms, so the reader will know how to evaluate them in his demonstrations, but more often these comparisons, as we will see with the *Ars demonstrativa*, are developed through a systematic elaboration of a combinatorial mechanism general to the entire work. In the ternary Art they have become imbedded into the techniques of ‘mixing’ Principles and Rules.

As for the Art as a method for generating demonstrations and answering questions, which was after all its origin and purpose, this will be the principal object of study in the rest of this book.

⁶³ This is similar to the problems of algebra before the modern notation of Viète and Descartes, where to express $x^2 + 4xy - 2y$ one had to say “a quantity squared, plus four times the same quantity multiplied by a second quantity, minus twice the second quantity.”

CHAPTER TWO

THE QUATERNARY PHASE

General Remarks

Not only did the Art, as mentioned above, go through two quite different phases, but within each phase it underwent many changes and developments. The quaternary period is divisible into two cycles, each with its foundational work followed by others offering commentaries and applications. The first cycle is based on the opening work of the Art, the *Ars compendiosa inveniendi veritatem* (ca. 1274), the second on the *Ars demonstrativa* (ca. 1283), and these are followed by a third stage of transitional works with innovations preparing many features of the next phase. The ternary period opens with the *Ars inventiva veritatis* (1290), takes on some major changes and additions with the *Tabula generalis* (1293–4), and receives its definitive formulation in the *Ars generalis ultima* (1305–8), with its accompanying abbreviated version in the *Ars brevis* (1308). To present all these changes would have led the reader down a maze of side-paths, with the risk of not seeing the wood for the trees, of losing the central point of this book, which is the functioning of the Art. Instead it seemed preferable to explain the most characteristic works of each period, and to take up other formulations, when necessary, as variations on themes with which the reader was by then already familiar.

With the quaternary period, this meant concentrating on the *Ars demonstrativa* (*AD*), which, judging by the number of commentaries Llull wrote on it, was clearly the work which he felt showed that stage of his system in its most presentable form. In addition, the *AD* has a far clearer, more linear organization than the *Ars compendiosa inveniendi veritatem* (*ACIV*), making it much more ‘user friendly’. The *ACIV*, however, will not be entirely neglected: in the next chapter, its main differences with the *AD* will be studied, along with the transitional works we just mentioned.

Moreover, in the *AD* (as opposed to the *ACIV*) we find a systematic division between ‘first figures’—normally the circular ones—and ‘second

figures’—the triangular ones.¹ Here the systematization is such that it enters clearly into the domain of the modern mathematical theory of graphs. To see what that means, we will give not a formal explanation of the theory, but only a brief, intuitive presentation, enough to allow the reader to understand how Lull uses it.

*Rudiments of Graph Theory*²

Graphs originated as a method of displaying connections between points of a network, such as those of a utility company distributing gas, water and electricity to the houses of a town, or of a transportation company connecting many different towns. To see how this works on the simplest scale, let us imagine four towns, which we will label B, C, D, and E, connected by the network of roads represented by Figure 1a, with one extra road circling C, perhaps because of some special vistas for tourists.

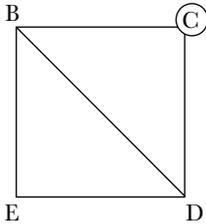


Fig. 1a

	B	C	D	E
B	0	1	1	1
C	1	1	1	0
D	1	1	0	1
E	1	0	1	0

Fig. 1b

Figure 1b shows another way of representing this situation. The fact that one can drive from B to C or from C to B, means that the compartments B-C and C-B we have 1 in them; and because one cannot go directly from C to E or from E to C, means that the corresponding compartments have 0. The road that starts and ends in C allows us to

¹ The only first figure which is not circular is the Elemental Figure. See p. 96 below for the lack of any such systematic distinction in the *ACIV*.

² The reader can consult, for a formal presentation, Deo 1974 and Foulds 1992, or, for a more popular presentation, Trudeau 1993. I would like to thank Miquel Bertran, professor of mathematics at the Universitat Ramon Llull of Barcelona, for having read this section and offered valuable suggestions and corrections.

put a 1 in the compartment of C-C, while all the other compartments B-B, etc., where we cannot take this route, have 0.

Now for a bit of terminology: Figure 1a is what mathematicians call a ‘graph’, B C D E are ‘vertices’, the lines connecting them are ‘edges’, and the one leaving and returning to C is a ‘loop’. Figure 1b is a matrix, and because of its exact representation of the relations in Figure 1a, it is called an ‘adjacency matrix’. The fact that in our examples one can travel all the roads in both directions means that any compartment, such as for example, B-C, will have the same number as its inverse, C-B. As a result, if we draw an imaginary line between the upper left to the lower right corners of Fig. 1b (that is through the compartments of B-B, C-C, D-D and E-E), we will see that the matrix is symmetric on either side of this diagonal.³

If we take the graph Fig. 1a and connect every vertex (and, for the moment, remove the loop), we get one that is ‘complete,’ as in Fig. 2a. Such a complete graph is symbolized by mathematicians as K_4 .

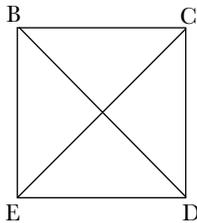


Fig. 2a

	B	C	D	E
B	0	1	1	1
C	1	0	1	1
D	1	1	0	1
E	1	1	1	0

Fig. 2b

As a result of this completeness, the matrix of Fig. 2b now has 1 everywhere except along the diagonal. If, in addition, the graph had loops at every vertex, Fig. 2b would then have 1 everywhere.

As we will see in a moment, Lull’s Figure A is merely an expanded version of Fig. 2a, with 16 instead of 4 ‘vertices,’ which makes this

³ If our roads were one-way, meaning that one could, for example go from B to C, but not from C to B (at least directly), that would give a matrix with 1 in the compartment of B-C and 0 in that of C-B. The result would be what mathematicians call a “directed graph” (often abbreviated to “digraph”), one whose adjacency matrix would not be symmetric. Lull, however, as we will see, only uses non-directed graphs with symmetric matrices.

figure an exact K_{16} .⁴ He also uses Fig. 2b, but altering the form in four perfectly legitimate ways, ways that in fact are mere notational variants. In the first place he only uses one of the (symmetrical) triangular halves of the matrix, thus saving himself the repetition of its mirror image. Or to put it in Lullian terms, he is not interested in distinguishing between the compartment of Goodness–Greatness and that of Greatness–Goodness. Secondly, since he is uninterested in non-connections, he omits the diagonal if it has zeros. Thirdly, having omitted the zeros, he uses letter pairs instead of ones to represent the edges. Lastly he presents the compartments in a different order. As a result, he would represent (the upper right portion of) a figure like Fig. 2b with the equivalent half-matrix of Fig. 3 below.

B C	C D	D E
B D	C E	
B E		

Fig. 3

B B	C C	D D	E E
B C	C D	D E	
B D	C E		
B E			

Fig. 4

If he uses repetitions (the ‘loops’ of the graph), which he does only, as we shall see, in the cycle of the *Ars demonstrativa* of the quaternary phase, then we get Fig. 4.⁵

Just to get straight the relationship of all this to the related field of combinatorics, let me say that Fig. 3 represents all the possible combinations of 4 letters taken two at a time without repetitions, and Fig. 4 represents the same with repetitions.⁶

⁴ In fact, Trudeau 1993, 28, as an illustration of a complete graph, has a drawing of K_{16} that is a perfect replica of Lull’s Figure A, only lacking that letter in the middle and the names of the dignities around the circumference.

⁵ Because of his reordering, repeated letters appear in the top horizontal line of his half matrix, instead of being along a diagonal.

⁶ Without repetitions the formula is $n!/[(n-r)!r!]$, where n is the number (in this case of letters), r is how many things we take at a time, and $!$ means the factorial. So in this case, $n! = 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$, $(n-r)! = (4-2)! = 2! = 2$, and similarly $r! = 2$, thus giving us $24/2 \cdot 2 = 6$ for the formula, which is the number of binary combinations in Fig. 3. With repetitions, the formula is $(n+r-1)!/(n-1)!r!$, which in this case gives $(4+2-1)!/3! \cdot 2! = 5 \cdot 4 \cdot 3 \cdot 2/3 \cdot 2 \cdot 2 = 120/12 = 10$, which is the number of compartments of Fig. 4.

Another aspect of graph theory that should be taken into account is the fact that a graph is called ‘connected’ if one can go from any vertex to another by walking along a series of edges; if not, it is called ‘disconnected.’ So clearly the graphs of Figures 1a and 2a are connected. A disconnected graph often looks like two separate graphs, as in the two triangles of Fig. 5a,

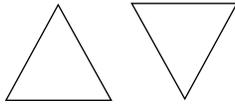


Fig. 5a

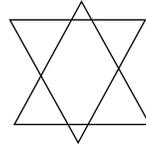


Fig. 5b

which can be redrawn as two overlapping triangles as in Fig. 5b. In order to understand that the two representations are identical from the mathematical point of view, the reader must realize that the edge-crossings of 5b are for these purposes irrelevant, or in other words, they are *not* vertices; mathematically the two figures represent the same disconnected graph, which we could symbolize as $D_{2/3}$, to indicate that it is a disconnected graph with two subgraphs with three vertices each. As we will see, Llull uses both kinds of graph, connected (K) and disconnected (D).

A third variety of graph that Llull uses is one with vertices but without connecting lines, which is called a ‘null graph’. We will see this when we come to the little Figures of Y and Z.

Two last points should be made about Llull’s use of graphical devices. The first is that his figures, or at least the ‘graphical’ ones as opposed to the matrices, are invariably drawn inside circles. This is just his way of showing that all the concepts of that graph form a single set, or that the triangles, for instance, of his Figure T are the disconnected subgraphs of a single graph, and should not allow the reader to confuse these circles with those of the single genuinely revolving figure present in most versions of the Art.

Secondly, even though he never uses trees in central works of the Art, a brief explanation might help avoid some of the confusions that have appeared in the literature concerning his trees. Mathematically a tree is a graph in which there is no path one can take that starts and ends at the same vertex (without back-tracking). The result is the typical branching structure we associate with a tree. Now with Llull, the trees

of the *Tree of Science* are genuine trees, as is the modified version of that of Porphyry in the *Logica nova*. On the other hand, those of the *Book of the Gentile and the Three Wise Men* are not trees in the mathematical sense; they have no paths one can travel from root through trunk to branch to flower. Only their flowers are operative, and there used merely as a more decorative and less alarming representation of the half-matrices of the Art with their successive binary compartments.⁷

The Terms and Figures of the Art

In the *Lectura compendiosa super Artem inveniendi veritatem*, Llull divides the principal figures into two groups: S and T, which are ‘active’, and the others, A, V, X, Y and Z, which are ‘passive’ (*MOG* I, vii, 41: 473). These last five constitute the material with which the Art works, while the first two provide the tools with which the ‘artist’, as Llull calls the user of his system, can study and manipulate them. So it would seem easier to begin with the simpler passive figures, before going on the more complicated psychological and relational ingredients of the two active figures, S and T.

Figure A

As the reader can see in the section of color illustrations between pages 92 and 93, this figure is a circle around which are written the sixteen concepts:

goodness	wisdom	glory	simplicity
greatness	will or love	perfection	nobility
eternity	virtue	justice	mercy
power	truth	generosity	dominion

⁷ The tree of the *Arbre de filosofia d'amor* (*ORL* XVIII, facing p. 72) is basically of the same type as that of the *Tree of Science*, even though it is not drawn that way in the manuscripts. That of the *Arbre de filosofia desiderat* (*ORL* XVII, 405) is closer to that of the *LN*. Those at the beginning of five works in *ROL* XX are like those of the *Book of the Gentile*, but now with flowers of ternary relations. That of the *Principles of Medicine* (*SW* II, facing p. 1120), is more a diagrammatic relation of the different parts of medicine. It might be worthwhile to examine in greater detail all of Llull's trees from the point of view of graph theory.

Since all the vertices of the round figure are interconnected, it is what we now know to call a ‘complete graph’, one which we can notate K_{16} .⁸ Lull’s description, as might be expected, is less mathematical. “This First Figure is circular, with A in the center . . . [It] is made up of sixteen compartments, with transverse lines extending from one compartment to another to indicate that all these compartments are concordant with one another, without any contrariety existing between them. In these compartments are written the sixteen dignities we understand to exist in God,” which he then goes on to name (*SW* I, 320).

A few comments on this statement might be helpful. In Lull the word ‘compartment’ (*camera* in Latin, *cambra* in Catalan) refers to any little box in which concepts are written.⁹ On the circumference of this figure there is one concept in each box. As we will see, in the discourse of the Art, these compartments are usually not unary, but binary, and sometimes they even have more than two concepts or letters.

Speaking of letters, those just inside the outer rim of this and of all the other figures—with the notable exception of Figure S—are only there for additional possible combinatory manipulation in connection with the Demonstrative Figure. They are never used in the actual discourse of the Art, where Lull only mentions the concepts of Figure A by their full names.¹⁰

As for the terms of “concordance” and “contrariety” Lull introduces here, they are, as we will see, components of Fig. T. It is important to understand that this concordance between concepts is represented by the lines of the figure (the ‘edges’ of our graph), because this is what will distinguish this complete graph from the disconnected graphs of other figures, as well as from the figures of the ternary Art.

As for the concepts contained in the figure, in his earliest works, such as the *Book of Contemplation*, the *ACIV* or the *Book of the Gentile*, Lull’s preferred designation for the divine attributes was “virtues,” even though this had involved some extra explanation, as when, in the *ACIV*, he said: “We put A for God, to whom we attribute sixteen virtues—not accidental but essential (we don’t mean to speak here of the theological

⁸ See n. 4 above.

⁹ In more popularizing works, usually ones involving trees, the compartments are depicted and referred to as “flowers”, undoubtedly to make the whole business look less alarmingly technical.

¹⁰ For the lack of these letters in the cycle of the *Ars compendiosa inviendi veritatem*, see Ch. 3, the paragraph preceding n. 14.

or cardinal virtues).”¹¹ By the time of the *AD*, however, ‘dignities’ became the standard Lullian word for the divine attributes.¹² This word, in addition to its usual meaning, was also the common scholastic translation of the Greek *axioma*, a connotation Lull might have found suggestive, even though the latter applied to propositions, not to terms. Finally it should be remarked that in no works of this period does Lull offer any explanation or definition of any of the individual concepts

For the reader’s convenience, here are the equivalents in Latin (and Old Catalan when their similarity might not be immediately apparent). We have also included under the last four dignities, the versions of the *ACIV* which differ from the *AD* (respectively before and after a slash).

bonitas (bonea)	sapientia (savica)	gloria	misericordia/ simplicitas
magnitudo (granea)	voluntas or amor ¹³	perfectio	humilitas/ nobilitas (noblea)
aeternitas	virtus	justitia	dominium/ misericordia
potestas (poder)	veritas	largitas (larguea)	patientia/ dominium (senyoria)

Its corresponding triangular (half-)matrix reproduced on the next page, is what Lull calls the second figure of A.¹⁴

Since the correspondence between this and the first figure is exact (because the dignities are all concordant with one another), we have here an adjacency matrix. And because, as we said before, in most of the cycle of the *Ars demonstrativa* (and only then) Lull allows repetitions (the top line of the half-matrix here), we have combinations with repetitions, giving us 136 compartments.¹⁵

¹¹ See p. 95 below.

¹² For the varying designations for the divine dignities, see Bonner 1996. Other synonyms he occasionally uses are “qualities,” “nobilities,” “perfections,” “reasons,” “attributes” and “infinities.”

¹³ Will and Love are often interchangeable in the list of Lullian dignities.

¹⁴ The reader should be careful *not* to apply this terminology of “first” and “second” figures to the *ACIV*, where they mean something completely different; see p. 96 ff. below.

¹⁵ Using the formula of n. 6 above, $(n + r - 1)! / (n - 1)! r!$, we get $(16 + 2 - 1)! / (16 - 1)! \cdot 2! = 17 \cdot 16 / 2 \cdot 1 = 136$.

As we will see later, the discourse of the quaternary phase is almost entirely built up from these binary compartments of the second figures, which is why we will follow him in displaying them all.¹⁶

Figure V

The seven virtues and seven vices of this figure are:

faith	gluttony
hope	lust
charity	avarice
justice	pride
prudence	accidie
fortitude	envy
temperance	ire

As the reader can see in the section of color illustrations, the virtues are blue and the vices red, which colors are used in the text, where “blue V” simply means the virtues and “red V” the vices.¹⁷ Notice too the grid of blue lines connecting all the virtues and similarly of red lines connecting all the vices, which gives us a graph that can easily seen to be an expanded version of Figure 5b above. This makes V a disconnected graph, with two subgraphs of seven vertices each, which we would notate $D_{2/7}$.

Llull’s explanation says that the figure is circular and “has transverse blue and red lines, and in the middle there is a V half red and half blue, to show that it includes both virtues and vices. The blue transverse lines going from one compartment of the virtues to another signify and show that the virtues are all in accord¹⁸ in two ways: one is when they simply accord with one other, and the other is when they accord

¹⁶ We will only omit those of the Figures of Theology, Philosophy and Law (for which see p. 55 below), since they involve applications to fields external to the Art.

¹⁷ Llull only omits the colors when coupling V with positive or negative concepts, in which case it is understood to stand for virtues or vices respectively. Thus the reader of the *AD* will frequently run into the compartments \overline{EAVY} and \overline{IVZ} , with the first meaning, as he will learn when we study Figures S, Y, and Z, “the rational soul loving God, virtues and truth”, and the second “the rational soul hating sin and falsehood”.

¹⁸ “Are in accord” is synonymous with “are concordant” from Fig. T. Notice again the equivalence between “concordance” and the edges of our graph. This is why there is no line joining a virtue to a vice.

with each other in being contrary to the vices. And the same is true of the vices, which have red lines extending from one compartment to another, which vices accord with each other and are contrary to the virtues” (*SW I*, 328).

The Latin (and Old Catalan) equivalents of the components of this figure are:

Virtues (blue V)		Vices (red V)	
fides (fe)	prudentia	gula	acedia
spes (esperança)	fortitudo	luxuria	invidia (enveja)
charitas	temperantia	avaritia	ira
justitia		superbia	

The concepts of this figure, along with those of Figure A, form the backbone of many of Llull’s more popularizing works, such as the *Book of the Gentile* or some of his collections of sermons, where he limits the visible presentation of the Art to these more immediately accessible concepts, omitting (or disguising) the use of the other figures.

The corresponding half-matrix is reproduced on the next page.

Since the second figures of the *AD* are the springboard for juxtaposing and studying *all* the possible binary combinations of the concepts listed, and not just the concordant ones connected by lines, here we no longer have a one-to-one correspondence of function between the two figures, and thus the second is no longer an adjacency matrix. Here, because of the lesser number of original components, we have 105 compartments of binary combinations.¹⁹

Figure X

Figure X has sixteen concepts, which we will list in two series of eight paired concepts, which is the way Llull frequently uses them:

predestination	free will
being	privation
perfection	defect
merit	blame
supposition	demonstration

¹⁹ $(n + r - 1)! / (n - 1)! r!$ here produces $(14 + 2 - 1)! / (14 - 1)! \cdot 2! = 15 \cdot 14 / 2 \cdot 1 = 105$.

immediately	mediately
reality	reasons ²⁰
power	object

As can be seen in the section of color illustrations, Figure X is similar to Figure A in having all vertices joined, so again we have a connected graph of K_{16} , and this in spite of the fact that it is composed of pairs of opposing concepts. Or as Lull explains it: “In this figure there are transverse lines going from one compartment to another in order to show that its compartments are sometimes concordant with one another and sometimes contrary, and this both immediately and mediately” (*SW I*, 330). Notice here that in addition to concordance and contrariety, we also have “immediately” and “mediately”, two concepts from Figure X itself.

In an earlier work Lull says of X that it is “the figure of contraries and concordances, but metaphorically it is called the figure of predestination,”²¹ referring to its use in resolving apparent contradictions, the most important of which is that between two of the terms of said figure, that is “predestination” and “free will”.²² As we will see below, in the alphabet of the *AD*, Lull also calls X the figure of “objectification”, referring to the fact that its “task is to present objects to the rational soul,”²³ so that a person can judge among different kinds of contraries. In fact, the figure is entirely composed of pairs of contrary concepts written facing each other on the circumference.²⁴ This might have led us to expect a figure similar to V, with positive and negative components connected by blue and red lines respectively, giving a $D_{2/8}$. But no; Lull here puts them on the same plane, allowing the user of the Art to establish their relations of concordance or contrariety.

Since this is the figure whose contents changed most between the *ACIV* and the *AD*, we will give the Latin (and Catalan) versions of the two sets of components separately, each one in two columns with facing

²⁰ Thus in plural in the earliest manuscripts, and not singular as in *MOG* or *SW*.

²¹ *Lectura compendiosa super Artem inveniendi veritatem* (*MOG I*, vii, 44: 476).

²² We will see below how Lull works out this particular apparent contrariety.

²³ This last phrase from the same work and page cited in n. 21 above.

²⁴ And not—as pointed out in Rubio 2000—the way Salzinger reproduced the figure, with opposing concepts next to each other. Lull probably used the two colors here just to show this opposition more clearly. It should also be mentioned that the medieval Mss. of the *ACIV*, sometimes have figures similar to the one reproduced here, but sometimes have more complicated versions with two inscribed squares; see p. 98 below.

pairs of opposing concepts, and in italics those of the *ACIV* that do not appear in the *AD*, and vice versa (which are exactly half).

<i>ACIV</i>		<i>AD</i>	
<i>sapientia (saviea)</i>	<i>justitia</i>	praedestinatio	liberum arbitrium
praedestinatio	liberum arbitrium	esse	privatio
perfectio	defectus	perfectio	defectus
meritum	culpa	meritum	culpa
<i>potestas (poder)</i>	<i>voluntas</i>	<i>suppositio</i>	<i>demonstratio</i>
<i>gloria</i>	<i>poena</i>	<i>immediate</i>	<i>mediate</i>
esse	privatio	<i>realitas</i>	<i>rationes</i>
<i>scientia</i>	<i>ignorantia</i>	<i>potentia</i>	<i>objectum</i>

It should be born in mind that for “privation” Lull frequently uses the synonym of “non-being”, and for “defect” that of “imperfection”. It is these two terms of “being” and “privation” which, in “presenting objects to the rational soul”, can act as an ultimate Neoplatonic determinant, with the things in the positive columns of the above lists according with the first, and in the negative columns with the second. As we will see, this mechanism plays a central role in the validation of arguments.²⁵

Since this first figure, like that of A, is a connected graph, the corresponding half-matrix of the second figure reproduced on the next page is again an adjacency matrix, similarly with 136 compartments.

Figures Y and Z

These figures have one concept each, “truth” (in blue) and “falsity” (in red). Lull represents them as little circles, but graphically they could as well be represented as single dots or vertices. As we pointed out before, such diagrams are called ‘null graphs’.

Now that we have done with the ‘passive’ figures, we will move on to the two ‘active’ figures, starting with the less complicated of the two.

Figure T

This figure, with its five interlocking triangles, is, as the reader can easily see in the section of color illustrations, merely an expanded version of Figure 5b on p. 30 above, and it is therefore a $D_{5/3}$. As with Figure V,

²⁵ See p. 83 below. It should also be pointed out that one of the terms of the both versions of Figure X—*perfectio*—and five from the version of the *ACIV*—*sapientia, justitia, potestas, voluntas, gloria*—also appear in Figure A, but evidently with different roles.

Figure T is also color coded, and here also it is important to keep these colors in mind, since in texts of the quaternary phase Lull frequently refers to “the green triangle,” “the red triangle,” etc.

When this figure is given a descriptive name, it is called the “instrumental figure,” meaning that “without T nothing can be done in this Art, just as the smith without a hammer could not work at his forge.”²⁶ Its components are called “principles”, a term, as we will see in the ternary phase, destined for a wider signification.²⁷

Since this is the only figure which has secondary or auxiliary concepts of the fields to which the primary ones can be applied, and since all of this is perhaps hard to read in the abbreviated forms given in the figure among the color illustrations, we give them in a fuller form, along with explanations of the auxiliary terms in the three outer rings.

Triangle	Primary terms	Auxiliary Terms
blue	God creature operation	in terms of, unity, essence, and dignities in terms of sensual, intellectual, and animal in terms of intellectual, natural, and artificial
green	difference concordance contrariety	any of the three can be between sensual & sensual, between sensual & intellectual, or between intellectual & intellectual
red	beginning middle end ²⁸	in terms of time, quantity, or cause in terms of extremities, measurement, or conjunction in terms of final cause, termination, or privation

²⁶ *Introductoria Artis demonstrativae* (MOG III, ii, 19: 73), with almost identical wording in the *Principles of Medicine* (SW II, 1126). See also *Liber propositionum secundum Artem demonstrativam compilatus* (MOG III, viii, 2: 504) and *Compendium seu commentum Artis demonstrativae* (MOG III, vi, 3: 295).

²⁷ See the Alphabet of the *AD* on p. 64 below, where Lull says that “T [stands] for principles”. This is the word used for its components in the *Quattuor Libri Principiorum*; see MOG I, ix, 2, 57: 608, 663; x, 63: 729; xi, 31: 763, where they’re called “universal principles”, and xii, 2: 768. They are also “principles” in the *Ars universalis* (MOG I, viii, 4: 486). I have insisted on this point, because the peculiar coupling of Figure T with the Elemental Figure in the first cycle of the Art (see p. 97 below) has caused much confusion on this point, which is more important than a mere question of names.

²⁸ These three terms of *principium* (*començament*), *medium* (*mijà*), and *finis* (*fi*) have three sets of translations in English. First, there is the temporal one in the text. Second, there is the causal one of “origin,” “means,” and “purpose.” Third, they can be rendered as “principle” (e.g. *Començaments de medicina* = *Principles of Medicine*), “intermediary,” and “goal.” See also SW I, 326, nn. 19–21.

	majority	any of the three can be between substance & substance.
yellow	equality minority ²⁹	between substance & accident, or between accident & accident
black	affirmation doubt negation ³⁰	any of the three can apply to being, to non-being, or to possible & impossible

The auxiliary terms mean that, for instance, as Lull explains with the green triangle, “any difference (or concordance or contrariety) must be between something sensual and something sensual, as between a horse and a lion, and so on; or between something sensual and something intellectual, as between body and soul, or between a pupil and what is taught, and so on; or between something intellectual and something intellectual, as between the intellect and the will, or between the soul and knowledge, and so on.”³¹ Notice too that of the three concepts corresponding to the angle of “beginning” in the red triangle, the first two, “time” and “quantity” imply (as he makes plain in later versions of the Art) the complete list of Aristotelian categories, which include “quality,” “relation,” “action,” etc., and the last concept of that same angle, “cause” likewise implies the four Aristotelian causes.³²

It is perhaps worth pointing out that it is the green triangle which is most frequently used in the Art, and especially the two concepts of “concordance” and “contrariety” that we have already seen Lull using in his descriptions of other figures.

The description of the first figure is followed in the manuscripts by the usual half-matrix, in this case, as with Figure V, not reflecting the graph structure but rather presenting a chart of all possible combinations of its components, giving 120 compartments.³³

²⁹ One could also translate these three terms by their adjectival forms as “greater,” “equal,” and “lesser”.

³⁰ The Latin (and Catalan) equivalents of the basic terms of Fig. T are:

Deus	differentia	principium (començament)	majoritas	affirmatio
creatura	concordantia	medium (mija)	aequalitas	dubitatio
operatio	contrarietas	finis (fi)	minoritas	negatio

³¹ *Compendium seu commentum Artis demonstrativae* (MOG III, vi, 7: 299).

³² See below, p. 129.

³³ The combination with repetitions of 15 elements taken 2 at a time here gives $(15 + 2 - 1)! / (15 - 1)! \cdot 2! = 16.15 / 2 \cdot 1 = 120$.

Figure S

The figure of S, which represents the rational or intellectual soul,³⁴ is perhaps the most unusual figure in the entire Art. Although based on the three Augustinian powers of the (rational) soul—memory, intellect, and will—it is not made up simply of these powers, but rather of their *acts* and *combinations of acts*. As a result, it is a figure that begins by being dynamic and combinatory. Lull presents it, as we can see in the section of color illustrations, as a circle with four inscribed squares, making it once again a disconnected graph, one we could notate as $D_{4/4}$.

In this case, however, Lull's distribution of the figure is not perhaps as helpful as others, mainly because of the complicated relations not only within each square, but also between those of one square and another, and also because it is frequently hard to read. The reader might therefore find more helpful the following chart, in which the columns represent the squares of the circular figure, and the rows the different uses of each power (or combination of powers) of the soul:

				The act of
The act of	B	F	K	B F K
the memory	remembering	remembering	forgetting	= O
The act of	C	G	L	C G L
the intellect	understanding	understanding	not knowing	= P
The act of	D	H	M	D H M
the will	loving	hating	loving or hating	= Q
The act of	B C D	F G H	K L M	O P Q
	= E	= I	= N	= R

It is important to remember that Lull frequently refers to the concepts of the bottom row, represented by the letters E, I, N, R—which in fact represent the four squares of Lull's figure and the columns of our chart—as the “four species of S, which contain the individuals, which

³⁴ The terms are interchangeable. In *AD* it is the “intellective soul” (see *SWI*, 317), and in *ACIV* the “rational soul”.

are B C D, F G H, K L M, O P Q.”³⁵ As with the subgraphs of V and T, the squares of Figure S are colored differently, but in this case, Llull’s invariable use of these letters E, I, N, R to refer to them, makes the color distinctions redundant.

For the Latin (or Catalan) originals of these terms see n. 78 below. It should be pointed out that, of the last two participles, the first could be translated as “loving” or “liking”, and the second as “hating”, “not loving”, or “disliking”.

Three things about this figure might seem surprising. The first is that an original ternary structure, the powers of the soul, with obvious analogies to the Trinity (analogies of which Llull makes ample use), should be treated in a quaternary manner with four squares and their sixteen components. The second is why did Llull choose these few combinations out of many other possibilities. The third has to do with the surprising double meaning of the letter M.

The first is explained by the fact that its components refer not to static entities of memory, intellect and will, but, as I said before, to their *acts* and what they do in conjunction with one another, which involve an expansion from three to four in both the vertical and horizontal directions of our chart.

As for the second point, the simplest answer is that, given that the Art was devised as a vehicle of persuasion, and this above all in matters of faith, those are the combinations that for Llull were significant,³⁶ or as he puts it in the *AD*:

By means of E I one formulates in this Art questions, arguments, and solutions, as well as necessary propositions. N, however, is the vehicle for suppositions, faith, and belief, whereas R is the vehicle for doubts. (*SW I*, 322–3)

So E and I represents the position of someone who, to put it in modern terms, having the necessary information, either accepts or rejects a proposal. N is the “species” used for supposition—which explains our third query above about the dual nature of M—either loving or hating (i.e., with no preconceptions), and with K L indicating no memory or knowledge of the matter under investigation. N also represents belief,

³⁵ *ACIV* (*MOG I*, vii, 2: 434). This structure of four squares with a total of sixteen compartments makes for a clear parallel with the Elemental Figure, a parallel which was important for Llull. For this and for discussions about which figure came first, see n. 68 below.

³⁶ Cf. Pring-Mill 1968, 122–3, reprinted in Pring-Mill 1991, 230–1.

or perhaps one should translate the Latin word more literally and say “credulity”, since it could equally well apply to Muslims or Jews.³⁷ R represents a state of intellectual or spiritual confusion, with all positive and negative possibilities open.³⁸

Some of this might become clearer in discussing one aspect of Figure S which reappears constantly in the discourse of the quaternary Art, and that is the role of the letters F G. As Llull explains in the *AD*:

E always and continuously apprehends objects by means of B C D. I apprehends objects sometimes by means of F G H, and sometimes by means of F G without H; the reason for this being that F G, sometimes without H and sometimes with H, examines the universals, seeking the particulars according as they are lovable by D or hateful by H. (*SW* I, 322)

In the *Introductoria Artis demonstrativae*, Llull says that F and G can “either be used to produce hatred, indicated by H, or they can be used to investigate without H, that is without hatred” (*MOG* III, ii, 6: 60). In the *Compendium seu commentum Artis demonstrativae* he explains how “F and G, without the addition of either H or D, perform their investigative task...based on a previous supposition of N” (*MOG* III, vi, 3: 295).³⁹ So the task of F G is to take on an investigation free of any feelings of liking or disliking, that is to say, with no preconceptions about the matter under investigation. This is important, because time and time again in the *AD* the reader will see F G used as an investigative tool—“With F G examining the compartments, it is clear that...”

The reader should, however, be wary of giving fixed meanings to the various letters. Those I have given for N or R are their most natural referents, but depending on the object they are viewing, they can take on quite different roles, just as E/I are positive/negative when they are considering virtues, but negative/positive when considering vices.

³⁷ In the *Introductoria Artis demonstrativae*, *MOG* III, ii, 6: 60, he refers three times to “suppositio vel credulitas” when describing the role of the column of K L M N. In the *Ars universalis*, *MOG* I, viii, 3: 485 he refers to it as “the figure which has to do with faith and belief: *fides et credentia*.”

³⁸ See *MOG* I, viii, 4, 23: 486, 505, and *MOG* III, vi, 3: 295 where he says this figure represents confusion. See Rubio 1997, 87ff. and 134ff. for ample explanations of the configurations of S.

³⁹ In the *AD* (*SW* I, 381) he says that “The supposition that S makes with N is the operation that constitutes the beginning of the demonstration toward which F G are impelled.” See the discussions of the roles of F G and N in Rubio 1997, 92ff. and Ruiz Simon 1999, 217 n. 234, where it is rightly explained within a section on “The Inventive Function of Hypothesis”.

All this also shows us the main reason for preferring letters to words. F and G have the same lexical referents as B and C, but they don't have the same relational reference. The letters show the place of these concepts in the structure of the acts of the rational soul; words just represent themselves. This is why, as Rubio has pointed out, to substitute words for letters, as some authors have been tempted to do to facilitate reading the Art, can quickly falsify its meaning.⁴⁰ All this also amply shows how the combinatory mechanisms are no mere side-issue or curiosity with the Art, but instead are fundamental both to Llull's epistemology and to the ontology it investigates.

The half-matrix corresponding to S, like those of Figures V and T, does not reflect the graph structure of the first figure, but rather presents again all possible combinations of its components, and, with its 136 compartments, is therefore once more not an adjacency matrix.⁴¹

An abbreviated version of this Second Figure, one based solely on combinations of the four "species" of S, is one he uses, as we will see, to structure one of the distinctions of the *AD*.

EE	II	NN	RR
EI	IN	NR	
EN	IR		
ER			

A Sample of the Functioning of the Figures

Before going on to present the remaining figures, it might be worthwhile to stop a moment and give the reader some idea of how these principal figures function in the discourse of the Art, and above all how they do so in combination with one another. At the same time we can show the fluid, changing roles of the components of Figure S depending on the attitude of the rational soul to the object under scrutiny. From the *ACIV* we will take an example in which Llull tries to solve the apparent contradiction or contrariety between predestination and free will. The argument is presented as four successive stages—which he refers to as four "figures of X"—in a kind of dialectic oscillation.

⁴⁰ Rubio 1997, 100.

⁴¹ See n. 15 above for the number of compartments.

The First Figure of X

S in E I N R, with T in X enters the compartment of perfect wisdom in order to form the compartment of perfect predestination. As a result, when E contemplates A, the compartment of perfect predestination is formed by necessary reasons, because if E remembered and understood predestination not to exist, it could not remember or understand perfect wisdom in A.⁴²

Here we have all of S and its four “species”, with T making the necessary distinctions in X, checking a binary compartment of dignities from Figure A, to form a binary compartment from Figure X.⁴³ The point being that predestination follows necessarily from God’s perfect wisdom. But this causes a problem.

As a result of E remembering and understanding perfect wisdom in A, R falsely signifies in the First Figure the compartment of free will non-being.⁴⁴ And then N loves free will which it forgets and does not know, even when E in the First Figure enters the compartment of perfect predestination, and therefore I is interposed between E and N, with F G H attaching themselves to predestination, because of the fact that K forgets and L does not know free will for salvation or condemnation.

In other words, the attachment of E to that compartment of Figure A, which makes predestination necessary, must, as far as the rational soul is concerned, preclude the existence of free will. But N cannot accept this, since it loves free will, even without understanding or remembering it. This causes I to interpose itself between E and N, with the disastrous result that it remembers, understands, but now dislikes, predestination, because of the exclusion of the possibility of free will for salvation or condemnation.

⁴² The text of this and the other quotations in this section are from *ACIV, MOG* I, vii, 9–10: 441–2, but with corrections according to the best manuscripts (for the reasons explained in Ch. 3, n. 5 below). In the *Ars Universalis, MOG* I, viii, 16: 498 the figures are named, respectively, of affirmation, denial, doubt and resolution (*determinativa*). Our analysis is based on those of Rubio 1997, 93–94, and Ruiz Simon 1999, 147–8. For these four figures as sub-figures of X itself, see p. 98 below.

⁴³ Note the presence of “perfection” from Figures A and X in the first and second compartments respectively (see n. 25 above). Notice also the lack of linguistic differentiation between noun and adjective where perfect wisdom and perfect predestination are equivalent respectively to perfection wisdom and perfection predestination. This is a matter that will be explained in the chapter on Logic, in the section on Predication.

⁴⁴ “*Non esse*” in the mss.; *MOG* has “*privationis*”, but as explained on p. 39 above, the two are synonyms in Figure X.

The Second Figure of X

The second figure of X is the contrary of the first in S, since as soon as E enters A by means of perfect justice, B remembers and C understands free will...

The text continues in a mirror image of the first figure. By substituting justice for wisdom in Figure A, and free will for predestination in Figure X, what starts out all for free will, gets turned against it by the lack of consideration for predestination. Which brings us to

The Third Figure of X.

Since S finds itself confused in the First and Second Figures of X, with R joined to each of E I N, it therefore forms this third figure, which is called the figure of doubt, composed as it is of the first and second figures. For just as fire strongly signifies dryness and water humidity, thus the wisdom of A strongly signifies predestination and the justice of A signifies free will equally strongly.⁴⁵ But C G cannot at one and the same time understand perfect predestination or perfect free will, and thus C G are in doubt, and don't dare to affirm or deny either predestination or free will, and all of S is perverted into R...

or in other words, into confusion and doubt, for when R remembers, understands and loves free will, it forgets, does not understand and hates predestination, and vice versa. The solution to this dilemma is found in

The Fourth Figure of X.

We place S, under the heading of E, in A, in the compartments of perfect power and perfect will, and all the other compartments of A, even though it will be enough to give examples of only a few. Now when C affirms these two compartments in A, it then affirms the compartment of predestination and free will. When, however it denies the compartment of predestination and free will, then it denies the compartments of perfect power and perfect will, for unless both predestination and free will exist,⁴⁶ no B can remember, nor any C understand that in A there is perfect power or will in wisdom and justice, and it [any B or C] could even remember and understand that in A there does not exist perfect wisdom or perfect

⁴⁵ As the reader can see from the Square of Elements on p. 59, dryness and humidity are the secondary qualities of fire and water, thereby making a closer analogy with the following relations between A's wisdom and justice with predestination and free will.

⁴⁶ Instead of "exist" the Latin has "*essent in esse*", "are in being", which refers to "being" as opposed to "privation" or "non-being" of Figure X. See Saranyana 2000, 333–6, for the syntagma "being in being".

justice, which is impossible, because it cannot remember and understand to be in A something that is not in A.

The affirmation and denial of the second sentence above are components of the black triangle of Figure T, which has now entered the fray to resolve its third component, which is doubt. In fact it realizes that when in the first figure T affirms predestination, then it is in Y (= truth), and when it denies free will it is in Z (= falsehood), while in the second figure the opposite is the case. The crux of the problem is that what S

believes when it is in the first figure, namely that free will does not exist, is in fact L, which does not understand free will in this first figure, which L is in Z, when it imagines itself to be C and to be in Y. The contrary ensues in the second figure, and thus when E puts the first and second figures in B C in this way, in the fourth figure it remembers and understands more than in the first or second figures, and with the other compartments A within itself causes wisdom and justice to accord with one another. And thus I puts F G in accord with E, and H hates the deception of those false significations which S received, through its weakness, in the first and second figure.

Notice how Llull has used a technique of comparing concepts or pairs of concepts (in ‘compartments’) from Figures A and X, using T to evaluate these comparisons, and S to present the possible attitudes of the subject doing the evaluating, in order to arrive at Y or Z. In this example T plays an unusually small role with its black triangle of affirmation, doubt and denial. Its green triangle, so predominant in Llull’s proofs, is here only implicit in the concordance of the concepts of the compartments such as perfect wisdom and in the contrariety of those of free will non-being, as well as the concordance and contrariety of various of these compartments with each other. It is S, however, that plays a major role here, shifting time and time again its perceptions of the unfolding arguments.

A General View of the Figures

As Josep Enric Rubio has amply demonstrated,⁴⁷ each one of the figures we have just studied has its roots in the earlier *Book of Contemplation*. This is even true of what is perhaps the most surprising figure for anyone

⁴⁷ Rubio 1997.

first studying the Art, Figure S. If Rubio is right—and I strongly feel that he is—in emphasizing this prehistory of the Art, what then is new about the *ACIV*, the work with which Lull initiated his adventures into the Art? I would say four things:

- (1) The basing of his discourse on a limited number of concepts organized in a few, clearly defined sets, and imposing a strict ban on any external input, thereby making it a self-sufficient, “compendious” Art. By contrast, the *Book of Contemplation* imbeds the seeds of these figures—along with many other groups of concepts⁴⁸—in the midst of a much more general discourse. Nor is the content of these groups always fixed: many of the concepts of Figures A and T, for instance, are indeed found in the work, but often dispersed and not organized in clearly defined sets with fixed groups of components.
- (2) The precise structuring of each figure, with visual representations in the form of graphs and half-matrices, showing their strict delimitation, internal relations, and combinatorial possibilities. In the *Book of Contemplation* we have usually just chart-like lists accompanied by letters of the alphabet.
- (3) The relationship between the figures, as in the example we have just given, which is so typical of the quaternary Art. They do not function alone, and at the same time this relationship is fluid and changes according to the circumstances of the arguments proposed, all of which Lull could not clearly formulate or delineate with the more amorphous presentation of the *Book of Contemplation*.
- (4) The use of (1), (2), and (3) to construct a generative system, that is one which, starting from a finite, limited set of concepts can generate arguments on all possible subjects. This is what made it, in medieval terms, “inventive”, and it was the possibilities offered by such a generative aspect that fascinated sixteenth- and seventeenth-century figures.

⁴⁸ There are a total of thirty-seven such groupings of concepts in the *Book of Contemplation*, in chs. 328–364. The reader should not be misled by the use of the word *figura* in the opening paragraphs of chapters 362–4 of the *Book of Contemplation*, where it refers to a letter standing for a concept (or, as in 364.2, for a group of concepts). This is one of the standard Latin meanings of the word as “a written symbol or character” (*Oxford Latin Dictionary*, s.v.). In other words, it there refers to individual letter symbols, not to geometric structures displaying internal relationships.

So one could say that, while there are few individual components of the *ACIV* that are new with respect to the previous *Book of Contemplation*, the way those components are structured and the overall structure they create are extraordinarily new and different.

The Remaining Figures

At the beginning of his career, with the *ACIV*, Llull wrote four short works to explain how his Art applied to the fields of Theology, Philosophy, Law, and Medicine.⁴⁹ With the *AD* these topics were incorporated into the structure of the work, and given one figure each.⁵⁰ As we will see in a moment, the principles of the first three fields are almost identical in the two works; only the fourth changes its role dramatically, going from a specific treatment of medicine to a more general one of elemental theory.⁵¹ The presentation of these contents as separate figures is Llull's way of displaying the specific principles of what, for medieval thinkers following Aristotle, were called 'subalternate' sciences. By doing it in this manner, however, he was contravening two clear doctrines of his time: the first was Aristotle's dictum that there could be no such thing as a general science, and the second the medieval formulation of theology as a superior science whose principles came from divine revelation. That he was fully conscious of what he was doing—and this at the very beginning of his trajectory—is clear from the prologue to the *Principles of Theology*.⁵² There, after listing the sixteen

⁴⁹ They are often referred to collectively as the four *Libri principiorum*, which include the *Principles of Theology*, *Philosophy*, *Law*, and *Medicine*, all published in *MOG* I, ix–xii: 607–814. This edition was reprinted by Pring-Mill in Lullus 1969, and, as this book went to press, the four works received a critical edition in *ROL* XXXI. The only one extant in a Catalan version is the *Principles of Medicine*, printed in Llull 1989, given a critical edition in *NEORL* V, and translated in *SW* II.

⁵⁰ The first three are represented by circular figures in the *AD* like the one given below, and the last by the elemental figure given further along. In the *Libri principiorum* dependent on the *ACIV*, what are called "figures" are, for the first three works, in fact just "alphabets" assigning their components to the letters from B to R (each accompanied by binary half-matrices), and for medicine a complicated double tree (see *SW* II, facing p. 1120) representing the old and (Llull's) new medicine, in the second of which is imbedded a variant of his elemental figure, along with three triangles of Figure T and one square of Figure X.

⁵¹ Elemental theory does not in fact appear in the body of the *ACIV*, except as a sub-figure of Figure T! See p. 97 below for details.

⁵² *MOG* I, ix, 1–2: 607–8. For a fuller discussion of the matter of subalternation in Llull, see p. 278ff. below.

specific principles on which the work is based, he says that they must “follow the conditions and rules of A S T V X Y Z,” clearly indicating its subalternate relation to the general science of the Art. As for the second, in the same prologue, he says that “theology is a subject about which nobody until now has tried to prove things by necessary reasons”, and that this is precisely what he proposes to do with the figures of the Art he has just mentioned.

Since the principles of these subjects, and especially of the first three, are not necessary to an understanding of the functioning of the Art, we will do little more than list their contents in our exposition. The same, however, is not the case with the field of medicine, based as it is on the Elemental Figure, whose importance will become clear when we explain its role in greater detail after presenting the other three figures.⁵³

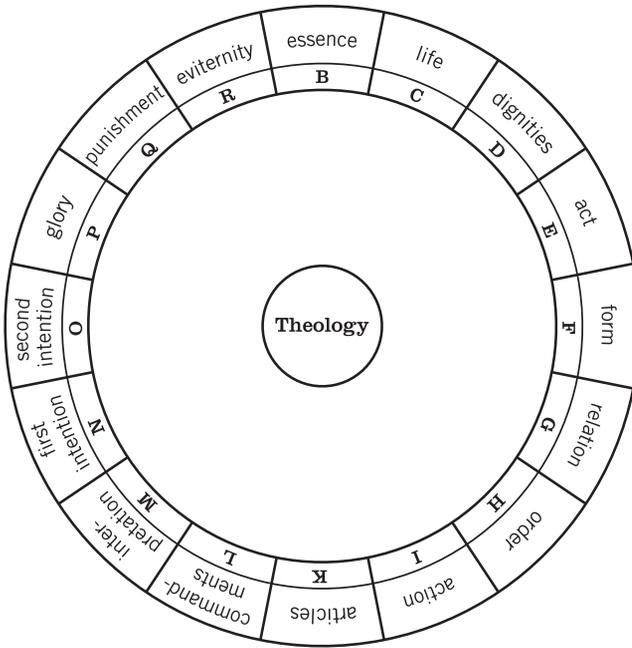
The Figures of the Principles of Theology, Philosophy and Law

The figures of these three fields to which the Art can be applied are identical in form, so we will just exhibit the first, that of Theology.

Since the concepts of each of the three figures—as the reader can see from the lists below—are not presented for reasons of comparison one with another in the same way as those of the Art, there are no connecting lines. To put it another way, if they are treated in pairs, it is more to study the results of such coupling, not to ascertain which are concordant with one another (thus producing lines). These figures are therefore, like figures Y and Z, null graphs, that is graphs with vertices and no edges. Whereas Y and Z had only one vertex each (and could therefore be notated N_1), these have sixteen vertices (giving N_{16}).

The concepts of these three figures are listed below, with those of Theology in two columns corresponding to the two versions of the quaternary phase.

⁵³ Note too the position of these figures in the text of the *AD*: the Elemental Figure is presented after Y and Z, and before the final Ninth or Demonstrative Figure, that is, within the main body of the Art. Theology, Philosophy, and Law, however, are added after the Ninth or Demonstrative Figure, and he begins his explanation of the Figure of Theology by saying “This figure has been added to the Art...”



THEOLOGY⁵⁴

<i>ACIV</i>	<i>AD</i>
essence	essence
dignities	<i>life</i>
<i>operation</i>	dignities
articles	<i>act</i>
commandments	<i>form</i>
<i>sacraments</i>	<i>relation</i>
<i>virtue</i>	order
<i>knowledge</i>	<i>action</i>
<i>love</i>	articles
<i>simplicity</i>	commandments
<i>composition</i>	interpretation
order	first intention
<i>supposition</i>	second intention
interpretation	<i>glory</i>
first intention	<i>punishment</i>
second intention	<i>eviternity</i>

PHILOSOPHY LAW

first cause	formal law
motion	material law
intelligence	law
firmament	common law
form	special law
matter	natural law
nature	positive law
elements	canon law
appetite	civil law
power	customary law
habit	theoretical law
act	practical law
mixture	nutritive law
digestion	comparative law
composition	ancient law
alteration	modern law

⁵⁴ As with Figure X we have indicated the differences between the two versions of the Figure of Theology by putting in italics those that only appear in one of the versions, which are exactly half of each list, again as with Figure X!

There is no need to display the second figures, since we will not be dealing with them in our exposition, and since they are, with their 136 compartments, identical in form to those corresponding to previous first figures with sixteen components, such as those of A and X.⁵⁵

The Elemental Figure

The First Elemental Figure, as the reader can see in the section of color illustrations, consists of four quadrangles—one for each of the four elements—of sixteen compartments each.⁵⁶ Lull then explains:

Fire is assigned the color red, air blue, water green, and earth the color black, and this is so the colors differentiate the compartments, insofar as a certain color is better suited to one element than another. The elements are measured in degrees, as is clear from the compartments of this figure, which compartments represent suppositis as well as mixture and digestion, as is explained in the *Principles of Philosophy* and *of Medicine*. The simple and compound forms can be found in these compartments, each form having its own matter.⁵⁷

To the figure Lull usually presents, I have added (in parentheses) the quality of each element: fire was hot, earth dry, and more surprisingly, water was cold, and air moist. The degrees mentioned in the previous quotation are the standard medieval measurement of the intensity of the elements in compounds, which Lull systematizes in his own way. An example Lull frequently uses is pepper, which he says is hot in the fourth degree, dry in the third, moist in the second, and cold in the first. We will see more of the functioning of this figure when we deal with “Conditions” and “Questions”.

⁵⁵ For the second figures, see *SW I*, 335–7.

⁵⁶ Each of these quadrangles is similar to a kind of graph called a lattice, but since the relationship is not one-to-one, this is a subject which will not be treated here.

⁵⁷ *SW I*, 332. See n. 34 there for “supposit”, which refers to something that subsists by itself. For a good presentation of the layout of the Elemental Figure, see Damberg 2000, in which degrees are discussed on p. 84, and mixture and digestion on p. 97.

of theology, law, and nature, as well as with the other sciences by which the intellect is uplifted to understand.⁶⁰

So medical (that is, elemental) theory can not only help us understand the other three fields of theology, philosophy and law, but even other components of the Art itself, as he explains in the *AD*:

This Elemental Figure is of primary importance to the Art, for by means of it the artist is led to knowledge of the other figures. This is because in the workings of nature are signified the intrinsic and extrinsic works of A S V, as can be seen by letting T examine the Elemental Figure and A S V with X Y. This is why, in the Art, the Elemental Figure is used to give various kinds of similes, examples, and metaphors, according to the conditions of the Second Distinction.⁶¹

Notice that it is the “workings of nature” that are useful as heuristic analogues, or, in other words, not so much the individual elements as the functioning of their interrelations, which is what is brought out by the Elemental Figure.

There was also a medieval tendency, especially noteworthy in Llull’s contemporary and fellow countryman, Arnold of Vilanova, to consider an analogical relationship between physical and spiritual medicine, that of the soul as well as that of the body,⁶² something Llull makes quite clear, as we will see, in Mode 16 of the *AD* on “Healing”.⁶³ What is new in Llull is the application of elemental theory to so many other fields, and, as we have just seen, even to the Art itself.

A graphical figure invented by modern scholars but based on explanations in a later work of Llull’s will help clarify certain other important relationships among the elements and their qualities as foundations for other analogical uses.⁶⁴

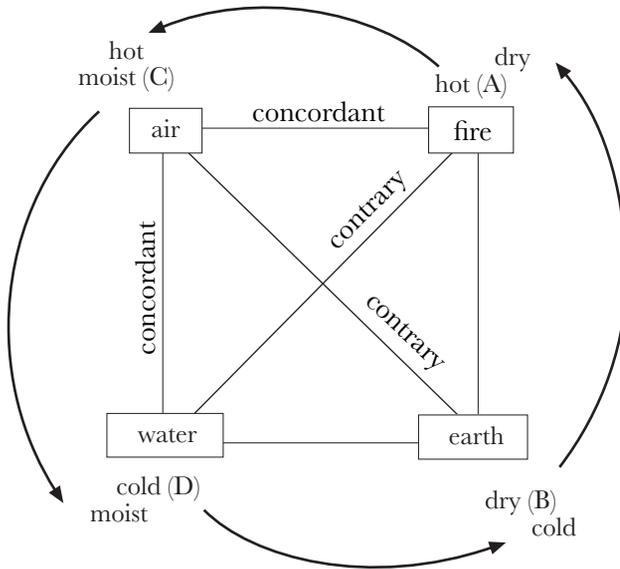
⁶⁰ *SW* II, 1128. Cf. Yates 1954, 130 (= Yates 1982, 27), as well as Pring-Mill in Lullus 1969, xxvi; Pring-Mill 1955–6, 237; Pring-Mill 1963, 38; and Pring-Mill 1972, 318 (all except the first reprinted in Pring-Mill 1991, 169–170, 144, and 244). To see how this elemental exemplarism fits in with Llull’s more general use of metaphor, see below, Ch. 6, the section on “Signification and Metaphor”.

⁶¹ *SW* I, 333. For a detailed analysis of this passage, see Pring-Mill 1972, 321ff. (= Pring-Mill 1991, 249ff.)

⁶² See Ziegler 1998, and especially pp. 179–180 where he refers to Llull’s application of medicine to religious discourse.

⁶³ See below, p. 76. Llull also wrote a *Medicina de peccat*.

⁶⁴ The Lullian explanation can be found in the *Compendium seu commentum Artis demonstrativae* (*MOG* III, vi, 60–61: 350–1). The figure itself is in Yates 1954, 149 (= Yates 1982, 48), and Pring-Mill 1961, 63, and Pring-Mill 1972, 321 (= Pring-Mill 1991, 58 and 248).



As we can see there, each element has its primary or “proper” quality, as Lull calls it. In addition, each also has an “appropriated” quality it takes from its clockwise neighbor: the proper heat of fire appropriates dryness from earth, etc.⁶⁵ This business of proper and appropriated qualities arises in many other situations in Lull’s universe, and it is often explained analogically with reference to elemental theory. We will see how it is codified in the ternary phase under one of the Ten Rules or Questions—that of “quality”.⁶⁶

In the more restricted field of logic, Lull uses this square of elements as an analogical substitute for the traditional square of opposition. The reason for this is that he is less interested in different extensions of qualities (“all X”, “some X”, “no X”) and their various relations, than in the two intensional aspects of proper and appropriate qualities,

⁶⁵ In the four-part Elemental Figure displayed in the section of color illustrations, these qualities are represented by the colors of the rows, with the first row of each quadrangle representing the quality proper to that element, and the second row the appropriated quality. Thus the proper quality of fire is heat (red) and its appropriated quality dryness (black), and so on. See again Dambergs 2000 for a detailed explanation.

⁶⁶ See p. 140 below.

and of concordance and contrariety, things which will be explained in greater detail in the chapter on Logic.⁶⁷

Finally, there were similarities of structure (both figures consist of four squares) and function between the Elemental Figure and Figure S which Lull exploits, especially in the transitional works studied in the next chapter. Even in the *AD* he says that “the Elemental Figure... is the mirror and image of S and its powers”.⁶⁸

The Universal or Demonstrative Figure

In the *AD*, for the first time, Lull introduces into the Art a revolving figure—and which is therefore *not* a graph.⁶⁹ It is the Demonstrative or Ninth Figure.⁷⁰

As he explains in the *AD*, “It is composed of six revolving circles along with a wheel situated in the middle in which there is a single triangle made to stand for the five triangles of T. It is made up of circles, each inside the other . . . , with a pin in the middle to keep the circles in place.”⁷¹ As can be seen in the illustration, immediately adjoining the central disk of Figure T, we have two circles each with the four elements. Then come two circles each with seven letters representing Figures A S T V X Y Z, and then two outer circles with the “Alphabet of S, and

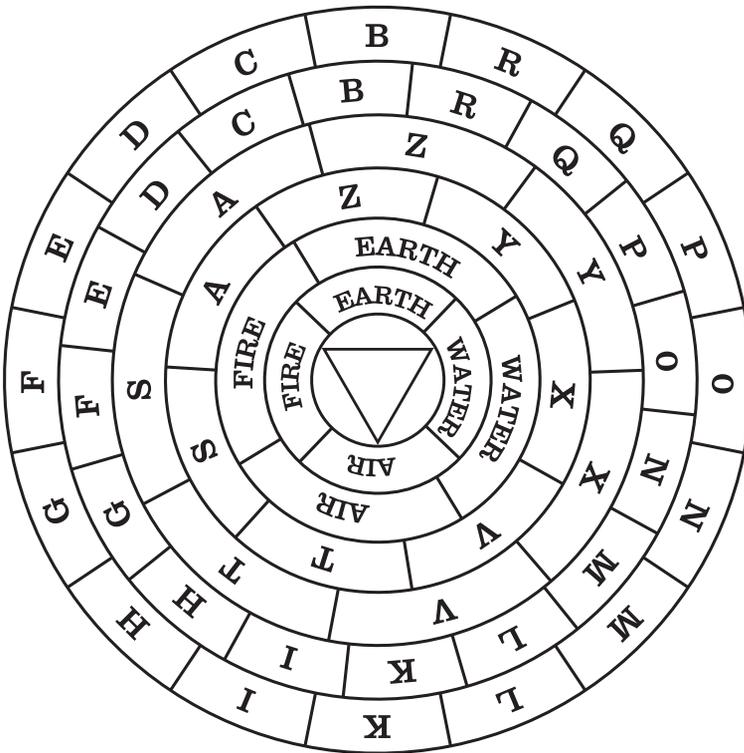
⁶⁷ See p. 196 below.

⁶⁸ *SW I*, 369 (with n. 57) and 376–8 (with nn. 69–70). Cf. Gayà 1979, 26–27 and 28–29. See Ch. 3, n. 34 below for a later presentation in the cycle of the *AD*. For a while there was a fairly heated controversy between Platzcek and Pring-Mill on which of the two figures came first and therefore inspired the other. See *SW I*, 309 and n. 9 for references, and Rubio 1997, 126–131 for a more recent and complete treatment of the subject.

⁶⁹ The reason Lull resorts to a revolving figure, as we said before, is that graphs can only deal with binary relations. With an n -ary relation of $n > 2$ (here $n = 6$), he has to resort to this kind of device.

⁷⁰ “Ninth” because it was preceded in the text by the seven figures of A S T V X Y Z plus the Elemental Figure. It was followed by the figures of the Principles of Theology, Philosophy and Law. In other works of the Quaternary Phase he calls it the “Universal Figure”.

⁷¹ *SW I*, 333–4. In manuscripts, in fact, these rotating figures were made by drawing the outside circle on the manuscript itself, and having the inner circles each on a separate piece of parchment one on top of the other, all with a string holding them in place, so they could be rotated one with respect to the others. In early printed editions, such as those of Zetzner (cf. Lullus 1996), the outer circle is printed in its proper place in the text, and the inner circles on an added back page, leaving the reader to cut them out and put them where they belong.



Demonstrative Figure

of A T V X, as well as of the Principles of Theology, Philosophy, and Law.” What he means by the last quotation is that the sixteen letters of B through R, can here not only stand for the concepts of Figure S, as they do in the Alphabet of the Art (see below), but also for those of any of the other seven figures mentioned. He is thus making use of the letters inscribed just inside the circumference of these other figures—as we said before, the only time he does so—to stand for the concepts themselves, expanding enormously the combinatory possibilities of the Art. Lull says that “this figure contains and includes all the other figures of this Art...; moreover, all the compartments of the other figures can be formed by revolving the circles of this figure in the right way.”⁷² It

⁷² SW I, 334.

is therefore a figure of figures. Moreover, its alphabet, as Llull himself suggests, can even be adapted to “many other principles of science” outside the realm of the Art.

The standard Second Demonstrative Figure of the *AD*,⁷³ as the reader can see, has twenty-eight compartments containing all possible binary combinations, with repetitions, of the seven letters A S T V X Y Z.⁷⁴

AA	SS	TT	VV	XX	YY	ZZ
AS	ST	TV	VX	XY	YZ	
AT	SV	TX	VY	XZ		
AV	SX	TY	VZ			
AX	SY	TZ				
AY	SZ					
AZ						

Note that this second figure corresponds to the middle two of the six rotating circles of the Demonstrative Figure. Llull has not given a second figure for the outer two, because it would merely repeat that of Figure S (see p. 47 above), nor one corresponding to the inner two circles, which would repeat that of the Elemental Figure (see p. 57 above).

This second figure is likewise a summation of all the previous second figures, one in which the compartments with repeated letters imply the possibility of expanding them to include the second figure of the letter repeated, so that the compartment of $\boxed{S\ S}$ or $\boxed{T\ T}$, for instance, could

⁷³ I say “standard” because it’s the one he presents in Distinction I of the *AD*, whereas the one he actually uses in Distinction II is an expanded version, based on other possibilities of the rotating figure. In subsequent works of the cycle of the *AD* he presents other variations. See for instance *Liber propositionum secundum Artem demonstrativam compilatus* (*MOG* III, viii, 7: 509), where it is called the “Figura Communis,” and the much more complicated version called the “Figura Universalis” in *Introductoria Artis demonstrativae* (*MOG* III, ii, 7, 12, 25: 61, 66, 79), along with the graphic figure preceding the work).

⁷⁴ $(7 + 2 - 1)! / 2! = 8 \cdot 7 / 2 \cdot 1 = 28$.

imply the Second Figures of S or T. Curiously enough, however, the Second Demonstrative Figure lacks any reference to the four elements present in the First Demonstrative Figure and which are used, along with this Second Figure, in the systematic organization of lists of proofs or explanations.

This brings us to a secondary but important use of the Second Demonstrative Figure. In Distinction II (“Conditions”) of the *AD*, for instance, the compartments of this figure are “conditioned”—or in other words, their use explained—by starting with the compartments of $\boxed{\text{A A}}$ followed by $\boxed{\text{A S}}$, and continuing systematically down the first column, then the second column, and so on, of the Second Demonstrative Figure, until he reaches $\boxed{\text{Z Z}}$. In the midst of this process, he stops twice, to use expansion devices at the head of the second and third columns. At $\boxed{\text{S S}}$ he goes through the shorter Second Figure of S, before continuing with $\boxed{\text{S T}}$, etc.⁷⁵ and at $\boxed{\text{T T}}$ he carries out an extraordinary investigation of each triangle of Figure T by means of each of its other four triangles, before going on to $\boxed{\text{T V}}$, etc.⁷⁶

Distinction IV (“Questions”) is organized similarly, but without the interruptions at $\boxed{\text{S S}}$ and $\boxed{\text{T T}}$. In both of these distinctions, however, he *precedes* these chains of compartments with those of the Second Elemental Figure, starting with $\boxed{\text{fire fire}}$, $\boxed{\text{fire air}}$, etc., making it even more surprising that the Elemental Figure goes unmentioned in his discussion of the Second Demonstrative Figure. As we will see, this systematizing use of the Demonstrative Figure will play a more important and somewhat different role with the Fourth Figure and the Table of the ternary phase.

Lastly one should keep in mind what, as we said above, Llull himself suggests, that the alphabet of this figure can even be adapted to “many other principles of science” outside the realm of the Art. This is important, because for Llull the Art is not enclosed in its own shell, but is full of further possibilities, of adaptations which the user is invited to try. The *AD* is thus a work which is open in the same spirit as Felix’s wish at the end of the *Book of Wonders* that those who had learned the proper use of his message should go forth proclaiming it to others. The user of the Art is invited to do the same.

⁷⁵ He uses here the abbreviated version of the Second Figure of S, the one with only ten compartments which follows the larger one on p. 47 above. See *SW I*, 364–379.

⁷⁶ *Ibid.* 385–392.

*The Prologue of the AD**The Alphabet of the Art*

In the *ACIV* Llull presents the Alphabet in the same way he does the terms, simply by referring the reader to the figures drawn amid the text. At the very beginning of the prologue of the *AD*, however, after two brief opening sentences, we are presented with:

A stands for God.

B stands for memory remembering; C for intellect understanding; D for will loving; while E stands for the act of B C D.

F stands for memory remembering; G for intellect understanding; H for will hating; and I the act of F G H.

K stands for memory forgetting; L for intellect not knowing; M for will loving or hating; and N the act of K L M.

O stands for the act of B F K; P for the act of C G L; and Q for the act of D H M; whereas R stands for the combination of O P Q.

S stands for the intellectual soul;⁷⁷ T for principles; V for virtues and vices; X for predestination or objectification; Y for truth; Z for falsehood.⁷⁸

The first thing to note about this alphabet is that the letters from B to R refer to concepts *interior* to Figure S, showing again the importance Llull concedes that figure in the quaternary phase. To put it another way, of the 23 letters of the alphabet, 16 are interior to one of the remaining 7.⁷⁹ The second thing is that those remaining seven refer to *entire* figures. This means that in the quaternary phase (as opposed

⁷⁷ As we saw above, n. 34 “Intellective soul” is synonymous with “rational soul.”

⁷⁸ *SWI*, 317. The original Latin (and Catalan, whenever they’re significantly different) terms are: “A ponitur esse Deus, B memoria recolens (membrant), C intellectus intelligens (enteniment entenent), D voluntas diligens (volentat amant), E autem ponitur esse actus ipsorum B C D, F ponitur esse memoria recolens, G intellectus intelligens, H voluntas odians (desamant), I vero actus ipsorum F G H, K ponitur esse memoria obliviscens, L intellectus ignorans, M voluntas diligens vel odians, N autem actus ipsorum K L M, O est compositum de actu ipsorum B F K, P es compositum de actu ipsorum C G L, Q est compositum de actu ipsorum D H M, R est compositum de actu ipsorum O P Q, S ponitur esse anima intellectiva, T vero principia (començaments), V virtutes et vitia, X praedestinatio seu objectio, Y veritas, Z falsitas” (*MOG* III, ii, 1: 93; *ORL* XVI, 3–4). See Ch. I, n. 55 above, for this Alphabet in the miniature of the *Breviculum* reproduced there.

⁷⁹ 23 letters represent the entire medieval Latin alphabet: our 26 minus any distinction between I/J or U/V, and lacking a W.

to the later ternary) Lull almost *never* uses letters to refer to any of the components of Figures A or T; with one exception, they are only referred to by their names (“goodness,” “concordance,” etc.).⁸⁰ So where in the ternary phase B can stand for “goodness” or “difference,” in the quaternary it can *only* stand for “memory remembering.” I belabor this point, because it has caused a certain amount of misunderstanding in the literature on the Art.

Methods of Proof

Immediately after this presentation of the Alphabet, the Prologue of the *AD* continues with:

There are three kinds of demonstration. The first is by equivalence (*per aequiparantiam*), that is to say, when a demonstration is made by means of things equal to one another, as for instance when one demonstrates that God cannot sin because his power is of the same essence as his will, which in no way desires to sin, and this will is of the same essence as justice, which is completely opposed to sin, which accords with injustice. And since the divine dignities are all equal in essence and nature, one can clearly construct a demonstration by equivalence; and the same follows for the virtues, properties, and entities of creatures.⁸¹

The second kind of demonstration is when an effect is proved by its cause, as in: when the sun is shining, it must be day. The third kind of demonstration is when the cause is proved by the effect, as: it is day, therefore the sun must be shining. It is with these three kinds of demonstration that this Art is constructed, and the first is stronger than the others, since it is based on the dignities of God; and the second is stronger than the last. (*SW I*, 317–318)

The last two are the classic Aristotelian proofs by cause (*propter quid*) and effect (*quia*). It is perhaps not out of place to insist that although Lull’s examples of these two look like the *topoi* of Stoic logic, such as one finds in Boethius and many others, they are fundamentally different.

⁸⁰ The exception involves Figures A, T, V, or X, which are indeed drawn with accompanying letters in the manuscripts, but this is solely for use in the Demonstrative Figure as we explained in the previous section. They are never used in the text, as they do not form part of the alphabet of the Art.

⁸¹ I have underlined the dignities. Their mutual concordance is what was represented by the cross lines of the graph of Figure A. But notice that the last sentence clearly indicates that this demonstration by equivalence is not usable only with the divine dignities. Notice too how the example he gives here involves the same problem of predestination vs. free will given in the our previous demonstration from the *ACIV*.

Stoic logic is formal propositional logic which would phrase the above second kind of demonstration as: “If the sun is shining, it is day. The sun is shining; therefore it is day”, which could be written: “If x then y , x , therefore y ”, or in symbols $x \rightarrow y$, $x \vdash y$, because the important point is the formal relationship between the various propositions of the argument, which could in fact be anything symbolized by x and y . Lull’s formulation is a logic based on meaning not on form. “When the sun is shining, it must be day” is valid because of our own observations about the causal relationship between sunshine and daytime. It has nothing to do with the form of its constituent clauses; to try to symbolize it at $x \rightarrow y$ would be meaningless.⁸²

It was these traditional demonstrations by cause and effect that Lull knew he had to avoid in dealing with the Divinity, which was why he developed the first kind, which he called the *demonstratio per aequiparantiam*. Once he had hit upon it, it remained absolutely central to his endeavors, and above all to his attempts to prove the Articles of Faith, and as such we will see it reappearing again and again in the course of this book.⁸³ Notice too how he says it is stronger than the other two; this does not mean that he disdains the other two, which he uses often when they are appropriate.⁸⁴ But it does allow him to step into realms forbidden to philosophers steeped in a more Aristotelian tradition. In fact one of his rare outbursts of anger is against people who are scandalized

⁸² Lull’s formulation, however, was not unique; for other medieval examples see Ruiz Simon 1999, 251 n. 301.

⁸³ Its first appearance with this name and as a basic tool of the Art is in the *AD*. It had, however, appeared in the *Liber principiorum theologiae* (*MOG* I, ix, 4: 610), where it was called *demonstratio necessaria* and only explicitly used in the little two-page distinction on the dignities in which it is formulated. A previous vaguely similar use, in which a demonstration based on infinite things is contrasted with two others based on inclusion, can be found in the *Compendium logicae Algazelis* (Lohr 1967, 118–119) = *Lógica del Gatzell* (*ORL* XIX, 30–32). The name might have been suggested by a passage from Peter of Spain (1972, 34–35) where it is used, albeit in a different context, but with some aspects remarkably analogous. There he gives three types of relatives, those that are *secundum aequiparantiam*, “such as similar being similar to similar, equal being equal to equal, a neighbor being a neighbor of a neighbor. Then there are those that are superior, such as lord, double, triple; others are beneath something else, such as a serf, one-half, one-third, since some are below others and others above them.” These last two relations could be considered analogous to *propter quid* as causing (from above), and *quia* as the effect (below, on the receiving end).

⁸⁴ This is part of what Ruiz Simon has called the “additive” nature of many aspects of the Art, for which see p. 296 below.

by the title *Ars demonstrativa*, assuming that he is referring to traditional Aristotelian demonstrative methods.⁸⁵

Conditions

In his little dictionary of philosophical terms called the *Taula d'esta Art*, Llull defines "condition" as "the mixing of principles, with some conditioned by others according to their definitions and properties."⁸⁶ In the quaternary phase there are as yet no definitions or explicitly presented properties of the basic terms of the Art, but this conditioning by means of their mixture is used throughout the Art. It gives the ground rules as to how to combine principles and what results are implied by such combinations, and is therefore essential to the functioning of the Art.⁸⁷ In the quaternary Art what is conditioned are usually the binary compartments of the second figures.⁸⁸ In its first cycle—that of the *ACIV*—no work is without them, and the *Ars universalis* is almost entirely devoted to "conditions": over a hundred pages with 464 second-figure compartments "conditioned". With the *AD* this is done by systematically introducing T into each compartment. Or as Llull explains at the

⁸⁵ The complaint is registered in a little note at the beginning of the manuscript of the *AD* which Llull sent to the Doge of Venice, for which see p. 187 below.

⁸⁶ Bonner and Ripoll 2002, s.v.

⁸⁷ An interesting suggestion of possible connotations of these conditions for contemporary readers has been made in Ruiz Simon 1993, 82–86. If the Art represents a Lullian recycling of the Aristotelian/scholastic topics, then Llull's conditions correspond to those which, according to Boethius of Dacia, distinguish dialectical (and demonstrative) syllogisms from the syllogism *simpliciter*, and the terms of the triangles of Figure T correspond to their "differences", so as to produce something equivalent to the maxims or maximal propositions of the topics. Ruiz Simon points out in this connection similarities to some of Aristotle's injunctions in the *Topics*: "Such then are the various ways in which you can argue from the greater and the less and the like degrees," "Next, you must argue from affirmations and negations," "For we say 'alien to' and 'contrary to,' but when we use 'different,' which is a genus of these terms..." (*Topics*, 115a25, 136a5 and 125a2).

⁸⁸ Sometimes they are unary, as in the *Principles of Theology, Philosophy, or Law*, but even there they are conditioned by the other principles of the Art. The *Principles of Medicine* (*SW* II, 1124–5) has some referring to the elemental degrees and others resembling those of the *Book of the Gentile*. This last work (*SW* I, 114–5) uses binary compartments (the 'flowers' of the trees), but conditions them by more general, or what one might call meta-Artistic, considerations (cf. n. 129 below). In any case, Llull cited them in many subsequent works (see Bonner 1978, 53 and *SW* I, 115, n. 17) and Le Myésier, in the margin of the *Electorium*, wrote: "In these conditions of the trees lies the entire virtue of all of Ramon's Arts" (see Hillgarth 1971, 386).

end of Distinction II of the *AD*, “Which Treats of Conditions”, this distinction shows and exemplifies “the art and doctrine according to which one knows how to put T into the compartments of this Art, with F G surveying the compartments through which they move towards the object of this Art.”⁸⁹

This conditioning by means of T, although not absent from works of the cycle of the *ACIV*, is extraordinarily systematized in the *AD*. The succession of compartments is taken from the Second Demonstrative Figure, along with the implied second figures described on p. 62 above. This means he starts off with the 10 compartments of the Second Elemental Figure implied in its two inner circles, and then does the 28 compartments of the Demonstrative Figure itself, but inserting, when he comes to $\boxed{S S}$, the 10 compartments of the abbreviated Second Figure of S. Each of the resulting 48 little chapters is divided into five paragraphs, corresponding to the five triangles of Figure T, which are “introduced” successively into the compartment being studied to reveal its meaning. To see how Lull uses this technique of “conditioning”, we will give examples corresponding to each of these three sections.

Some Examples of the “Conditions” of the AD

First an example from the first section based on the Second Elemental Figure:

The compartment of $\boxed{\text{Fire Earth}}$.

We will take the second paragraph, where Lull introduces into the compartment the green triangle of “difference” (= “diversity”), “concordance,” and “contrariety.”

2. There is diversity between fire and earth in that fire receives and earth gives; for fire evacuates earth by receiving dryness from it, and earth fills up fire by entering into it. Earth, when it proceeds from fire and enters air with heat, is concordant with the action of fire, with which fire it is contrary when it enters it with cold and when the fire itself enters into the air with dried heat. (*SW I*, 343)

The first sentence explains the active/passive nature of elements depending on their place in the square of the elements as represented on p. 59 above. The second sentence involves the secondary actions of

⁸⁹ *SW I*, 414. For the role of F G as the principal investigative tool of the *AD*, see p. 46 above.

the qualities shown by the curved lines in the same figure, with fire transmitting (along with its own heat) the dryness of earth to air, and earth transmitting the cold of water to fire.

Now some examples from the Second Demonstrative Figure:

The compartment of A A.

The fifth section of this compartment corresponding to the black triangle of Figure T, that of “affirmation,” “doubt,” and “negation” reads:

5. A exists, for if it did not exist, no good would exist without evil, no greatness without littleness, no eternity without beginning; and the same thing would be true of perfection, which would not exist without imperfection, nor would justice exist without injustice, nor nobility without baseness, and so on for the others. But since goodness, greatness, etc. are concordant with being, and their opposites with privation, therefore one should not doubt that A exists, nor should one deny the existence in it of goodness, greatness, etc.; because if there were no goodness, greatness etc. in A, then it would be impossible for A to exist, since this existence is in accord with no being in which there is not immense goodness, greatness, etc., and in which, through bonification, there is no goodness in greatness, nor, through magnification, any greatness in goodness, and so on for the rest, which bonification is so great, etc., and which magnification is so good, etc., that it could only accord with a being that is A.⁹⁰

The first sentence is essentially an affirmation (based on the impossibility of negation) of A's (God's) existence. The beginning of the second sentence (“But since goodness...”) discusses the problems of doubt, and the second part of the same sentence (“because if there were no goodness...”) discusses the problems of negating that existence.

I chose this paragraph not only as an illustration of what Lull means by “putting T into the compartments of the Art,” but also because, it will permit us to understand the significance of the first compartment of a proof we will analyze later on,⁹¹ where he uses what he has explained here to deduce the consequences of the affirmation or negation of A's existence, or in other words as the starting point for his proof of the existence of God.

Now an example from the mixing of the “species” of Figure S:

⁹⁰ *SWI*, 356. Note this foretaste (here still only used with the Divinity) of the cor-relatives which will be explained in the next chapter.

⁹¹ See p. 80 below.

The compartment of $\boxed{\text{E I}}$.

I have chosen this one to illustrate how the passage quoted on p. 18 above is easily resolved, and how Lull uses the shorthand device explained in n. 17 above, of omitting the color (blue or red) with V when the context makes it clear whether he is speaking of virtues or vices. Here he introduces the red triangle (here using its terms in the sense of “origin”, “means” and “goal”) into the compartment.⁹²

3. The goal of E is A V Y, and thus E is the origin of I, like the cause of an effect, with A, however, existing as the means, through creation and the influence of grace, by which E sets I against V Z; but when E is in V Z and I in A V Y, then E is the origin of I, contrary to the purpose for which I exists, and I is then the means by which E exists contrary to its own origin and purpose.⁹³

This a further example of how S does not contain a set of concepts with fixed applications, but how it can represent the flexible, changeable nature of the rational soul, depending on the goals to which it is directed, and its attitude towards those goals.

And finally an example of the combination of two figures, and of a new treatment of the problem of predestination and free will.

The compartment of $\boxed{\text{S X}}$.

We will take the fourth section devoted to introducing the yellow triangle of “majority,” “minority,” and “equality” into this compartment. I will follow concepts with superscript letters to remind the reader to what figures they belong.

4. Predestination^X is greater^T in E^S than free will^X is in N^S according to object^X, reality^X, and reason^X, and because of this majority^T and minority^T N^S is more inclined to deny free will^X than E^S is to transform itself into N^S. And the same is true when E^S takes free will^X and N^S predestination^X as their objects^X, with S having Y through majority^T at one moment, and Z through minority^T at another. And since all of A cannot fit into S, E^S is transformed into N^S, and N^S into E^S by means of the black triangle^T and the compartment of $\boxed{\text{predestination free will}}^{\text{X}}$; moreover the equality^T of the yellow triangle^T consists in the very transformation of E^S into N^S and N^S into E^S, with S changing its objects^X which at one and the same moment cannot be included in the second acts of S itself, and which

⁹² See n. 28 above.

⁹³ *SW* I, 367. The compartment of $\boxed{\text{E I}}$ of the *Ars universalis* has a similar discussion, but there in terms of Lull’s doctrine of first and second intention (*MOG* I, viii, 20: 502).

cannot enter or remain, in different ways, beneath a single species of S. (*SWI*, 382)

Notice the evolution of the argument from that of the earlier version presented above. One can still detect similarities to the stages of the previous argument, but since the version of Figure X presented in *AD* no longer has the terms of “wisdom” and “justice” it had with *ACIV*, Llull has had to modify the basis of his argument.⁹⁴ As for the “second acts”, in the *Logica nova* Llull explains that a first act is when something potential becomes actual (as when the tree existing potentially in the seed actually becomes a tree), and a second act is when that thing carries out its destined operation (as when the tree bears fruit, or as is the case here, the intellect understands, the will loves, etc.).⁹⁵ The crux of the problem is, as he states here, because “all of A cannot fit into S,” which means that although S can understand both sides of the question, it cannot do so *simultaneously*. In other words there are things about God beyond the normal understanding of the rational soul, and that can only be comprehended, as Llull shows in other places, by S attempting to exceed its own limitations.⁹⁶ When we come to the Questions of the *AD*, we will see how the rational soul’s inability to grasp both sides of the question is explained in a somewhat different way.⁹⁷

This should be enough to give the reader a feeling of how Llull “conditions” each of the compartments, to launch the reader on a further step of his user’s manual.

⁹⁴ See above, p. 39, how they are present in the list of *ACIV* and absent from that of *AD*.

⁹⁵ *ROL* XXIII, 19; *NEORL* IV, 7.

⁹⁶ This negative feeling about the ability of the rational soul to solve the apparent contradiction of simultaneously understanding predestination and free will is to a certain extent limited to the *AD* and perhaps to its immediate satellite works. We have seen how he solved it through four dialectical movements in the previous *ACIV* and its satellite works, and then at the end of the quaternary and the beginning of the ternary phase (from the *Compendium seu commentum Artis demonstrativae* and *Quaestiones per Artem demonstrativam seu inventivam solubiles* of 1289, to the *Declaratio Raymundi per modum dialogi* of 1298) he solved this problem through what he called “transcendent points”, which permit the intellect to rise above the other senses, or even above its own natural capacities. See Ch. 5, n. 170 below for a brief explanation. The best modern treatment of the topic is in Ruiz Simon 1996, 28–32; 1999, 159–168; and 2005, 190–1 (this last for its origin in Proclus and Pseudo-Dionysius). See also Carreras y Artau 1939–43, I, 545–6.

⁹⁷ See the passage corresponding to n. 134 below. Ruiz Simon has pointed out to me that after the presentation of the Second Figure of X in the *AD*, Llull gives the crux of the problem: “Predestination and Free Will are concordant in reality through A, whereas they are opposites according to C L”, which is remarkably similar to the solution offered at the end of the discussion of the “four figures of X” in the *ACIV* (see p. 51 above).

Intentions

The next step of his user's manual, "the Third Distinction, which treats of intention," begins with the following explanation:

Since this distinction concerns intention, we should begin by explaining that there are two kinds of intention, first and second. First intention is final cause, and second intention is that which is directed towards the end, which second intention accords with beginning and middle, while the first accords with final cause. And thus first intention accords with majority and second with minority, through which majority and minority the beginning is impelled through the middle towards the end. This being the case, the object of this Art is therefore the end, that is the first intention, which end is to know and love A, while the second intention exists in the terms of this Art, insofar as it accords with them.⁹⁸

The second intention is thus the instrument or means which permits one to arrive at the first intention, the final cause or goal. This formulation of Llull's is quite different from that current in medieval logic, where a term of first intention was a concept, such as "animal" or "man", and one of second intention was a concept of a concept, such as "genus" or "species. Llull's use of the terms is consonant with that in Arabic thought, a usage which he might have amalgamated with scholastic reflections on natural science originating in Aristotle's *Physics*, and especially section II, 8, with its references to "final cause" (as in the previous quotation), and with the example from a tree and its fruit (as in the following quotation).⁹⁹ The Art is thus not only the instrument (second intention) showing the reader how "to know and love A" (the first intention), but also that for explaining the final causes of each of the sixteen modes listed below.

Usually these intentions just involve two steps, but it can also work as a ladder with many steps:

⁹⁸ *SW I*, 415. Llull wrote an entire book on the subject: *Libre d'intenció*, *ORL XVIII*, 1–66; *Liber de primera et secunda intentione*, *MOG VI*, ix: 537–560. Notice how here Llull explains the difference between the two intentions using the red and yellow triangles (along with "concordance" from the green triangle) of Figure T.

⁹⁹ For an Arabic source for Llull's use of the concept of two intentions, see Urvoy 1980, 146–7, 345, and Lohr 1986a, 14–15, and for the use of the concept in scholasticism (and occasionally, but more rarely, the same terminology in this sense), see Ruiz Simon 2002, with a title taken from the same passage of the *Tree of Science* quoted in the next paragraph.

such as the tree which exists for the sake of its fruit, and the fruit which is produced so that man can live from it, and man who is there to know, love, praise, honor, and serve God.¹⁰⁰

The mechanism can even bifurcate, as in the following more important theological example:

The purpose of the Incarnation is so that God become man, and this is the first and principal intention why God is incarnate. And the second intention is so that man become God, and this second intention is first in comparison with the redemption of mankind, which redemption is by second intention.¹⁰¹

This kind of presentation allows Llull to avoid the either/or dilemmas so frequent in scholastic discussions which oblige medieval thinkers to take positions on one or the other side. It permits him to say that both can be valid, but within an acceptable scale of values. In this way he can relativize certain doctrinal confrontations, or indeed even make them unnecessary by replacing them with what Robert Hughes has aptly called an “asymmetrical synthesis”.¹⁰²

This doctrine of the two intentions is of prime importance for Llull, and he repeatedly claims that many of the problems of the world come from people confusing one for the other (their worldly for their spiritual well-being, etc.). His most basic formulation, one essential to his entire endeavor, and one that he repeats in many places in his works, is:

The ultimate reason, that is, the principal reason why God created man is so that man can know and love God. And God’s second intention in creating man is so that man can participate in glory with God eternally without end.¹⁰³

This holds for most of Llull’s efforts—including, of course, the Art—their primary purpose is to try to induce people to know and love God, to which personal salvation is secondary.

¹⁰⁰ *Tree of Science*, Elemental Tree, Flowers, Hundred Forms, no. 45.

¹⁰¹ *Disputació de cinc savis*, ed. Perarnau, ATCA 5 (1986), 75.

¹⁰² Hughes 2005–6, 18, dealing precisely with the problems of the Incarnation treated in the previously quoted passage.

¹⁰³ *Book of the Gentile and the Three Wise Men* (SW I, 199–200). The *Llibre d’intenció* in the note above begins with the same formulation. In the *Book of the Lover and the Beloved* (DI, 217) the beloved complains about believers who either fear him because of possible eternal punishment or love him hoping to be granted glory, whereas hardly anyone loved him purely for his goodness and nobility. See Bonner 1993, 20–21.

What he does in this Third Distinction of the *AD* is to pick sixteen intentions, or “modes” as he calls them here, applications or uses to which the Art can be put. Each mode begins with words about “how to”, “instructing to”, or that such-and-such an Artistic combination “shows how to” correctly carry out Remembering, Contemplating, Preaching, etc. These are the second intentions showing the reader “how” to tread the correct path leading to the mode of the first intention.

After giving the following list of sixteen modes, Lull adds that “many other modes can be found in this Art, modes which, by the example of those given here, can be understood according to the doctrine and rule of the Art”, which is yet another example of his *opera aperta*.¹⁰⁴

- | | |
|------------------|-----------------|
| 1. Remembering | 9. Interpreting |
| 2. Understanding | 10. Solving |
| 3. Willing | 11. Judging |
| 4. Believing | 12. Teaching |
| 5. Contemplating | 13. Disputing |
| 6. Discovering | 14. Counseling |
| 7. Guiding | 15. Accustoming |
| 8. Preaching | 16. Healing |

Some Examples of “Intentions” in the AD

Before giving our examples, I recommend that the reader keep to hand the fold-out chart between pages 92 and 93, so as to be able to sort out the meanings of the chains of compartments with their explanations. It should also be kept in mind that the binary compartments have been “conditioned” by the corresponding section of the *AD*, a few examples of which will be found above. For a “conditioning” of all the compartments, the reader must consult the long Distinction II of the *AD*.¹⁰⁵

The first three “intentions” or “modes” of our previous list are, of course, the powers of the soul. But Lull is not interested here in embarking on yet another presentation of Figure S, but rather in explaining

¹⁰⁴ *SWI*, 415. For his *opera aperta*, see pp. 92, 293–6 below. In the *ACIV* Lull similarly starts off a list of 16 “universal modes” with an explanation of first and second intention, but there the modes are philosophical or theological, not strictly speaking uses to which the Art can be put (see *MOG* I, vii, 11–17: 443–9). They are followed by a list of 30 “special modes” which are uses of the Art, many of them identical to those of the *AD*.

¹⁰⁵ *SWI*, 338–414.

the proper use of each of these powers in accordance with the primary purpose for which man was created: to remember, understand, and love God. The second mode of “understanding”, for instance, he investigates using eight compartments: fire earth | A S | S S | S T | S V | S X | S Y | S Z, which he treats one by one. Under the first compartment he discusses B and C (remembering and understanding from Figure S) in terms of medieval medical theory, pointing out the damage that can be done to them by excessive heat and dryness. Under the compartment of S X, Llull discusses again the business of predestination vs. free will, giving us now, as Carreras y Artau pointed out, a foretaste of the doctrine of the transcendent points.¹⁰⁶

By the compartment of S X C knows how to understand loftily in X when it understands the compartment of predestination and free will, in which C understands that perfection exists, as well as understanding that it does not understand all the perfection of the compartment which N loves. And thus, by the unsurpassed understanding that S has in C, as well as by the supreme will that it has in N, it knows how to achieve very subtle understanding of the other compartments of X, by using T to survey them. (*SW I*, 419)

Mode 5, “Contemplation”, treats an aspect of Llull’s endeavor that was never overlooked amidst all the other aims of the Art.

In order for a person to contemplate A, F G must examine the compartment of T T, so that, in accordance with the rule of the triangles, E may examine the compartment of A A, H may examine the compartment of V Z, M may surpass D in loving A, and R may be destroyed by the compartment of S S in order for the compartments of X X and Y Y not to be absent in contemplation. (*SW I*, 422)

Mode 8, “Preaching” treats the application of the Art to a subject Llull will deal with frequently in his output.

This Art provides instruction for preaching, as is indicated by the following compartments A S | A T | A V | A X | A Y | A Z | fire air. The first of these means that E has as object the compartments of A, and this compartment denotes the third, with E having blue V and I red V. The fourth compartment signifies how X affords material for the preacher, so that he may praise A, according as is indicated by the fifth and sixth compartments. The seventh compartment signifies how the preacher presents examples in accordance with the Elemental Figure, while the second compartment

¹⁰⁶ Carreras y Artau 1939–43, I, 545 n. 50. Cf. n. 96 above.

signifies how T exists in all these above-mentioned compartments for the purpose of giving knowledge and love of A. (*SW I*, 428)

Notice how he uses Figure X, with its contrasting concepts, to give the preacher material with which to work, and how he suggests examples from elemental theory, something he will work out so extraordinarily in the “Arbre exemplifical” of the *Tree of Science*.¹⁰⁷

Mode 9, “Interpreting”, follows on that of “Preaching” since it is the classic medieval technique for the construction of sermons on *themata*, but it is one which Lull will, as usual, work out in his own way.¹⁰⁸ Here he explains that:

This Art has been devised for the interpretation of the Scriptures of A, which texts can be interpreted according to the signification of the following compartments: E A V Y | I V Z | M A V Y Z. The first of these indicates that the interpretation should be such that E has A V Y as object, for no Sacred Scripture contradicts the compartments of A, blue V, or Y, nor is it against the E which is a creature so that it may have A V Y as object and impel I toward V Z, which are in disaccord with A V Y. (*SW I*, 429)¹⁰⁹

Mode 13, “Disputing” begins one of its sections with a particularly blunt statement about the use of authorities:

In every disputation one must necessarily dispute by authorities according to N, and by reason according to E I. (*SW I*, 433)

Or, in other words, arguing by authorities, people neither remember nor understand the subject at hand, and they either love or hate it (all this from N) depending which side of the fence they’re arguing from. Disputing by reason, on the other hand, makes them confirm arguments which are positive (E) or disprove those which are negative (I).

Mode 16, “Healing”, makes clear something we mentioned earlier:

This Art provides instruction whereby a person can know how to heal physically and spiritually, as is indicated by the compartments of

¹⁰⁷ The connection between elemental exemplarism and preaching is explicitly stated in the little prologue to the “Arbre exemplifical”.

¹⁰⁸ See Bonner 1993 and 2005.

¹⁰⁹ The reader should remember what was said in n. 17 above, that V, when coupled with E A Y, refers to the virtues (the “blue V” of the example here), and when coupled with I Z, to the vices.

fire water R E I N E V Z I A V Y M A V Y Z, which compartments signify bodily and spiritual illness, as a result of which signification, health is signified by the contraries of these compartments. For the first of these compartments signifies that fire and water are contrary to one another, which contrariety causes physical illness without the intermediate tempering of air and earth...

The second compartment signifies illness of S, since R, in proportion to what it contains of E I N, represents the imperfection of said E I N... (SW I, 436)

This is just a small portion (each one occupies a page or more) of six of the sixteen “modes” or applications to which the Art can be put, but it is perhaps enough to give the reader at least a taste of how this aspect of Llull’s system works.

Questions

In the *AD* Llull states that the fourth distinction is that of “questions in which the Art is exemplified”, a statement he repeats in many places.¹¹⁰ In the ternary period he goes so far as to define the Art as “a general artifice for the solving of questions”.¹¹¹ So the questions with which every work of the Art ends, and which occupy between one quarter and two-thirds of each of these works, are by no means minor afterthoughts, as they are sometimes treated; they are instead a principal goal of the Art, and this in two ways.¹¹² They are first of all the area in which the Art is worked out or “exemplified”, they are the instrument for studying and practicing the Art, thus making it, as Lola Badia has pointed out, ‘interactive’, a remarkable quality for a medieval system. In the second place, the questions show the range of

¹¹⁰ SW I, 318. See also the *Ars inveniendi particularia in universalibus* (MOG III, vii, 1: 453), *Liber propositionum secundum Artem demonstrativam compilatus* (MOG III, viii, 40–41: 542–3), *Compendium seu commentum Artis demonstrativae* (MOG III, vi, 1: 293), and *inventiva veritatis* (MOG V, 2).

¹¹¹ *Lectura Artis quae intitilatur Brevis practica Tabulae generalis* (ROL XX, 344, where he repeats the statement twice on the same page). See the similar formulation at the beginning of the *Ars brevis* (SW I, 579; DI, 297). I use the less idiomatic “solving” instead of “answering” to indicate that what Llull offers are reasoned solutions, not just “yes” or “no” answers.

¹¹² As examples of the percentages of works they occupy: *AGU* and *AB* 25%, *Tree of Science* 41%, *AD* 54%, *TG* 62%. Works have occasionally been edited without them, as if they were appendages to a theoretical treatise, rather than the other way around, where the Art is a practical treatise which gives a method for answering the questions.

problems to which the Art can be applied, and how it can be used to solve them.

Answering the questions constitutes the investigative procedure by which the particulars (the subjects of the questions themselves) can be found (this is the *inventio*) in the universals (the terms or combinations of terms—i.e. the compartments—of the Art).¹¹³ Notice too how they conform to the black triangle of Figure T in resolving doubts by affirming or denying the question posed. Finally it should be pointed out that Llull is here following the standard scholastic practice whose texts are for the most part, formulated as a series of questions asking “Whether such-and-such is the case”. One has only to open to almost any page of Aquinas’ *Summa theologica* to see the predominance of questions beginning with *Utrum*.¹¹⁴ There, however, the similarity ends, since Llull precedes his questions with a long theoretical treatise (the text of the Art) explaining his new method, which is what he then uses to answer them.

In the *AD*, Distinction IV, after a little prologue with the single question as to “Whether this Art is demonstrative?” which we will look at on the next page, is divided into two parts, the first with 38 questions answered by chains of compartments followed by detailed explanations, and the second one with 1041 questions answered only by chains of compartments. The first part follows the model of Distinction II which “conditions” it, and is thus organized according to a pattern in which the first (binary) compartment of each answer follows the same pattern described on p. 67 above, giving us two sections, one based on the 10 compartments of the Second Elemental Figure, and another based on the 28 compartments of the Second Demonstrative Figure.

Near the prologue to this Distinction, Llull gives us a little explanation on how to use the chains of compartments:

It is the condition of the compartments by which questions are solved that every compartment following the first must be directed toward that first compartment through concordance or contrariety, and this rule holds for all the questions presented here. (*SW* I, 439)

¹¹³ For the business of “particulars” and “universals” in Llull, see Ch. 6, n. 47 below.

¹¹⁴ “Whether” is by far the most usual manner of formulating questions, but Llull sometimes also asks “what?”, “in what way?”, or “which (of several possibilities)?”, etc. With the ternary Art this mechanism becomes codified with the ten Questions and Rules.

This means that the first compartment of the solution, which is taken from one of the compartments “conditioned” in Dist. II, is fundamental, and the following compartments are related to it through the green triangle of Figure T, with its concordance and contrariety.

Finally, he ends his little prologue to Dist. IV with a general, but important question.

Question: Whether this Art is demonstrative?

Solution:

E A V Y	I V Z	M A V Y Z	R A V Y Z
---------	-------	-----------	-----------

.

F^S remembers that, with G^S surveying A S T V X Y Z, the first two compartments accord with the compartment of majority end^T whereas the last two accord with that of minority doubt^T, and thus C^S understands that this Art is demonstrative. For if it were not, the first two compartments would not surpass the last two in dignity, which surpassing is self-evident in this Art, in which each universal is sufficient for the finding of many particulars in said universals by necessary reason, as we have shown in many places in this Art. (Ibid.)

The first two compartments express symbolically the aim of the Art, to get the user, or rather the S of the user, to remember, understand and love (E) God (A), virtues (blue V), and the truth (Y), while remembering, understanding, and hating (I) the vices (red V) and falsehood (Z). With the last two compartments he either has (not necessarily reliable) beliefs (M) or is confused (R) about God’s (A) connection to virtues and vices (V) as well as to truth (Y) and falsehood (Z).¹¹⁵ This is why the first two compartments are in “majority” with respect to the “minority” of the last two compartments, as well as being the “end” of the Art as opposed to the “doubt” of the last two. This condition, which must be true for any argument involving the divinity, is the basis for the demonstrative nature of the Art.¹¹⁶ As we said before, the “universals” are the terms of the Art (or the combination of same in a “compartment”), and the “particulars” are the things that can be deduced from them. And it is this argumentation, which often proceeds, as in this case, by *reductio ad absurdum*, which produces what he calls his “necessary reasons”. All this will be discussed in greater detail in the following section where we analyze

¹¹⁵ Note that when V is followed by both Y and Z, it could refer to either virtues or vices.

¹¹⁶ This condition could be read in line with the Neoplatonic conditions of the *Book of the Gentile*, for which see notes 88 above and 129 below.

A sample question from the Demonstrative Figure

We will begin studying how Lull answers the questions of the Art by analyzing in some detail the first question from the Demonstrative Figure:

1. *Question:* Whether God exists?

Solution:

A	A		being	perfection		privation	imperfection		S	V		Y	Z
---	---	--	-------	------------	--	-----------	--------------	--	---	---	--	---	---

.

When F^S remembers this question, G^S examines the first compartment and then the others that follow it, and by their meaning it understands A to exist in this way, namely, that if A exists, the second and third compartments are contrary^T to one another, whereas if it does not exist, imperfection^X accords^T with all being^X, and all perfection^X accords^T with privation^X, which accords^T with all imperfection^X. This, however, is impossible, and therefore there exists some perfection^X and some being^X without privation^X and imperfection^X, and such perfection^X and being^X is A.

It is, moreover, evident from the fourth and fifth compartments that A exists, for otherwise it would follow that E^S could accord with V Z and I with V Y. Since, however, E^S accords with the second compartment and I^S with the third, this is impossible, for then majority^T would accord with the third compartment and with V Z, and minority^T with the second compartment and with Y, which is clearly impossible. (*SW I*, 444)

Lull's answer offers two demonstrations of God's existence. The first begins with the usual presentation of F G,¹¹⁷ implying that F remembers the previous conditioning of the first compartment of

A	A
---	---

, where the introduction of the black triangle of Figure T (affirmation, doubt, and negation) was used to discuss problems of the existence of God.¹¹⁸ The demonstration then applies two concepts of the green triangle of Figure T, "concordance" and "contrariety", to two pairs of opposite concepts from Figure X, "being"/"privation" and "perfection"/"imperfection". The demonstration of the first paragraph is divided into two parts, a positive and a negative one. The positive one says that if A exists, then there exists a being in which the second and third compartments are contrary to one another, that is to say a being without any privation (a synonym of non-being), and whose perfection is without any imperfection, which, clearly, is as it should be. The negative one says that if A does not exist, then no being is perfect, which means that all being

¹¹⁷ See p. 46 above.

¹¹⁸ See p. 69 above.

has some (that is, accords with)¹¹⁹ imperfection, and all perfection has some (that is, accords with) privation (or non-being), which last accords with all imperfection. The contradiction of perfection according with imperfection allows us to conclude that “there exists some perfection and some being without privation and imperfection, and such perfection and being is A.”

The second demonstration uses a similar argumentation, but now using the last two compartments, breaking down the S of the fourth compartment into its “species” of E and I, and introducing the two concepts of “majority” and “minority” from the yellow triangle of Figure T. The reader should also remember the contextually alternating referent of V, where V Z means “vices and falsehood” and V Y “virtues and truth”.

Notice too how this demonstration is based on the general condition mentioned above, “that every compartment following the first must be directed toward that first compartment through concordance or contrariety.”¹²⁰

If now we try to analyze the techniques Lull is using here, we will see first of all that he is not using the standard scholastic technique of answering the question of “Whether” (*Utrum*) by explaining what others have said about it, answering their arguments, and then drawing a conclusion (*determinatio*).¹²¹ Nor is he working according to the Euclidean model, where a group of preestablished principles are used to prove successive new principles (theorems). Instead he presents the question as a hypothesis, and draws out the implications of assuming this hypothesis to be true or false. The positive one merely shows that the argument is valid, and that therefore the premise leads to no inconsistency, whereas the negative form uses the classic *reductio ad absurdum* to show that if the negation of a premise leads to an impossibility, this proves the premise. Rather than using principles to work towards the thing to be proved, he is, as it were, working backwards, starting with the hypothesis to test it against the principles.¹²²

¹¹⁹ A synonym, as should be clear, of “is concordant with”.

¹²⁰ See p. 78 above.

¹²¹ The final *determinatio* can use methods of argumentation such as the *reductio ad absurdum*, but Lull’s is on a different basis, and lacks the whole stage-setting, as it were, of the for-and-against previous authorities.

¹²² For an analysis of these statements from a logical point of view, see Chapter 6, p. 272 below.

This is why the role of “supposition” (*sub-positio* is the literal Latin translation of the Greek *hypo-thesis*) is so important in Llull.¹²³ Note that in the *AD* it forms, along with “demonstration” one of the opposing pairs of Figure X; or, as he puts it in that work, “supposition is the beginning and demonstration the end” (*SW* I, 407). Note too that it uses the letter N from Figure S, which assumes no previous recollection or knowledge, but is open to “loving or hating”.¹²⁴ It is through supposition that “doubt” can be replaced by “affirmation” or “negation”, all of the black triangle of Figure T.¹²⁵ Finally, Llull often equates supposition with belief or faith, and this in two senses. The first is that no debate is possible if there is no faith in the possibility of the supposition proposed. Or, as he puts it, “faith and supposition are the light and path of the exaltation of the intellect”.¹²⁶ The second is that this is what permits him to take the Articles of Faith, Biblical passages, Church fathers, etc. as suppositions that must be true, and since any truth must be affirmable or demonstrable as such (otherwise it would be an inferior truth), this is what he proposes to do with the Art.

It should also be clear how much the argument is based not on individual terms or concepts, but instead on successive comparisons of terms or groups of terms (i.e. compartments) by means of components of Figure T (concordance, contrariety, majority, minority). It is not a proof which studies one thing as the cause of another (*propter quid*) or of the second as the effect of the first (*quia*), but one which operates *per aequiparantiam*, i.e. investigating the relationships of terms which are “equiparant” or of similar weight.¹²⁷ At the highest level this means the comparison of the “equiparant” divine dignities, but in fact it works at other ontological levels, as we can see by the chains of comparisons in this demonstration. Llull can even include unequal terms in this method, by the simple device of pointing out their inequality through the use of the appropriate term of Figure T (“majority”, “minority”, etc.).

¹²³ Llull’s use is unrelated to the medieval (semantic) theory of supposition which has to do with what a term stands for or refers to; it is much closer to the modern meaning of “hypothesis”. He later codified it under the *demonstratio per hypothesis* (see p. 233 below).

¹²⁴ See the *Lectura super figuras Artis demonstrativae* and the *Compendium seu commentum Artis demonstrativae* (*MOG* III, iv, 21: 225, and vi, 3: 295 respectively).

¹²⁵ See the first of the two passages cited in the previous note. The most complete modern treatment of Lullian supposition is Rubio 1997, 94–101.

¹²⁶ *Ars inveniendi particularia in universalibus* (*MOG* III, vii, 4: 456). See also *Introductoria Artis demonstrativae* (*MOG* III, ii, 6: 60) where he equates supposing and believing.

¹²⁷ See Ruiz Simon 1999, 238ff.

Although these comparisons are not based on meaning in any classical definitional sense, they do use something quite similar to the semantics of modern logic. Just as there any proposition can be given a truth value, or in other words can be mapped onto T or F, 1 or 0, with Llull concepts or terms (not propositions) can be assigned a kind of ontological—moral positive or negative value, which in Platonic (or Neoplatonic) fashion could be mapped onto Being or Privation. As propositions in modern logic which have the same truth value are called equivalent, so concepts of the Art which coincide ontologically—morally are called concordant. Notice how in our demonstration “perfection” accords with “being” and “imperfection” with “privation”, and of course, the first two are concordant with (blue) V and Y, and the second two with (red) V and Z.¹²⁸ This mapping is not usually spelled out or even mentioned, but simply given as something to which any thinking Christian, Muslim, or Jew would assent without difficulty.¹²⁹

These Lullian demonstrations, therefore, test an original hypothesis (i.e. the particular being sought) solely with components (or universals) of the Art. This is what makes his system both “compendious” and “inventive”, or what we would now call “generative”. But it is generative not in the sense of building complex concepts out of simple ones, which was the Leibniz program; it is generative only in the sense that all manner of demonstrations can be generated from a finite, limited set of ‘primitive’ concepts, those displayed in the figures of the Art. In the example under discussion he uses 13 components of the quaternary Art, 9 from the chain of five compartments, and 4 from Figure T.

Another aspect of Llull’s demonstrations in the quaternary phase should be emphasized, which is the presence of Figure S. It is limited

¹²⁸ And thus ultimately mapped onto truth or falsehood (Y or Z). It should be remembered that for Llull, as with most Christian Neoplatonists, sin and non-being accord, meaning that sin does not exist in itself (God could not have created it), but rather as a deviation or insufficiency of virtues (the standard Neoplatonic trope was *malum privatio est*). Notice how in the fifth miniature of the *Breviculum* the virtues are represented by labelled human figures, whereas with the vices the figures have disappeared, leaving just the labels. For a good summary of the Neoplatonic position, see Scouteris 1989. On the positive side, Martin 2004, 32, says that “The [Neoplatonist] tradition is notorious for conflating with reality ideas that modern philosophers carefully distinguish: necessity, spirituality, moral goodness, beauty, substantiality, eternity, truth—to list a few.” See also *DI*, 179.

¹²⁹ There are exceptions such as the ten (or twelve) “conditions” of the tree in the *Book of the Gentile and the Three Wise Men*, where the comparisons are “conditioned” in a clear Neoplatonic hierarchy. See n. 88 above.

in the demonstration we have just presented, but usually much stronger, as we shall see in others (we saw its extraordinary importance in the arguments concerning predestination and free will presented earlier). This presence of (the acts of) the three powers of the soul as mapped out in Figure S guarantees an unusual psychological component to his demonstrations. They are not purely intellectual; the intellect is invariably accompanied by the memory and the will, which last can sometimes be quite as crucial in charting the course of the argument.¹³⁰ Behind this, of course, is Lull's insistence that faith (allied with the will) and reason (guided by the intellect) should not be separated, but worked out in what one might anachronistically call holistic terms. Neither can work without the other, and one of the tasks of the Art is to act as a guide in how to combine the two.

As to the relative importance of these two powers of the soul, Lull says in the *Book of the Lover and the Beloved*:

The lover asked Understanding and Will which one was closer to his beloved. They both started running, and Understanding reached his beloved before Will. (*DI*, 192)

This is confirmed by the fifth miniature of the *Breviculum* in which the three powers of the soul (along with the seven virtues below them) hang by ropes held by the hand of God atop the "tower of faith (*fiduciae*) and eternal truth, love, and science". All three are represented as human figures, but Intellect is in the center, higher up (i.e., nearer to God), larger, and endowed with angel's wings, whereas the smaller figures of Will and Memory don't hang independently, but instead are holding on to the tips of Intellect's wings. In the *Tree of Science*, however, the same question is asked as that of the *Book of the Lover and the Beloved*, where it is answered with a reference to the *Book of the Gentile*, whose protagonists argue purely intellectually. But then when the three powers of the soul rise up towards God, the intellect, which had gone first, could no longer bear the heat of the sun, and asked the will to take the lead.¹³¹ Lull often exemplifies the relation between the two with the simile of faith being like oil which floats on the water of understanding: the higher our understanding, the higher faith can rise

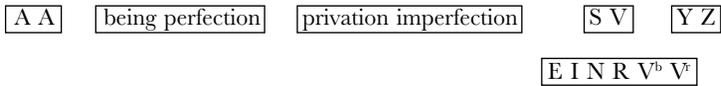
¹³⁰ See Gracia 1975, 33–34, on this aspect of Lullian demonstrations. Arnaldez 1993, 326, remarks that Lull's "position is very original: one cannot think without willing, nor think well without willing well."

¹³¹ "Arbre exemplifical", III, 6 (5,2 in the *ROL* edition).

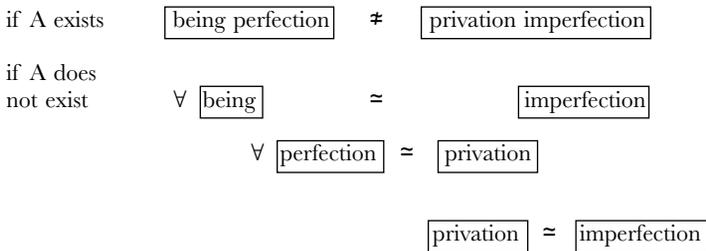
on top of it.¹³² Faith too is what the Art supposes and then confirms. So the relation between them is delicate, but for our purposes here, it is enough to realize how inseparable they are, and how their varying roles are mapped by Figure S.

The demonstrative procedure we have just analyzed is an example of what Llull calls his *rationes necessariae*, or “necessary reasons”. These argumentative strategies undergo major alterations in the ternary phase of the Art, and are changed almost completely in the post-Art phase. Their purpose, however, remains constant: to ensnare Muslims and Jews into debates on unobjectionable bases, and to offer a method that will avoid Ecclesiastical censure for trying to prove the Articles of Faith.

It might be interesting to end with a schematic display of this proof showing its structure and how it somewhat resembles a distant precursor of modern mechanical theorem proving. We start with the original chain of compartments, breaking down the penultimate compartment into the species of S, E I N R, as well as into blue and red V (virtues and vices).



Then, keeping concepts and compartments (more or less) vertically aligned, arbitrarily using the symbols \approx and \neq for the two components of the green triangle of Figure T, here expressed as “is concordant with” and “is contrary to”, and using the usual symbols \forall and \exists for the quantifiers “all” and “some”, we get for the first of the two demonstrations:



¹³² *AGU* (ROL XIV, 276). Llull discussed the subject in many works—it was one about which he was clearly concerned—but he treated it most especially in the *Disputatio fidei et intellectus* (ROL XXIII, 224–279), analyzed below on p. 207 ff. (although there principally for the logical techniques it introduces). See also the passage quoted on p. 164. Of the large secondary literature on the subject, particularly good are Colomer 1986 and Colomer 1997, 145–151.

but since this is impossible, there must therefore exist

$$\exists \boxed{\text{being perfection}} \quad \text{without} \quad \boxed{\text{privation imperfection}}$$

Then for the second demonstration, adding the modal operator \diamond for “it is (or would be) possible that”, we get:¹³³

If A does not exist $\diamond (E \approx (\forall^r Z))$

and $\diamond (I \approx (\forall^b Y))$

but this is impossible, because instead of the correct situation of

$$\boxed{\text{being perfection}} \quad \approx \quad E$$

$$\boxed{\text{privation imperfection}} \quad \approx \quad I$$

we would have one in which $(\boxed{\text{privation imperfection}} \quad \forall^r Z) \approx$ majority
 and $(\boxed{\text{being perfection}} \quad Y) \approx$ minority
 which is clearly impossible.

Other examples

To broaden the reader’s perspective on Llull’s methods, we will give a few more sample solutions, with little or no commentary, since the reader should now better be able to navigate for himself. We will start with a question for which we presented an earlier solution.

11. *Question:* Whether one can simultaneously understand predestination and free will?

Solution: $\boxed{S X | A T | E N}$.

The second compartment signifies that A and majority^T are concordant^T, as well as signifying the same for creature^T and minority^T, and by this meaning of A T is indicated the solution to this question in the first and third compartments. For S X means that A’s work is greater in predestining^X and in judging by means of free will^X than E^S is capable of understanding, and therefore E^S cannot understand simultaneously the entire compartment of $\boxed{\text{predestination free will}}^X$; but when E^S understands

¹³³ I have not transcribed the two appearances of “and” in the left margin below by the usual “ \vee ”, even though they in fact are equivalent to the modern logical conjunction. This was partly to conserve the vertical alignment of the components of Llull’s compartments, and partly to avoid a formula, which, for the non-mathematician, might seem unhelpfully complicated, one which would come out to:

$$\diamond (E \approx (\forall^r Z)) \vee \diamond (I \approx (\forall^b Y)).$$

predestination^X, then N^S through supposition^X, belief, and object^X, conserves free will^X until such time as E^S finally takes it as object, and abandons predestination^X, and the same happens with N^S and predestination^X.¹³⁴

The solution is an extension of that of the ‘conditioning’ of the first compartment studied above (p. 70). Notice how we have the same shifting roles of E and N we had under the treatment given this same question in the *ACIV* on p. 49 above, but now with a slightly different treatment of the impossibility of understanding the two simultaneously.

13. *Question*: Whether the soul, which is good, is subject to falsehood, which is evil?

Solution:

S	Z	S	Y	S	V	fire earth	water earth
---	---	---	---	---	---	------------	-------------

.

With F^S remembering the fourth and fifth compartments, G^S understands that, just as dryness is the passive subject upon which is carried out the transformation of wine into vinegar as a result of the transformation of heat into cold, so E I N are the subjects in which is carried out the transformation of Y into Z contrary to the red triangle^T, there being a difference between S and B C D, as well as between B C D and E I N, to which B C D are subject, beneath which B C D lies S. (*SWI*, 449)

Here we have elemental exemplarism used to show how truth (Y) can be transformed into falsehood (Z), contrary to their relation of majority/minority (of the red triangle), to operate a transformation (or misuse) of the powers of the soul, in which E I N influence B C D (causing them to remember, understand, and love the wrong thing), thus affecting all of S.¹³⁵

19. *Question*: Whether each intellect is one with others?

Solution:

V	V	S	S	A	S	S	T	Y	Z	E	A	V	Y	I	V	Z
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

.

The first compartment signifies in the sixth and seventh that the green triangle^T—designated by the fourth and fifth compartments—is present in the second compartment, for otherwise it would follow that the third and fifth compartments would be destroyed, since A would then be without justice^V and there would be no diversity^T and contrariety^T in the fifth compartment, which is impossible, and therefore G^S understands this solution, as has been explained. (*SWI*, 451)

¹³⁴ *SWI*, 448–9. “Belief” is not explicitly a term of the *AD*, but Llull is probably referring to N itself; see the passage quoted on p. 45 above.

¹³⁵ Pring-Mill 1963, 50 and n. 146 (= Pring-Mill 1991, 154 and n. 93) points to this question as one of seven of the *Ars demonstrativa* which use elemental exemplarism.

Here Llull is arguing against the Averroist doctrine of monopsychism, which upheld the unity of the potential intellect, one common to all men and whose cognition each individual merely shares. It was an issue which caused immense controversy in thirteenth-century scholastic circles, mainly because it precluded the possibility of individual salvation, which is precisely the point Llull makes here, when he says that God could no longer exercise (individual) justice.¹³⁶

It might be instructive to give one additional example, this one a proof of the existence of God from the *Book of the Gentile and the Three Wise Men*, not only to compare it with the one we have just studied, but also to show how it is only a very thinly disguised version of the Art. Instead of a chain of compartments, here Llull starts—as he did in the Conditions of the *AD*—with a single binary compartment, which he has previously presented as the ‘flower’ of a tree. The passage is in quotes because one of the wise men is speaking.

6. Love Perfection

“Love,^A perfection^{A,X} are in accord^T with being^X; and being and perfection are in accord with one another, as are nonbeing^X and defect^X. Now, if nonbeing and defect are in accord with being and with perfection in man and in the things of this world, how incomparably more fitting it would be for being and perfection to be in accord in something that had no nonbeing or defect. And if this were not the case, it would follow that being and perfection could be in accord in nothing without their contraries^T, nonbeing and defect, also being present. But this is impossible, and by this impossibility it is demonstrated to the human understanding^C that there exists a God in whom there is no nonbeing or defect, and in whom there is being and perfection...¹³⁷”

Questions concerning elemental theory

Because of its importance at this stage of the Art, it would be wrong not to present Llull’s answers to some questions from elemental theory:

¹³⁶ See, for instance, Kretzmann et al. 1982, 613–5, and Marenbon 1987, 68–71.

¹³⁷ *SWI*, 122–3. Note that “perfection” is present in both Figures A and X. In the binary ‘flower’ of the heading it is taken from Figure A, but the way he contrasts it with “defect” in the proof makes his use of it closer to that of Figure X, and this is the way we have labelled it. Here we have only labelled the first appearance of a concept with a superscript letter identifying the figure to which it belongs. In subsequent questions and answers we have omitted this labelling, since the terms used are simpler and the reader is by now more experienced.

6. *Question:* Whether air has greater concordance with water in earth or in fire?

Solution:

air	water
-----	-------

fire	earth
------	-------

.

N has doubts about solving this question, and it impels F to survey the compartments so that L may be transformed into G, and since F remembers the disposition according to which T enters the compartments, G understands that in fire the quality of air is passive, and in earth it is active, through the intermediary of water, whose quality in earth is active, for which reason air has greater concordance with water in earth than in fire. (*SW I*, 442)

From Figure S we can see that the transformation of L into G involves moving the act of the intellect from not knowing to knowing. The rest of the text can easily be understood from the figure of the rotation of the qualities of elements on p. 59. “Passivity” and “activity” are explained from the qualities on the receiving or giving end of the counterclockwise arrows of that figure.

9. *Question:* Whether natural appetite impels a power more strongly towards an extrinsic than towards an intrinsic object, or vice versa?

Solution:

water	earth
-------	-------

S	X
---	---

Y	Z
---	---

.

L thinks that water, by allotting cold to earth, receiving moisture from air, and mortifying fire in itself, as well as in earth and in air, has the noblest object to which it can be impelled. But then F remembers that in this world S cannot attain perfection, which it seeks with E I N, in which perfection it would be with E continually, if such perfection existed in itself. Hence, as a result of these things that F remembers, G perceives the solution by understanding that S is in Z when it thinks it can find its perfection better within itself than outside itself, in which extrinsicality, so to speak, exists A and its compartments, in which A and its compartments, E is in Y, when it seeks the perfection of S therein. (*SW I*, 443)

Again we have L misinterpreting the rotary motion of the qualities of the elements of the figure on p. 59, or rather misinterpreting their significance, which has to do with the concept of “perfection” from Figure X, which is what S cannot attain within itself. To put it another way, S is in Z (falsehood) when it tries to do so; it can only attain Y by trying to do so extrinsically, with the goal and help of A (God).

Lastly one question in which the analogy between the Elemental Figure and Figure S—the two figures, as we have said before,¹³⁸ similarly constructed with four squares—is brought out in a curious way:

¹³⁸ See n. 68 above.

5. *Question:* In what way do accidents come from physical substance?

Solution:

air	air
-----	-----

 | S S |

fire	water
------	-------

 |

water	earth
-------	-------

.

Once G has understood the question, it examines the compartments, as a result of which examination F remembers how the elements and S have certain similarities in their operation. Hence G solves this question metaphorically, for from S there issues virtue in B C D, from which is formed E, which is composed of the accidents proceeding from substance by means of B C D, and therefore F remembers that from the generation which takes place in the first, third, and fourth compartments, there comes forth quantity, quality, relation, etc., since, through generation, substance is brought into existence, and this substance is the being and source of the accidents, which remain in it, it being a subject with respect to them. (*SW I*, 441–2)

Through the use of ‘metaphor’, Lull shows how the ‘similarities’ between the acts of the powers of the soul and generation within the Elemental Figure can be used as an explanatory tool. The term “virtue” used here in connection with B C D, means their “power, capacity” (in the same way herbs are said to have the “virtue” of curing such-and-such an ailment), or in other words their acts which conjointly produce E. “Quantity, quality, relation, etc.” refer to the nine Aristotelian accidents brought forth by the similar generation of elemental substance.

Concluding remarks

As we said before, the questions we have just studied are samples from a first lot of 38, which, as Lull explains in the little prologue to this section in the *AD*, are just the tip of an immense iceberg:

The questions of this Art are of three sorts. The first is that of 38 questions; the compartments indicating the solutions of these are accompanied by an explanatory text, and thus are dealt with in a usual way. The second group is dealt with in a more subtle fashion, since the compartments indicating the solutions to the questions are not explained by any accompanying text, but are just given by themselves, so that the questions may be solved by their meaning alone. The third and last method, however, concerns external questions which do not appear in this volume, but which could only be settled by the Art, by seeking their solution among the compartments of the above-mentioned 1,080 questions. And if the artist is skilled, he will solve them immediately by means of the compartments that best accord with them; or else he should make up other compartments himself, according to the method displayed in the compartments of this work. And this third method is subtler than the other

two, as well as being the goal toward which the Art is directed, and the general goal of the entire Art, which is in C G.¹³⁹

So, with the remaining 1041 questions, no explanations are offered. To give merely five examples:

Question: Whether God forms the acts of the powers while the soul uses them?

Solution:

H	I	S	T	S	V	S	X	fire	fire	fire	air
---	---	---	---	---	---	---	---	------	------	------	-----

. (*SW I*, 478)

Question: Whether usury is licit?

Solution:

order	commandments	hope	charity	accidie	envy	fire	fire	fire	air
-------	--------------	------	---------	---------	------	------	------	------	-----

. (*SW I*, 530)¹⁴⁰

Question: Whether air and vapors differ in essence?

Solution:

motion	mixture	digestion	composition	air	air	fire	air	air	water
--------	---------	-----------	-------------	-----	-----	------	-----	-----	-------

air	earth
-----	-------

. (*SW I*, 539)

Question: Which is the surer cure for a sick person, that by similarity or that by contrariety?

Solution:

motion	digestion	appetite	composition	fire	fire	fire	air	fire	water
--------	-----------	----------	-------------	------	------	------	-----	------	-------

. (*Ibid.*)

Question: Whether, for a similar crime, a townsman should be more punished than a peasant?

Solution:

special	nutritive	E	I	N	R	mixture	digestion	being	privation
---------	-----------	---	---	---	---	---------	-----------	-------	-----------

majority	minority
----------	----------

. (*SW I*, 559)

From this brief sample it should be clear that Llull is not proposing the Art only as an apologetic method for the demonstration of Christian truths, but as a more general technique of answering all manner of questions. And this is, of course, what makes the Art generative: starting from a finite, limited number of concepts, it can test all manner of hypotheses, and thus answer any question.

As for trying to solve such questions, this is perhaps where the utopian ideas of the self-taught man are most in evidence. To expect a student

¹³⁹ *SW I*, 438–9. The figures for the number of questions don't add up because Llull forgot to mention the one at the end of the Prologue to this section, the first one given above on "Whether this Art is demonstrative", giving the correct $1 + 38 + 1041 = 1080$.

¹⁴⁰ From the "Questions concerning the Principles of Theology", from which come the concepts of the first compartment. The next two questions with their solutions are based on the Principles of Philosophy, and the last on the Principles of Law, for all of which see the lists on p. 55 above.

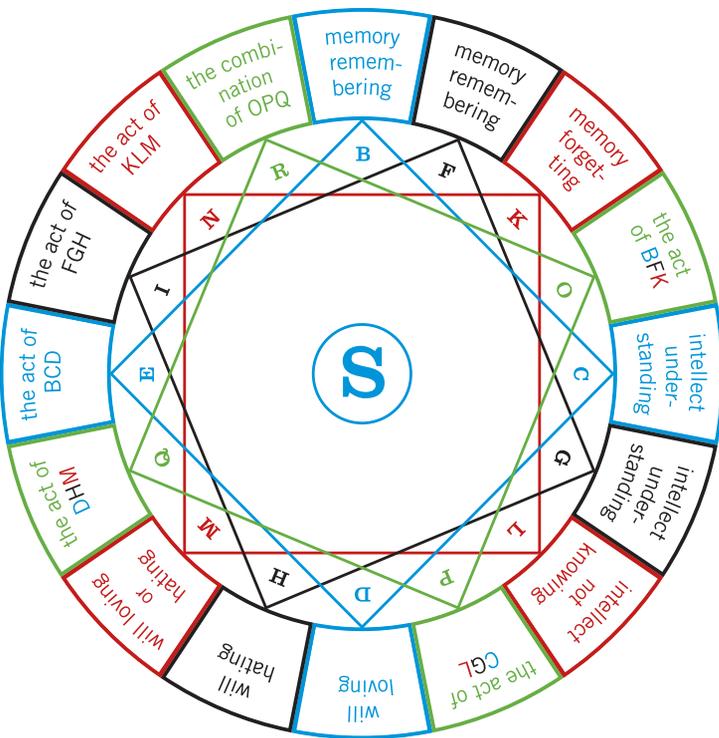
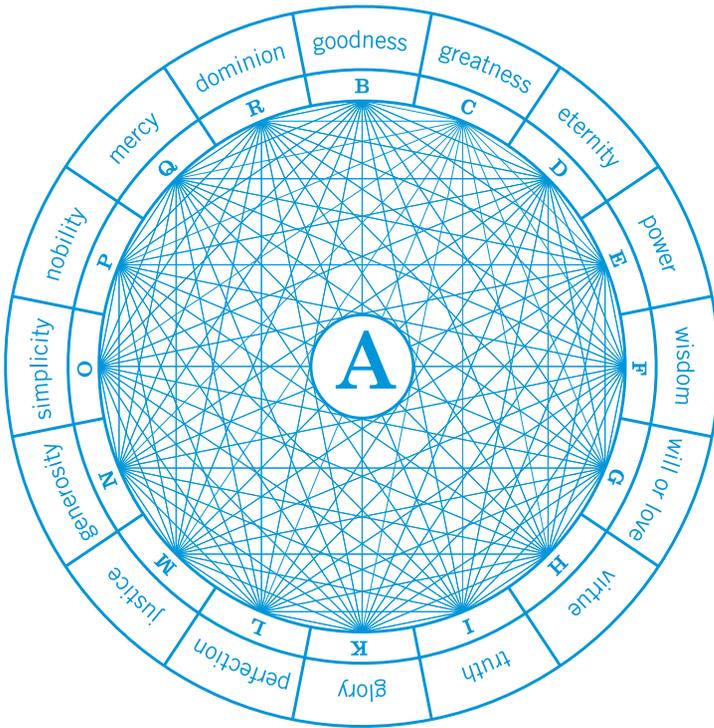
not only to assimilate and memorize all the previous mechanisms of the Art, but then to have to work out solutions to these uncommented chains of compartments himself, is perhaps asking more than most people are willing to attempt. The matter is further complicated by a lack of “Conditions” for the extra figures of the *AD* (those of Theology, Philosophy, and Law), as well as a feeling that the choice of possible interpretations of each compartment is singularly aleatory, or at least involves a laborious process of trial and error to make sure one is on the right track.¹⁴¹

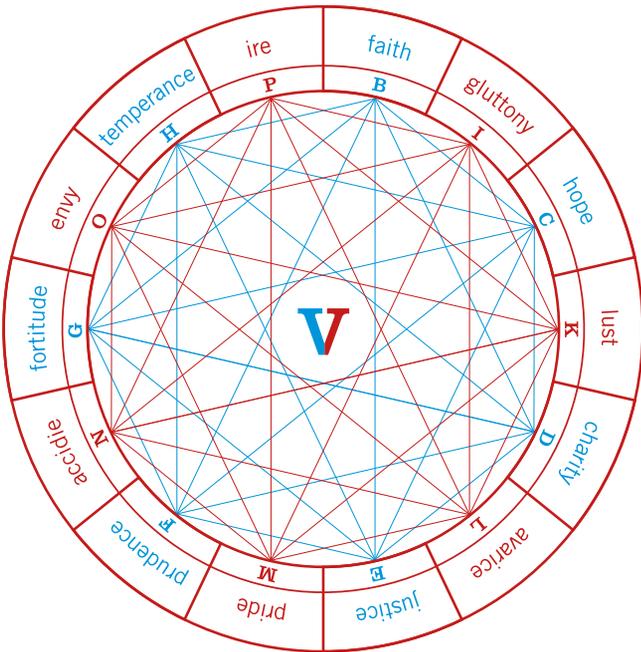
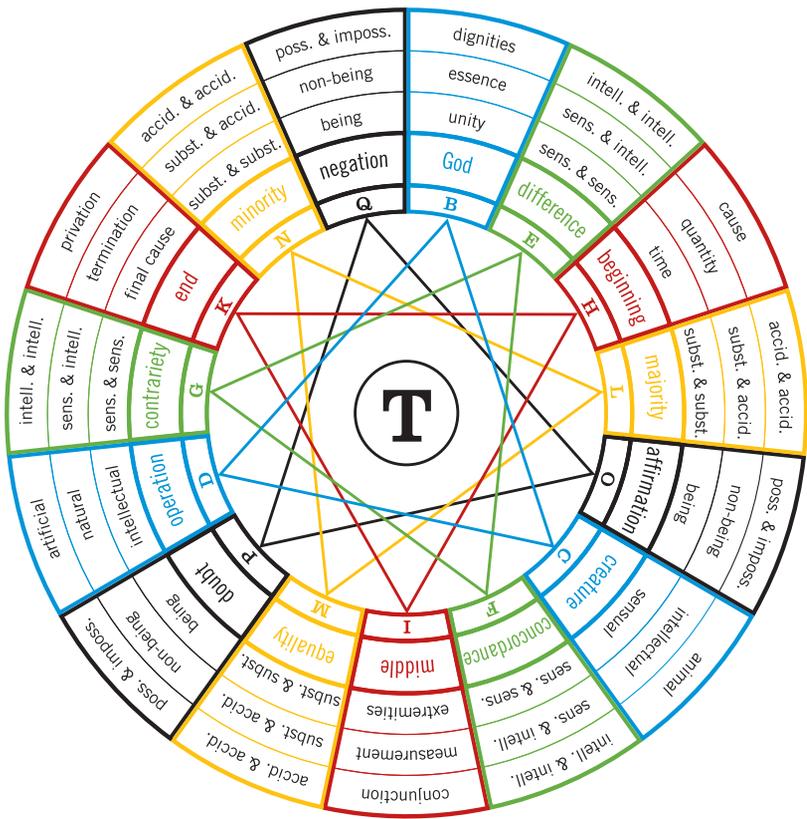
Lastly, there is the third method of “external questions which do not appear in this volume”, which could be solved either by following a model chosen from similar questions of the *AD*, or by the user “making up other compartments himself”. On the one hand, this is one of the many fascinating instances in which Llull presents his Art as an *opera aperta*, which does not end with the pages of the book the reader has before him.¹⁴² On the other hand, with these “external questions”, the reader, or “artist” as Llull calls him, might feel like someone set afloat in a small boat with a sextant, compass and pocket watch, with the assurance that will be enough to complete the task of mapping the coast of a new continent.

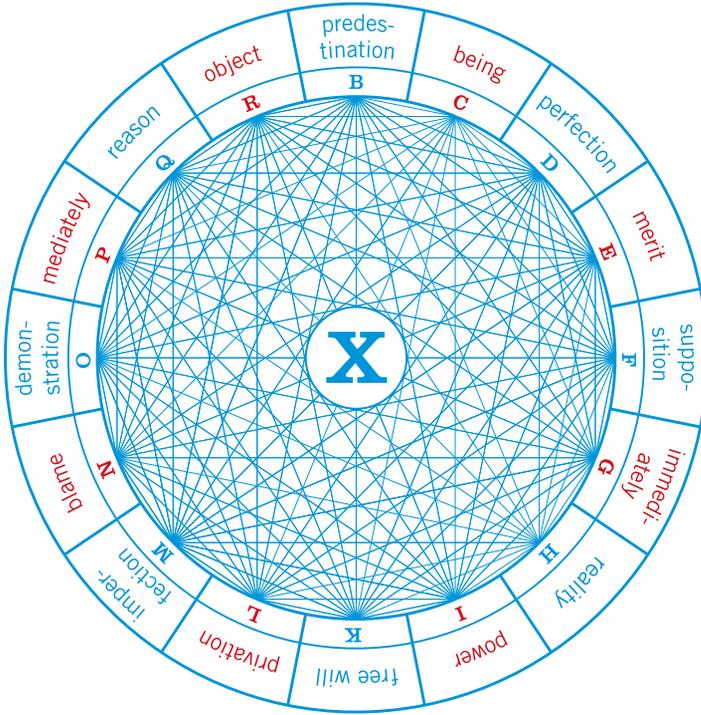
These are some of the things Llull set about changing; but before explaining the new Ternary Art, we must trace the road he used to get there.

¹⁴¹ People self-taught tend to assume that others better trained than they would find their projects unproblematic. The utopian nature of this particular project of Llull’s is, I think, corroborated by the fact that, among the many manuscripts of works of the quaternary Art, we have none (as far as I know) with commentaries offering solutions to these questions. Llull’s many followers were perhaps more enticed by the theory of this version of the Art than by its practice.

¹⁴² See pp. 293–6 in Chapter 6 for more about this aspect of Llull’s work.







TRUTH



FALSEHOOD

The Figure of Fire (heat)

fire	air	water	earth
air	fire	earth	water
water	earth	fire	air
earth	water	air	fire

The Figure of Air (moisture)

air	fire	water	earth
fire	air	earth	water
water	earth	air	fire
earth	water	fire	air

The Figure of Water (cold)

water	earth	air	fire
earth	water	fire	air
air	fire	water	earth
fire	air	earth	water

The Figure of Earth (dryness)

earth	water	air	fire
water	earth	fire	air
air	fire	earth	water
fire	air	water	earth

Elemental Figure

CHAPTER THREE

CHANGES IN THE ART DURING THE QUATERNARY PHASE, AND THE TRANSITION TO THE TERNARY PHASE¹

General Remarks

In the second-to-last miniature of the *Breviculum* reproduced on p. 20 above, Llull's disciple, Thomas Le Myésier, complains about "the confusion caused by the meanings of the alphabet of the *Ars demonstrativa* and its sixteen figures, which confound the mind." The quaternary Art, however, was not only very complicated but remarkably variable, as if in a state of continual experimental flux, during the fifteen or so years of its existence. The Art of the ternary phase, in addition to having much simpler foundations (only four figures), adopted, in the words of Jordi Gayà, a modular form,² which permitted the addition of a module or the substitution of one for another, in a much more ordered progression. The fluctuating state of the quaternary Art in part might have been because of outside criticisms or suggestions, but it was also because Llull had a vision of the truth as something accessible by many different paths. The fact that he tried one and then soon after another does not necessarily mean that he thought the first was wrong or had to be rejected. One path might indeed have proved more effective than another, but the fact is that Llull spent much of his career offering new proposals with a generosity which, for those of us who want to study his works, can be quite disconcerting. Perhaps it was the result of a decision to sow many seeds to reap the largest possible harvest.

The changes during the quaternary phase of the Art can be divided into two groups, those preceding and following the *AD*, corresponding to the two cycles of that phase. The first group involves mainly the *ACIV* and its differences with the *AD*. Within the second cycle some are of a purely experimental nature and others part of a gradual transition to the ternary phase. These second changes are greater than Llull seemed

¹ This chapter is a reworking and expansion of Bonner 2003a, which was largely based on Ruiz Simon 1986 and Rubio 2002.

² *ROL* XX, xlii.

willing to admit. In the *Vita coetanea*, in the passage quoted on p. 5 above, he merely states that, during his first visit to Paris, “having observed the attitude of the students there”, he returned to Montpellier where he wrote the *Ars inventiva veritatis* (the first work of the ternary phase), in which “he used only four figures, eliminating—or rather disguising, because of the weakness of human intellect which he had witnessed in Paris—twelve of the sixteen figures that had formerly appeared in his Art.” The impression that redoing the house was just a matter of a bit of cleaning and throwing out some superfluous furniture, and that his Parisian audience was chiefly to blame for it all, can only be justified by imagining that Llull was trying to palliate possible confusion and discouragement among his followers who had already expended so much intellectual energy trying to master the quaternary Art. The fact of the matter is that the changes involved a major reorientation, the components of which, as we will see, developed over the course of several years, and which were as much the result of inner demands of Llull’s system as of outside pressures.³

We will accordingly divide this chapter into two main sections, the first dedicated to the cycle of the *ACIV* and its differences with the *AD*, the second to developments within the cycle of the *AD* along with the changes that would lead to the ternary phase. To these we will add a brief third section on the two opening works of the ternary phase which are still transitional to its final form as a general Art.

The Cycle of the ACIV

The organization of the foundations of the Art in the ACIV

With the first work of the Art, the *ACIV*, Llull’s Art burst onto the philosophical and literary world of his time with what could only be described as explosive force. The reader should try to imagine the effect of the opening paragraphs on a reader accustomed to reading

³ This does not mean that Parisian students hadn’t found Llull’s system strange, nor that other outside pressures did not exist, nor that friends and disciples hadn’t formulated some of the difficulties mentioned at the end of the last chapter, nor that the number of figures and terms did not indeed “confound the mind”, in the words of Thomas Le Myésier. It just means that Llull was simplifying a much more complicated process and blaming it too exclusively on others.

Aquinas, Bonaventure or other theologians of the time.⁴ After a little invocation, it says:

This *Compendious Art of Finding the Truth* is divided into five figures, namely A S T V X. And by using these five, anyone can find the truth in a compendious fashion, and by contemplating God adhere to virtues and root out vices. This Art, moreover, teaches how to propose questions and to resolve their doubts by necessary reasons.

Here begins the prologue of the five figures.

We put A for God, to whom we attribute sixteen virtues—not accidental but essential (we don't mean to speak here of the theological or cardinal virtues)—out of which are formed a hundred and twenty compartments, in which lovers of this Art can arrive at a knowledge of God, and, by necessary reasons, propose and solve questions, provided that S know how to form in each compartment one term as subject and another as predicate, as well as knowing how to consult all the compartments of A T V X, in a manner agreeable to Y and contrary to Z.

After several more sentences, this section ends with several “notes”, one of which says:

Note that with A and its compartments the user finds Y in S, T, blue V, and X, and with Y which is in S T V and X he gains knowledge of A.⁵

These quotations should make evident to what degree Llull is indeed speaking a new language,⁶ and how the novelty is made even more striking by the circular, non-linear nature of this first version of the Art. In this opening text he hasn't yet told the reader what all the letters refer to, why Y should be smiled upon and Z frowned upon, nor why V should sometimes be blue, nor what the words “figures” or “compartments” refer to. On the next page, to be sure, the reader

⁴ The text of the *ACIV* was written ca. 1274, the year of the death of both Aquinas and Bonaventure.

⁵ Our translation of this and subsequent passage of the *ACIV* is from the transcription to be published in *SL* 47 (in this case corresponding to *MOG* I, vii, 1–2: 433–4). We have done this because Salzinger, the editor of *MOG* I, tried, as it were, to clean up the text to make it more orderly and readable, principally by putting all the figures at the beginning, and adding titles for different sections (in addition to other minor textual changes). In doing so, while conserving the basic content of Llull's message, he disguised the significant and unusual flow of the original text, which is what Albert Soler and I have tried to bring out in the abovementioned article.

⁶ See Hames 2003.

will find (if the copyists have finished their job)⁷ the circular figure of A with the sixteen “virtues” around its circumference, followed by the corresponding half-matrix, from which he may be able to deduce to what the word “compartment” refers. Slowly he will be introduced to the other figures, but it isn’t till he has finished all ten pages of the prologue that he will be able to understand what it was that Llull was talking about in these opening sentences.

Its organization is also confusing. After discussing each letter and what it represents, he simply, as we already said with A, presents its two figures—the circular and the half matrix—with no clear distinction between them, or rather as if they merely constituted varying versions of the same figure. There is no systematic organization into “first figures” and “second figures” as in the *AD*. To give just three examples. With A there is nothing at all called a “second figure”. With S we find Llull explaining:

For the rational soul we put S, which has four squares as shown in the figure, which are E I N R, and which are the four species of S, each of which contains the individuals, which are B C D, F G H, K L M, O P Q, as shown in Figure S.

S is divided into seven figures: the first is of its species, the second is of fifteen compartments, the third of E, the fourth of I, the fifth of N, the sixth of R, and the seventh of the individuals of the species of S.

After the rest of the exposition of S comes the circular figure we know from the *AD*, followed by of seven rectangular figures having from 7 to 36 compartments each, with no more explanation than the simple names given in the passage just quoted.⁸

Figure T has a simpler but more surprising distribution. Llull explains that:

T consists of three figures, of which the first is made up of five triangles inside a circle. The second is made up of 16 compartments in which

⁷ This was indeed a problem, because in the *ACIV* the figures are not described in the text nor their terms listed; sometimes the reader is simply referred to the graphical figure, and sometimes not even that. In the cycle of the *AD*, however, they are described verbally and their terms listed meticulously. Thus, even if a manuscript was copied either without the figures or having them—as is all too frequently the case—left incomplete, the text could now be read without the visual representations of the circular figures.

⁸ The curious reader will find one in a satellite work, the *Ars universalis*, *MOG* I, viii, 2-4: 484-6, but this is of little help to someone just reading the *ACIV*, and who has not been given the reference.

the elements are written, and is the diagram (*impressio*) in which the third figure is exposed by means of tropology and allegory. The third figure is that in which T is used in the other figures of this Art.

Of these three the first and third correspond to the circular and half-matrix figures of the *AD*, but the second is the Elemental Figure! From the *AD* we're familiar with its metaphorical use (hence the "tropology and allegory" in our quotation), but it is none the less surprising to find it here under Figure T, as if its physical basis were almost completely secondary. This double use is why in the *Lectura compendiosa super Artem inveniendi veritatem* he calls T "the figure of principles or significations", explaining that the elemental part can be used "metaphorically".⁹ This in turn probably explains why in the *ACIV* the Figure T is simply called the "Figure of Significations".¹⁰

This placement of the Elemental Figure is also symptomatic of its minor role in the *ACIV*, where the above-quoted line about its being "made up of 16 compartments in which the elements are written" is the only description offered. In the rest of the work it is only used very occasionally.¹¹ This secondary role contrasts remarkably with that in the following cycle. We have seen its leading role in the *AD*, where the first "conditions" (Dist. II) and the first "questions" (Dist. IV), treat this figure and the elemental theory emanating from it. He then wrote a *Liber exponens figuram elementalem Artis demonstrativae*, a central portion of which was the much commented *Liber chaos*, devoted principally to elemental theory. It thus goes from being a kind of add-on in the cycle of the *ACIV* to being one of the principal foundations of the Art and of Llull's cosmology during the cycle of the *AD*, only to take on a much more subordinate role again in the ternary phase.¹²

⁹ *MOG* I, vii, 42: 474. See also the *Ars uniuersalis* (*MOG* I, viii, 4 and 9: 486 and 491).

¹⁰ These terminological distinctions are important, because a certain amount of scholarly energy has gone into explaining why the name "Figure of Significations" is applied to Figure T alone (i.e. without the accompanying Elemental Figure).

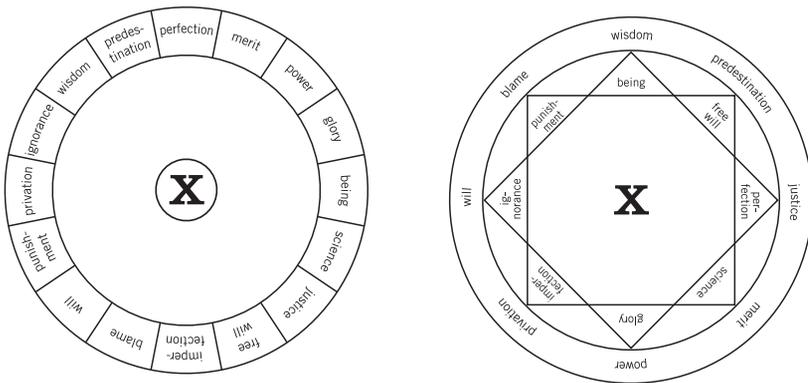
¹¹ In the *Lectura compendiosa super Artem inveniendi veritatem* it only occupies half a page (*MOG* I, vii, 42: 474), and in the *Ars uniuersalis* Llull gives it some 6 pages (*MOG* I, viii, 6–9, 47–49: 488–491, 529–531) out of a total of 123. The only exception is the *Principles of Medicine* (translated in *SW* II), which is, of course, entirely based on elemental theory; but this is a specific application, not a work of the Art.

¹² The figure itself disappears, and the elements become the bottom rung of the ladder of being which forms the Nine Subjects.

The last figure of the first cycle of the Art that requires comment is that of X. In the text of the *ACIV* he explains that it is divided into four “universal” figures:

The first is formed from the perfect wisdom of A. The second is formed from the perfect justice of A. From the first figure is formed predestination, and from the second free will. The third is of doubt and is formed from the first and second in S. The fourth is formed from A and all its compartments, as well as from X, blue V, T, and Y.

If the reader will look back at pages 49–51, he will recognize what we there called the “four stages” of his argument about predestination and free will. But in what sense these constitute “figures” is a bit mysterious. In the various medieval manuscripts of the *ACIV*, Figure X comes in two forms.



The second is surely an attempt to represent graphically the four figures Lull has just described, but, in spite of a perspicacious and interesting attempt on the part of Josep Enric Rubio to try to fathom the relationship, it is still not completely clear.¹³

We cannot leave this section without pointing out a minor but general difference between the figures of the two stages of the quaternary Art,

¹³ See Rubio 2000. As he points out, the figure in *MOG I* is a reworking by Salzinger of the first purely circular figure above. We have reproduced the figures here without color, since it would be meaningless without going into the details of the functioning of these figures, which can be consulted in Rubio’s article.

in that, except for Figure S, the letters just inside the circumference of the figures of the *AD*, those used only in its Demonstrative Figure, are absent from the figures of the *ACIV*. The reason is quite simply that the Art before the *AD* lacks a Demonstrative Figure.¹⁴

The preceding outline of the organization of the figures and terms of the Art, with its complications and with its paucity of explanations, should be enough to show how the first work of the Art can easily give the appearance of a sibylline labyrinth. At the same time, I feel—and this is an opinion I share with some other scholars—that it is the most adventurous, experimental form of the Art, and as such might one day repay further study. Only then could we judge whether its proposals were later abandoned as unacceptable because Lull decided they were logically—or Artistically—inviolate, or simply because of the objections of readers who found them unclear or too difficult.

Exploring new notations

Equally surprising and fascinating are two attempts at a total symbolization of the quaternary Art. They take place in the early *Ars notatoria* and in chapters 31–33 of the *Introductoria Artis demonstrativae*.¹⁵ Unfortunately, however, any minimally comprehensible explanation of their mechanisms (which are quite different from one another) would occupy a space quite out of proportion to their role in the development of Lull's Art, because the truth is that they constitute two experiments he never used or even mentioned in subsequent works.¹⁶ But all the same, I cannot resist the temptation to give the reader the briefest of glimpses of the *Ars notatoria*.

In that work he gives symbols not only for the components of the Art, but also for the parts of speech: pronouns, verb tenses, the five universals, the ten predicaments, etc. The symbols can be letters (such as those for Figure T, like the horizontal “f” in the example below), geometrical figures (crosses, vertical lines, circles or triangles with dots

¹⁴ For the letters just inside the circumference of the figures of the *AD*, see p. 32 above. The only thing vaguely similar in the previous period is the Universal Figure of the *Ars universalis*, but it is a much simpler, non-rotary precursor of the later Demonstrative Figure.

¹⁵ The first was edited with a very helpful, interpretive translation in Gayà 1978; the second was printed in *MOG* III, ii, 25–30: 79–84.

¹⁶ The only exception is the *Ars notatoria*, which gets a passing mention in one work, the *Ars universalis*.

in varying positions), or combinations of the two (like the concept from Figure X below). As an example, in the string of symbols

↵ a X.

the “f” pointing to the left represents “doubt” from the black triangle of Figure T (upside down it refers to “affirmation”, and pointing to the right “negation”), “a” represents “God”, and the last symbol represents “being” (*esse*) from Figure X.¹⁷ The entire string thus asks “Whether God exists?” This is the simplest of examples—the work ends with questions answered by strings of a dozen or two dozen symbols—, but it might give the reader the flavor of two notational ventures on Llull’s part that would surely repay further study, not only in themselves, but in terms of what might be implied by their alternative structuring of the Art.

Differences in the terms of the ACIV with respect to the AD

As for the terms of the Art, in our presentation of the *AD*, we pointed out the differing lists of concepts for the Figures of A, X, and Theology in the *ACIV*.

The one change we didn’t mention, since it concerns a single secondary concept of Figure T, might seem relatively unimportant. In the *AD*, the first term of the first (blue) triangle, God, is discussed in terms of his “unity”, “essence” and “dignities”.¹⁸ In the *ACIV* it was in terms of his “unity”, “trinity” and “virtues”. The change from “virtues” to “dignities” just reflects the different denomination of the divine attributes in these two cycles of the Art. The change from “trinity” to “essence”, however, is due to an important reorientation between the two cycles, one directed to eradicating from the Art any specifically Christian references.¹⁹ While in the *ACIV* the Trinity and Incarnation are proved, and extensively so, in the body of the work, from the *AD* on we no longer find any central presentation of the Art which mentions these

¹⁷ The 16 terms from Figure X of the *ACIV* (for which see p. 39 above) are represented by an “X” inside a box and distinguished by dots on and between the axes of the “X”, 8 outside the box and 8 inside.

¹⁸ See p. 41 above.

¹⁹ A change pointed out by Gayà 1979, 57, by Rubio 2002, 95, and by Ruiz Simon 1999, 365 n. 507.

two doctrines. In his *Quaestiones Attrebatenses*, in answer to a question by his disciple, Thomas Le Myésier, Lull explains why:

Question: Since God is trine (*trinus*) and incarnate, why do you have no question in this Art about the Divine Trinity or the Incarnation?

Solution: The Art is general to everyone, Christians, Saracens, Jews, and even pagans; but it would not be general to everyone if the Christian faith were to be found in it explicitly (even though it is there implicitly).²⁰

So instead of presenting the Art directly as an instrument for demonstrating the Articles of Faith, from now on it will provide the foundation for such demonstrations, which will only be carried out in other works: either in those in which the Articles of Faith are treated openly, or in those presented as commentaries on a central work of the Art, or even in literary works.²¹ It was a classic tactic of presenting foundations to which the adversary could not object, but which would have drawn him into a labyrinth from which he could not extricate himself without finally admitting to positions undeclared at the beginning: those involving the truth of Christianity. This is why Lull wanted to make the Art “general to everyone”, or, as Viola Tenge-Wolf has so aptly put it, “a religiously neutral universal science”,²² one which disguised but never lost sight of its fundamental role as a methodological support for his missionary/apologetic task.

We do, however, find in the Art the Lullian equivalent of the *preambula fidei*, that is to say, those doctrines which the Church considered provable by rational means, and which were acceptable to other religions of his time and of his geographical ambit.²³ It is a division to a certain extent exemplified in the *Book of the Gentile*, Book I of which proves things

²⁰ See Lullus 1746, 45–46, quoted in Hillgarth 1971, 161 n. 49. See the similar passages in the *Ars demonstrativa* (*SW* I, 425) and the *Art amativa* (*ORL* XVII, 8).

²¹ In *Felix*, for example, he speaks of “God in his essence, in his dignities, in his trinity, and in his unity” (*SW* II, 1081) calmly combining the formulations of Figure T from both cycles.

²² *ROL* XXVII, 30*.

²³ In the *AD*, for example, if we look at the general questions concerning God (*SW* I, 444), we will find “Whether God exists”, “Whether a blessed soul in Glory could have knowledge of God, even if God had no intrinsic work in himself”, “Whether in God there is equality of goodness and bonifying, greatness and magnifying, eternity and eternalizing with his wisdom and understanding, will and loving”, etc. In the questions on the Figure of Theology (*ibid.*, 520ff.), we find “Whether God’s essence is essentially without any difference from himself”, “Whether the divine dignities are in the essence, and vice versa, with no difference between dignities and essence”, etc. One finds a similar division between the *preambula fidei* and the Articles of Faith in Aquinas (see for

common to the three religions (the existence of God, the existence in him of the dignities, the existence of the resurrection), while the specific articles of the Christian are treated in Book III.

A difference in aim and title

One of the most important changes between the *ACIV* and the *AD* is revealed by the fact that, in the first work, the words “demonstrate” or “demonstration” do not appear at all, whereas “signify” and “signification” appear over and over again.²⁴ Figure T (which, as we have seen, at this point includes the Elemental Figure) is called “the figure of significations”, and the third of the Universal Modes, or instructions on use of the Art, is called “On Signification”. As opposed to this absence of any idea of demonstration in the *ACIV*, in the *AD* it is explained in considerable theoretical detail in the very Prologue of the work, with a new third kind of demonstration added to Aristotle’s classic two.²⁵

As Ruiz Simon has pointed out, this change is related to the change in title from *Ars compendiosa inveniendi veritatem* to *Ars demonstrativa*. The latter title seems to reflect Lull’s desire to present his epistemological device as a science which could be framed in terms set by the Aristotelian schema of the *Posterior Analytics*, the work of Aristotle which, according to the scholastic tradition, dealt with “demonstration”, that is with “necessary reasons”, as opposed to the *ars inventiva* or dialectic discussed in the *Topics*, which dealt with “probable reasons”.²⁶

Ruiz Simon has also pointed out how the “analytic elegance” of the passage quoted on p. 18 above from the *Introductoria Artis demonstrativae* is “far superior to the rare epistemological considerations found in the *ACIV*... In this first work of the Art, Lull shows little agility in the use of university terminology, and he still negotiates with difficulty the geography of the epistemological subtleties of scholasticism. He limits himself almost exclusively to offering the rudiments of his peculiar *ars inventiva*.”²⁷

example *Summa theologica*, I, Q2, Art. 2 and *Summa contra gentiles*, I.3), but with the major difference that the former are provable philosophically and the latter not.

²⁴ A search for “demonstr” in the first half of the work which I have transcribed drew a blank, whereas doing the same for “signif” drew 27 hits.

²⁵ See p. 65 above.

²⁶ The two previous paragraphs are a reworking of Ruiz Simon 1993, 97–98.

²⁷ *Ibid.*, 96.

*The Cycle of the AD**An experimental interlude after the AD*

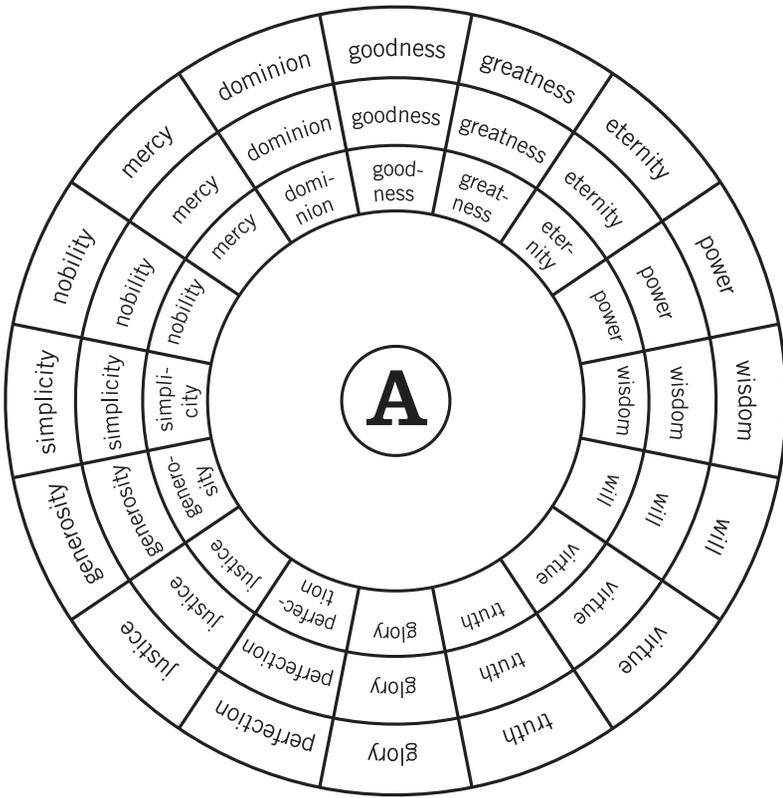
So far in this chapter we have described works of the first cycle of the quaternary Art, and how they differed from the following *AD*. Within that first cycle there were few changes, in contrast to the next cycle, where the ground seems to be constantly shifting beneath our feet. And the curious fact is that these changes were often disguised as mere commentaries on the *AD*. This process began with two closely related works, in which various components of the Art are restructured in a novel way: they are *Ars inveniendi particularia in universalibus* (*AIPU*) and the *Liber propositionum secundum Artem demonstrativam compilatus* (*PropAD*). The first thing that strikes the reader is the primary role allotted to the Figure T:²⁸ the *AIPU* treats *only* this figure, while the *PropAD* begins by explaining that:

Since in this Art T is instrumental to all the other figures of this Art and they all revolve around it, because without it nothing useful can be performed with the figures of this Art, hence in this work we will deal primarily with T in its role as an instrument. (*MOG* III, viii, 2: 504)

In these two works the first figures (with two exceptions which we will discuss in a moment) are identical to those of the *AD*; their principal novelty, and what most unites them, are the second figures, where the former binary mechanism is replaced by a ternary one. Since, as we explained before, graph theory only functions with binary combinations, he is forced to find some substitute for the half-matrices. He finds it in new second figures using three rotating circles or volvelles. As an example, here is the second Figure A from *PropAD*:²⁹

²⁸ This was pointed out by Rubio 2002, 88–89, 93–95.

²⁹ Adapted from Paris, Bibl. Nat., lat. 16113, fol. 52v.



As in the *AD*, where the second figures determine the form of the discourse of the rest of the work, here these new figures do the same with the discourse of the *AIPU* and the *PropAD*. For example, in the *AIPU*, the proofs of the existence of God begin “Compartment 1. Operation, End, Majority” “Compartment 2: Difference, Majority, Equality”, etc., following the route around the three circles starting from the initial position of “Operation, End, Majority”. Each question is thus answered with fifteen solutions resulting from doing the round of these three circles starting from an arbitrary initial position.

It should be noted that with these ternary combinations, however, Lull prohibits the use of repeated terms, explaining that

this is because in A T S V X etc., identical terms do not make a figure, as for instance in T with God God God, and similarly with others, no

figure can be produced; and in A with goodness goodness goodness, and similarly with others, no figure can be formed. This is because such a combination does not produce any meaning. (*MOG III*, viii, 2: 504)

But this injunction only applies to the ternary combinations; with the binary ones, as with the Elemental Figure, in the middle of which, as we will explain below, is preserved the half-matrix of the original second figure, he still permits the repetitions of fire fire, air air, etc.³⁰

There is a good possibility that Lull's introduction of ternary combinations was due to a realization on his part that presenting two members of the compartment being perfection as concordant, was equivalent to manipulating one compartment with three members, being perfection concordance. This mechanism is a clear precursor of the Fourth Figure of the ternary Art, as well as the Table derived from it, but here, either because it didn't produce the desired results, or perhaps because he found that, instead of simplifying, it complicated even more the handling of the multiple figures of the quaternary Art, he abandoned it in subsequent works of this phase. In the ternary Art, with its much smaller number of figures, and thanks to the way he set up the ternary mechanism, he could make it at the same time simpler and more general. But for the moment, it seems to have been little more than an experiment.

Another equally interesting innovation of the *PropAD* has to do with the contents of two figures, or rather of the two first figures of S and of the Elemental Figure. I will give them in the original Latin, because of the importance of the forms of some of the words.³¹

Figure S

recolentia
intelligentia
volentia
esse
forma
materia
conjunctio
simplicitas

Elemental Figure

igneitas
aereitas
aequeitas
terreitas
esse
forma
materia
simplicitas

³⁰ See, for example, *MOG III*, viii, 35 (537). In the last section, "De quaestionibus", he frequently answers with a ternary compartment followed by a binary elemental one, as for example *ibid.*, 41 (543): Deus operatio finis ignis ignis.

³¹ And as before, putting in italics those words that only appear in one of the two columns.

compositio	compositio
substantia	substantia
accidens	accidens
virtus	virtus
operatio	operatio
interioritas	interioritas
exterioritas	exterioritas
motus	motus

The first thing evident from these two lists is the change in denomination of the basic concepts: the three powers of the soul from Figure S, have gone from *memoria*, *intellectus* and *voluntas* to *recolentia*, *intelligentia* and *volentia*, and the four elements have gone from concrete terms, *ignis*, *aer*, *aqua*, *terra* to the corresponding abstractions of *igneitas*, *aeritas*, etc. Or, as Llull explains in the description of each of these figures, he is now giving for every concept its *essentia* instead of its *esse*.³² In part this is another response to the need to make the Art more general, abstract and further removed from its anchors in the everyday world of the senses. In addition, as we will see when we discuss the following work, it has to do with an orientation capable of handling the correlatives, which function with essences.

The second change, even more evident, is the addition of a series of new concepts, identical in the two figures (except for *conjunctio* absent from the Elemental Figure), an addition which now gives the two figures 16 concepts each.

These changes, however, don't cause Llull to abandon his previous formulations; instead he just incorporates them into the new one. In the case of Figure S, after explaining that its First Figure has the 16 concepts of our list around an exterior circle, he adds:

In the middle of this circle there are four squares of varying colors, by which are signified the powers of the soul and their acts, according to the letters of the alphabet written in these four squares, as can be seen in the figure. (*MOG* III, viii, 3: 505)

And he ends by giving the whole alphabet from B to R as found in the *AD*. So in fact the old first figure is drawn inside the new one. With the Elemental Figure the situation is similar, except that what he writes

³² Or its abstract vs. its concrete form (see these two words in Bonner and Ripoll 2002. s.v.).

inside the exterior circle with the 16 essences of the elements, is not the original first elemental figure, but the second, the half-matrix with the 10 binary compartments of the elements themselves.³³

This addition of identical concepts for the two figures responds to a desire on Llull's part to assimilate the figures one to the other, or rather, to make one the mirror of the other. In the same work he says:

The elements are a mirror of S (for just as the elements are ordered in nature, so the powers are ordered in S), and S is the mirror of A.³⁴

This was something Llull had been affirming for some time,³⁵ but here he made it more explicit within a new conceptual framework.

The correlative explosion in the Lectura super figuras Artis demonstrativae

Before examining the *Lectura super figuras Artis demonstrativae* (*LEAD*), some previous observations might be in order. Salzinger, in the Mainz edition, published a part of it, the *Liber chaos*, as a separate work. He seems to have been conscious of this situation, because at the place where the second should have been inserted into the first, as indicated in the manuscripts, that is, between the sections "De Figura Elementalī" and "De secunda Figura Elementalī", the pagination gives a mysterious "23 usque 31".³⁶ It would seem that the first modern scholar to discover the correct relation between the two works was Johannes Stöhr in an important bibliographical article published in 1957, but his warning

³³ *MOG* III, viii, 6: 508.

³⁴ *MOG* III, viii, 10: 512, with the relationship to A further explained *ibid.* 8: 510.

³⁵ Ever since the *Lectura compendiosa super Artem inveniendi veritatem* (*MOG* I, vii, 42: 474). See Ch. 2, n. 68 above for passages from the *AD* and other references.

³⁶ The *LEAD* is printed in *MOG* III, iv: 205–247, and the *Liber chaos* right after it, v: 249–292. The pagination is mysterious not only for somebody who doesn't understand what that might mean, but even for somebody who does, because the missing text occupies 44 pages and not 9 as implied by the phrase "23 usque 31"! These 44 pages for the *Liber chaos* against 43 for the rest of the *LEAD*, is a good indicator of the equal weight of the included and the including works. The *Liber chaos* is of undoubted importance for its elemental and cosmological formulations, as is clear from the bibliography it has generated since the time of Frances Yates. Salzinger might also have wanted to extract it from, as we will see, its theological surroundings, in order to make it fit in better with his own alchemical interests. It is only fair to admit, moreover, that since the beginnings of Lullism there has been a certain desire to treat the *Liber chaos* as an independent work, as in the catalogs of Llull's works ever since the *Electorium*, and in manuscripts since the fifteenth century (see the *Liber chaos* in the Llull DB).

went unnoticed for quite a while.³⁷ As Ruiz Simon has said, the *Liber chaos* has also suffered from being considered *the* Lullian formulation of elemental theory and its use as a cosmological foundation, instead of being *one* formulation—however important and interesting it might be—, significant aspects of which, as we will see in the following sections, Llull soon began to modify.³⁸

If we now look at the entire *LEAD*, we will see first of all that Llull abandons the two experiments of the previous works: the primary role of Figure T and the rotating ternary second figures. He thus returns to the traditional order of the figures, and he once again works with binary compartments, two things clearly necessary in a *lectura* of the *AD*. On the other hand, he conserves the new components of Figure S and of the Elemental Figure of the two previous works, that is to say the essences of their basic components (“recolentia”, etc. and “igneitas”, etc.) along with the same lists of supplementary concepts. While with Figure S these supplementary concepts play a minor role,³⁹ with the Elemental Figure they constitute the titles of the majority of sections studied there.⁴⁰

Although he has abandoned the ternary revolving circles, in the *LEAD* we find him suddenly using another ternary formulation, one which is not methodological, but ontological: his correlatives. These are the mechanisms that articulate Llull’s dynamic ontology, one in which being and activity are inseparable. With the correlatives he succeeds in giving this activity a general and always identical structure with—as we will see in a moment—its corresponding linguistic expression. He introduces these mechanisms right at the beginning of the work, with the opening phrase of his invocation (where I again leave in Latin the significant terms because of the importance of the linguistic form):

³⁷ Stöhr 1957, 51. Even Friedrich Stegmüller, Stöhr’s teacher at Freiburg and editor of the 1965 reprint of the Mainz edition, treated it as a separate work. Platzeck in his catalog of works from 1964, under No. 21a, says that the *Liber chaos* is “basically” a part of No. 21, the *AD* (as opposed to the *LEAD*, which he catalogued as a later work under No. 30).

³⁸ Ruiz Simon 1986, 79.

³⁹ They are in the section entitled “De secundo gradu S” (*MOG* III, iv, 8: 212), as a part of the arguments presented there, without any strict systematization.

⁴⁰ See *MOG* III, v: Esse—1: 249; Forma, Materia—6: 254; Simplicitas, Compositio—7: 255; Substantia—26: 274; Accidens—4: 272; Virtus—13: 261; Operatio intrinseca et extrinseca—12: 260, Motus—5: 253.

God (*Deus*) Father and Lord, who is *Deificativus* and *Deificans* eternally and immensely, Lord God (*Deus*) the Son, who is *Deificativus* and *Deificabilis* without end, Lord God (*Deus*) the Holy Spirit who is *Deificatus* and *Deificabilis* equally by the Father and Son with every perfection of goodness, greatness, eternity, power, wisdom, etc... (*MOG* III, iv, 1: 205)

Then under the second Figure A, he discusses the first “compartment”:

which contains two words, “goodness, goodness” (*bonitas, bonitas*), by the first of which we understand the divine essence, and by the second its act, that is to say *bonificare, magnificare*, etc., from which necessarily follows its being (*esse*), as well as its essence, *bonificativus, bonificabilis, bonificare, bonificatus*, and thus the other compartments of this figure should be similarly declined. (*Ibid.*, 4: 208)

Although this first ‘declension’ still has both potential and actual components in its active (*-ativus* and *-ans*) as well as its passive parts (*-abilis* and *-atus*), the second of each of which (*-ans* and *-atus*) disappear from the definitive formulation of the next phase of the Art, the essential mechanism is all here. As Jordi Gayà pointed out in his classic study, there are many antecedents of this correlative structure in previous works, formulations such as matter/form/conjunction, beginning/middle/end from Figure T, or doer/done/means.⁴¹ As for the dynamic ontology, in the beginning of Lull’s production it was pretty well limited to the divine activity, which internally produces the Trinity and externally the Creation.⁴² In earlier works we even find some of the characteristic verbal forms, or substantivized verbal forms, to express this dynamism.⁴³ But in the *LFAD* we find two innovations of great importance: for the

⁴¹ Gayà 1979, 41. See also Pring-Mill 1961, 140–2, reprinted in Pring-Mill 1991, 96–98. The last triad in our text is a limp translation of *agent/obrat/mitjà*. As for that of matter/form/conjunction, it is not only an antecedent, but remains one of the prime manifestations of the correlatives, as for instance in the trunk of the Elemental Tree in the *Tree of Science*.

⁴² And indirectly in the acts of the powers of the soul and in the operation of the elemental qualities.

⁴³ See above, Ch. 2, n. 90, for the terms “bonification” and “magnification” in the *AD*. Even in earlier works such as the *Book of the Gentile*, in the trinitarian proofs we find verbal forms of the dignities such as *poderejar* and *saviejar* (“empower” and “make wise”; see p. 284 below), or the *Llibre de demostracions* with forms such as *gloriejar* (*ORL* XV, 262; see Gayà 1979, 40). At the end of the *Compendium seu commentum Artis demonstrativae* (*MOG* III, vi, 160: 450) Lull refers to this correlative terminology as his “Arabic manner of speaking”; for what this means in terms of possible Arabic influence, see the excellent summary in Gayà 1979, 68.

first time Lull gives a (1) complete and systematic formulation of this ‘declension’ with the three grammatical forms derived from each noun: the active *-tivus* and the passive *-bilis* joined by the verb in *-are*; and (2) one that operates not only in the Divinity, but in identical fashion at all levels of reality, right down to the elements (as in the *Liber chaos*), allowing Lull to formulate a completely trinitarian vision of the world.⁴⁴ Lastly, it should be pointed out that in later works Lull goes so far as to reify these correlatives by abstracting their suffixes and referring to them simply as the “*tivum*”, “*bile*”, and “*are*”.⁴⁵

Another undercurrent of change

After the unfolding of the correlatives, the last change in depth during the quaternary phase comes from the new role of the dignities—or rather of the semblances of the dignities—as foundations for Lull’s cosmology. In the work we have just studied, Lull treats the substrate of sensible reality, chaos, as the product of the essences of the four elements. From the correlatives which constitute each essence, the composition of their four forms (*ignificativum*, *aerificativum*, etc.) produces a universal form, while that of their material components (*ignificabile*, *aerificabile*, etc.) produces a prime matter. Both of them together make up chaos, which acts as the material cause of all natural beings, and which in turn contains, besides Aristotle’s five universals and ten categories, all the other *rationes seminales*.⁴⁶

In a work written immediately afterward, the *Liber exponens figuram elementalem Artis demonstrativae*, Lull introduces an important novelty. Here we find, as Ruiz Simon says, an “attempt—half hidden in one of its pages—to implicate the divine dignities, by means of their semblances, in elemental theory, an attempt without precedent in Lull’s cosmological thought”. As Lull now explains it:

⁴⁴ Pring-Mill 1955–1956, translated into Catalan in Pring-Mill 1991, 161–189.

⁴⁵ As for instance in the *Liber de ascensu et descensu intellectus*, *Liber novus physicorum*, *Liber correlativorum immatorum*, and *Liber de possibili et impossibili*. For an interesting modern perspective on the correlatives, see Lohr 1987.

⁴⁶ This and the next section are in fact brief outlines of Ruiz Simon 1986, expanded with new material in Ruiz Simon 2005. Specifically this paragraph is from p. 81 of the first (corresponding to p. 168 of the second) and the quotations in the following two paragraphs are from pp. 84–85 and 86 (corresponding to pp. 168–9 of the second). See also Gayà 1979, 61–62.

In this plant are mixed the semblances of the elements, that is, the goodness, greatness, duration, power, etc., of fire with the goodness, greatness, duration, etc., of earth, of air and of water... (*MOG IV*, 7)

So the semblances of the dignities are no longer only present in each element, but a compound is formed by a mixture of these semblances.

At the end of the cycle, in two works clearly transitional to the ternary phase, the *Compendium seu commentum Artis demonstrativae* and the *Quaestiones per Artem demonstrativam seu inventivam solubiles*, the semblances of the dignities are not only present in the elements which enter into a compound, but now constitute the *essence* of the simple elements. While in the *Liber chaos* “universal form and matter were constituted, respectively, out of the forms and matters of the simple elements, in the *Compendium seu commentum Artis demonstrativae*, the form and matter of each of the simple elements are constituted, respectively, out of the forms and matters of the semblances of the dignities. As a result, the simple elements are obviously no longer considered as the first principles in the constitution of corporeal nature, but are now hierarchically subordinate to new first principles, the semblances of the divine dignities. This fact, as one might expect, is closely related to undeniable changes in considerations on the foundations and in the functioning of the Art”. In the last work of the cycle, the *Quaestiones per Artem demonstrativam seu inventivam solubiles*, he states the situation clearly:

Every real being, such as a man, a lion, a plant, etc., is substantially created and constituted out of the semblances of God, that is to say goodness, greatness, duration, etc.⁴⁷

In the meantime—in fact starting even earlier—a similar change had been taking place with respect to the powers of the soul. Already in *Blaquerna* Llull had insinuated the presence of the semblances of the dignities in the human soul. In *PropAD* he introduced a first change by using this presence to emphasize, no longer the distance between creator and creature as he had in previous works, but rather their concordance, which made Figure S the “mirror of God”. This was followed by another change, remarkably similar to that with the elements. If up to a certain moment the semblances of the dignities were present in the soul as accidental qualities, with the last work of the quaternary

⁴⁷ *MOG IV*, iii, 100: 116. All the same, the presentation in the *Quaestiones per Artem demonstrativam seu inventivam solubiles* is more changeable, as explained in Ruiz Simon 1986, 87–88.

phase, the *Quaestiones per Artem demonstrativam seu inventivam solubiles*, they become essential constituents of the human soul and/or its powers.⁴⁸ This permitted the rational soul to fulfill its ultimate end of approaching and understanding God, or as Llull puts it in that work:

Just as the soul is substantially made up of the divine semblances, so it must substantially understand God, in order that the goal of these semblances of which the soul is constituted be achieved by a substantial non-accidental operation, which is the intellect's objective attainment of God.⁴⁹

These two sets of changes in the structure of the elements and the rational soul had important consequences. The first implied, among other things, a transfer of the signifying function, which passed from the Elemental Figure to Figure A, that is to say that the significations of creation are now no longer realized on the basis of elemental exemplarism, but instead on that of the semblances of the divine dignities. In parallel fashion, the juxtaposing of Figure S with the Elemental Figure is now more or less abandoned; since Figure S has also become explainable by the presence of the semblances of the dignities, the recourse to analogies with the Elemental Figure is no longer necessary.

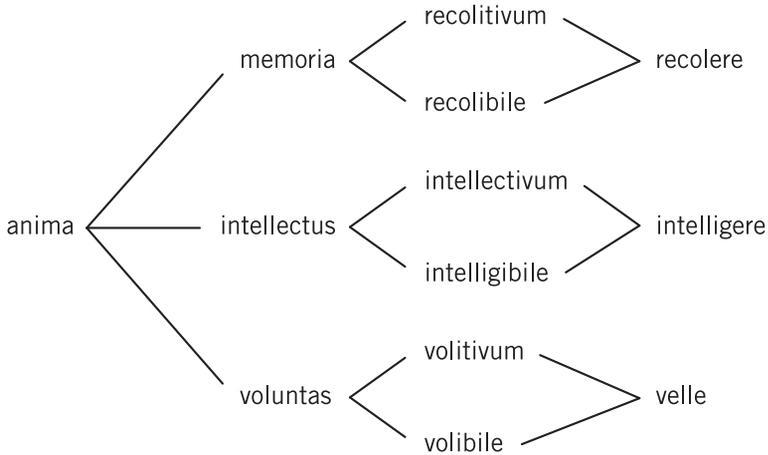
With the (semblances of the) dignities as constituents of all reality, and with the correlatives acting on all levels, instead of a simple affirmation of the world made in the image of God, what Llull now gives us is an explanation of the ontology of this image. And this ontology is necessarily dynamic—a fact much commented on in the bibliography on Llull during the last twenty-five years—in addition to being, as we have just explained, necessarily Trinitarian.

To see how this works in the case of the two figures we have been discussing, in the case of the powers of the soul, their new structure can be summarized as follows:⁵⁰

⁴⁸ Ruiz Simon 2005, 172, which see for the chronology of these changes, and for the *Art of Contemplation* citation from *Blaquerna* (*ENC* III, 117–8).

⁴⁹ *MOG* IV, iii, 87: 103, cited Ruiz Simon 2005, 173, the rest of which article discusses the implications for the beatific vision of God in Llull, and its sources in contemporary Neoplatonism.

⁵⁰ Which we give in Latin to avoid a search for forced translations (“recollective”?), and to avoid wrong connotations of others (“memorable”). As for the important role of Figure S as a mechanism for channelling an initial supposition or hypothesis, that is largely taken up, as we will see in the next chapter when we discuss the Questions and Rules, under the first question of “Whether” (*Utrum*).

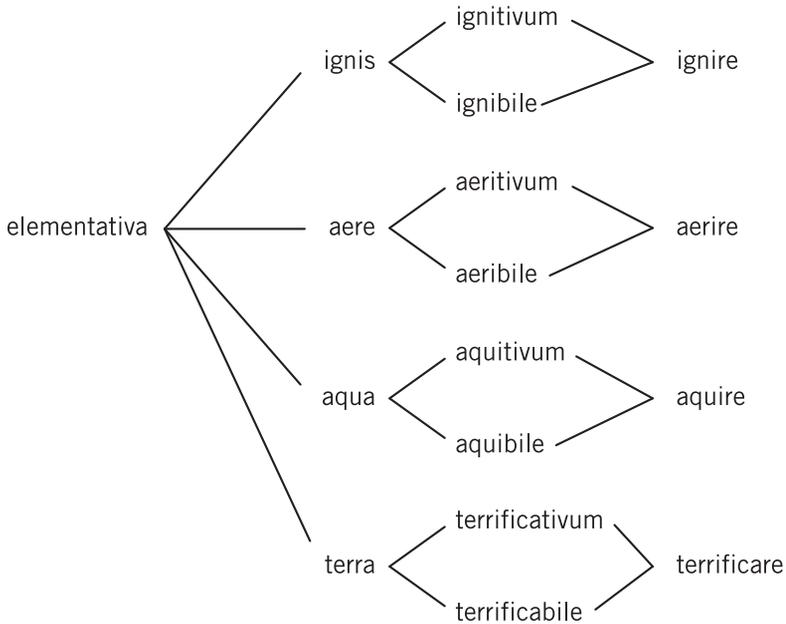


In the later *Liber de homine* (1300) Lull explains the foundation of this structure in the semblances of the dignities: “memory, intellect, and will are made up of essential, substantial and natural goodness, greatness, duration, power, virtue, truth and delight.”⁵¹ He then goes on to say that “memory is made up of essential, substantial and natural *recolitivum*, *recolibile*, and *recolere*; intellect of essential, substantial and natural *intellectivum*, *intelligibile*, and *intelligere*; and will of essential, substantial and natural *volitivum*, *volibile*, and *velle*. And this is so that the intellect can take those species which acquires by understanding (*intelligendo*), and place them in its own foundations, that is, so that in its own essential, natural and substantial understandable (*intelligibile*) it can make them understandable (*intelligibiles*) and understood (*intellectivas*).” (*ROL XXI*, 170–1; *ORL XXI*, 15–16)

This structure is similar to that of the elements, which can be displayed as:⁵²

⁵¹ *Delectatio* as a synonym for *gloria*.

⁵² These charts are taken from Pring-Mill 1961, 161–7, reprinted in Pring-Mill 1991, 106–9. As Pring-Mill points out, and displays in a chart on p. 167 (109 of the reprint), the correlatives can be regrouped so that all the *-tivum* come under “form”, all the *-bile* under “matter”, and all the *-are* under the “act” uniting them.



This structural similarity, however, is not limited to the rational soul and the elements, but now extends, as we have explained, to the whole ladder of being, uncreated and created. It is this new all-inclusive vision that will permit Lull to present a unified vision of the world in works such as the *Tree of Science*, where the semblances of the dignities appear as the roots of the trees, and hence as the foundation of all reality.

The last transformation of the quaternary phase

In the last work of this phase, the *Quaestiones per Artem demonstrativam seu inventivam solubiles*, we find a terminological reorientation that brings us one step closer to the ternary phase. After a little paragraph of introduction, Lull presents the

principles of this Art, which are goodness, greatness, eternity, power, wisdom, will, virtue, truth, glory, etc.; difference, concordance, contrariety, beginning, middle, end, majority, equality, minority. (*MOG IV*, iii, 1: 17)

These are the components of Figures A and T (without showing or even naming the figures themselves; the work in fact presents no

figures at all), reduced to nine each, and all referred to as “principles”, exactly as in the ternary phrase. With this he initiates a terminological distinction which will become characteristic, and which will be more fully explained in the next chapter. For now it is sufficient to say that “goodness”, “greatness”, etc., as foundations for the Art and the reality it describes, are now called *principia* (*començaments* in Catalan), whereas when speaking of God, as he does in the “*Quaestiones de Deo*” of this work, they are called “dignities”.⁵³

This work not only lacks the figures of the ternary Art, but also the characteristic definitions of the new phase. On the other hand, it does contain an application of these eighteen principles to a series of ten subjects—God, the afterlife, angels, the soul, the imaginative power, the sensitive, vegetative, elementative, and motive powers, and morality—a list almost identical to the one we will discuss at the end of this chapter under the *Ars inventiva veritatis*.

What happened to the other figures?

After the panoply of figures we have studied with the *AD*, this is a question the reader might well be asking. As will become clear when we deal with the Art of the ternary phase, the only figures retained from the quaternary phase are those of A and T. This means that S, V, X, Y, and Z, along with the Elemental and the Demonstrative Figures have been dropped by the wayside. Of the original nine (that is, without counting the more or less ancillary figures of Theology, Philosophy, and Law), seven have vanished. Where have they gone?

To begin with, the restructuring of the Art means that Figure V and the Demonstrative Figure have been recycled in a different form. The virtues and vices will now appear as part of the alphabet of the Art, and will be treated under the last of the Nine Subjects,⁵⁴ “Artifice”, there under the first of three headings: morality, the liberal arts, and the mechanical arts. The functioning of the Demonstrative Figure will basically be replaced by the ternary Fourth Figure, with its corresponding elaboration in the Table.

The quaternary Art functioned by making chains of comparisons, in which Figure X provided the principal criteria against which the

⁵³ *MOG* IV, iii, 2–54: 18–70.

⁵⁴ For which see p. 160 below.

resulting arguments could be evaluated. Since the ternary Art will now be based much more on the definitions of concepts—and by the combination or “mixture” of definitions—it will evaluate arguments by whether or not they agree with the definitions, and therefore it no longer needs these criteria.

The elements have become, like V, a rung on the ladder of being, the second-to-last of the Nine Subjects, and their separate figure disappears from the Art. Moreover, their formation from the semblances of the dignities has, as we have seen, made Llull articulate them in a new fashion, one which receives perhaps its most complete treatment in the Elemental Tree of the *Tree of Science*.⁵⁵

It is Figure S, however, the letters of which occupy most of the alphabet of the AD, and which form the backbone of almost all the discourse of that work, whose disappearance might seem most surprising. The functioning of the three powers of the soul, however, has, as we showed above, now been reworked into the correlative (trinitarian) structure that pervades all of being. The positive or negative sides of the powers of the soul, so crucial to Figure S, are now implicit in the functioning of the Art.⁵⁶

Llull’s new, totalizing structure permits a fundamental mixing of ontology and epistemology, where the latter can work its way vertically up and down the ladder of the former, as well as horizontally at each level with the correlatives, using of course, the mechanisms of the Art. Or, to put it another way, the investigating subject—consisting of the acts of the powers of the soul, formerly making up Figure S—is now identical in structure to the object being investigated, or rather both are parts of a single unified structure.⁵⁷

⁵⁵ In the “Branches” of the Elemental Tree he even explains how one can, for instance, derive from the elements the three dimensions of space. In the ternary phase elemental theory is only used extensively in medical works. The quaternary elemental figure even resurfaces in the *Liber de levitate et ponderositate elementorum* of 1294, but then the *Liber de regionibus sanitatis et infirmitatis* of nine years later organizes things differently. For the surprising and seemingly anomalous use of elemental theory in the *Logica nova*, see p. 196ff. below.

⁵⁶ As Llull explains in the AA (*ORL XVII*, 5–6; *ROL XXIX*, 121–2), referring the reader to the Rules of the AIV (see *MOG V*, 37–38).

⁵⁷ See the beautiful passage from the *Art amativa* (*ORL XVII*, 25–29; *ROL XXIX*, 134–7) quoted in Ruiz Simon 1986, 88–90, elucidating all these connections between the semblances of the dignities, form/matter/act, the various rungs of the ladder of being, the investigating subject of the powers of the soul, and the Art.

The psycho-sociological twists and turns which, as we saw in the previous chapter, can be so fascinating with this figure, have now either been dropped altogether, or subsumed in the much more general mechanism of inquiry found in the Questions and Rules. In general, one could say that separate purely epistemological figures such as X, Y, or Z, as well as S insofar as it is auxiliary to the epistemological process, are no longer necessary because the task of Lull's epistemology now becomes that of following his ontology. Even Figure T, which was such an important tool of inquiry in the *AD*, has now become—as will be explained below⁵⁸—more centrally ontological. That it continues to be used epistemologically is part of the mixture of the two which Lull now practices so fully.

In a more general way, the heterogeneous foundations of the quaternary Art, among which the task of the “artist” is to discover their analogical or metaphorical relationships, will now be replaced by the univocality of a single set of principles (the eighteen of the first two figures) studied by means of a very general set of Questions and Rules. Instead of a web of lateral connections, we will now see a vertical structure, symbolized, as we have said, by the *Tree of Science*.⁵⁹

The Last Stages of the Transition

The first two works of the ternary Art

The last stage of the transition was accomplished by the first two works of the ternary phase: the *Ars inventiva veritatis* (*AIV*) and the *Art amativa* (*AA*). They are curiously hybrid works, at the same time works of the Art, and, as Jordi Gayà has pointed out, applications of the Art, with the first guiding the intellect to the truth (*verum*), and the second the will to the good (*bonum*).⁶⁰ As if warning us not to consider these works as belonging to two different realms of his endeavor,⁶¹ in the prologue of the *AA* Lull explicitly explains how they are paired (we leave the two

⁵⁸ See pp. 131–3.

⁵⁹ Notice that it is the *Tree of Science*, in the singular, not a series of trees of different sciences. For the preceding paragraph, see Ruiz Simon 2005, 171.

⁶⁰ *SL* 45–46 (2005–6), 120.

⁶¹ A tradition that began with the Mainz edition, where the first is in *MOG V*, which contains only works of and on the Art, and the second in *MOG VI*, where it is followed by the *Tree of the Philosophy of Love* and the *Flowers of Love and Flowers of Intelligence*.

crucial terms in Catalan, because the first has no proper translation, and the second, as the reader has had cause to observe, has a much broader meaning than the modern “science”.⁶² There he says how he has compiled:

... this *Art amativa* in which *amància* is presented, just as was done with *ciència* in the *Ars inventiva [veritatis]*. For just as *ciència* comes under the heading of intellect, in the same way *amància* comes under that of will. And since *amància* is defective without *ciència*, and *ciència* without *amància*, this is why in this [study of] *amància* we use the principles and techniques of the *Ars inventiva [veritatis]* and we follow the manner of that science. This is why we adapt this *amància* to the *ciència* of the *Ars inventiva [veritatis]*, the better to treat it artificially [i.e., using the Art] and make it known by means of *ciència*.⁶³

This passage is important not only as a programmatic statement on the inseparability of the powers of the soul of intellect and will, but also for showing how and why the two works are intimately related, with the *AA* a structural calc of the *AIV*.

That these works indeed initiate the ternary Art is the result of two absolutely essential aspects of that phase. The first is the reduction of the figures from twelve (or sixteen) to four, something which since the time of Le Myésier has been recognized as marking the beginning of a new phase of the Art,⁶⁴ and the second is the presence of definitions of the eighteen principles. Moreover, with the *AIV* and the *AA* we find the form of the figures and the wording of the definitions presented in a way which was to remain absolutely standard throughout that phase.

Aside from these foundations, however, these two works have none of the other structures of the finished ternary Art from the *TG* on. First

⁶² In the *Taula d'esta Art* Lull defines the two terms as “*Amància* is said of the will which loves (*que ama*), and *ciència* is said of the intellect which understands (*enteniment qui entén*)” (*ORL XVII*, 389).

⁶³ *ORL XVII*, 4; *ROL XXIX*, 120. The medieval Latin translation is called *Ars amativa boni* and has some minor differences in the quoted passage. This passage was commented in Pring-Mill 1961, 132 and Pring-Mill 1968, 116, reprinted in Pring-Mill 1991, 224 and 92. I cannot resist quoting, in this connection, the opening statement of the final section of *Question* in the *AA*: “A question is the stimulation (*excitació*) of the intellect (*enteniment*), so that it can find rest in its understanding (*entendre*), and it is the stimulation of love, so that it can find rest in its loving” (*ORL XVII*, 236; the Latin text of *ROL XXIX*, 298, is slightly different).

⁶⁴ See the passage from the *Vita coetanea* quoted on p. 5 above which mentions the new basis of four figures, along with n. 55 somewhat later in the same chapter, with its reference to the new alphabet of the *Ars inventiva veritatis*.

of all, neither the *AIV* nor the *AA* have an Alphabet.⁶⁵ Their Rules and Questions, which might seem to relate to those of the new phase, in fact having nothing to do with the standard forms initiated with the *TG* several years later. In both works they take up specific subjects, with Rules in the *AIV* such as “Supposition”, “*Modus essendi et intelligendi*”, or “Demonstration”, in the *AA* such as “Definition”, “Simplicity and composition”, or “Reality and reason”, and one that appears in both works, “Transcendent points”.⁶⁶ If the reader will compare these Rules and Questions with those of the next chapter, he will see that they have none of the generality of the later models, nor are they paired with one another, as is the case from the *TG* on, where, for instance, the Question of “Whether?” is coupled with the Rule of “Possibility”, the Question of “What?” with the Rule of “Quiddity”, and so on.⁶⁷ In addition, another fundamental component of the ternary Art is not yet in place: the Table. In its place we have a holdover from the *AD*, in a chapter on Conditions, but carried out in an unusual way.⁶⁸ Instead of the earlier mechanism where T was successively introduced into the compartments of the rest of the Art, we have Table-like combinations derived from the Fourth Figure. Here, however, rather than a general method exemplified by multiple solutions of a single question, we find that the combinations generate chains of proverb-like short statements, somewhat reminiscent of the *Liber propositionum secundum Artem demonstrativam compilatus*, or even more of the *Flowers of Love and Flowers of Intelligence*.⁶⁹ As for the Subjects, they appear in the *AIV*, in a list similar to that given above under the *Quaestiones per Artem demonstrativam seu inventivam solubiles*, but not yet the definitive version proposed some three years later, and here treated as the rungs of the ladder of the

⁶⁵ That printed on *MOG* V, 13, is an invention of eighteenth-century editor; it is in none of the early manuscripts.

⁶⁶ Plus the “Majority of end (or goal)” of the *AIV* which is similar to the “Intention and end (or goal)” of the *AA*.

⁶⁷ See p. 137ff. below.

⁶⁸ Note that the *AIV* begins by saying that “This present Art descends from the *Ars demonstrativa*”, a reference which disappears from subsequent works of the ternary phase.

⁶⁹ The similarities can be seen by comparing the “conditions” of the *AIV* in *MOG* V, 14ff., with the “flowers” in *SW* II, 1233ff. The reader should be warned that the Table-like letter combinations in this section of the *AIV* are an invention of the *MOG* editors; in the manuscripts the little proverb-like phrases are only numbered, as with those of the *AA* printed in *ORL* XVII, 154ff., or *ROL* XXIX, 232ff.

Transcendent points (and thus not as a division of the Art itself, but of one of its Rules).⁷⁰

The *AIV* and the *AA*, therefore, lacking as they do the new Alphabet, the Table, the standardized Questions and Rules, the equally standardized nine Subjects, as well as the generality of the later works, are not yet fully representative of the ternary Art. So while they indeed initiate the ternary phase of the Art, they are still transitional to its fully developed forms.

Finally, this little excursus about the *AIV* and the *AA* has nothing at all to do with their importance; my only desire here was to treat their formal role in the progress of the Art. Indeed, from the point of view of content rather than form, they are notable as works in which Lull allows himself unusually extended explanations of different doctrinal and epistemological points of his system, such as the Rules mentioned on the previous page.⁷¹ One should also emphasize the Prologue of the *AA* in which Lull goes into fascinating details concerning the language(s) and dissemination of his work.

⁷⁰ The only difference is that the *AIV* lacks the section of “*Quaestiones alterius vitae*” (on the Resurrection, i.e., the afterlife) found in the *Quaestiones per Artem demonstrativam seu inventivam solubiles*. The *AA* treats these subjects in an abbreviated form under the same Transcendent points, as well as under other Rules. The definitive version of the Nine Subjects would be incorporated into the Art with the *Taula general* and the *Arbre de filosofia desiderat*.

⁷¹ The Rule of “Simplicity and composition” from the *AA* is the one referred to in n. 57 above, and of which Ruiz Simon says that it is a passage “singularly important for understanding the meaning of the Lullian Art as it crystallizes in the ternary phase”.

CHAPTER FOUR

THE TERNARY PHASE

General Remarks

In presenting the Art of the ternary period, I will do something similar to what was done earlier with that of the quaternary: concentrate on a single work, or rather one work and its companion. The main work was the culmination not only of the ternary phase, but of all of Lull's system, which is why he called it the *Ars generalis ultima* (*AGU*), explaining at the beginning of the prologue:

God, with the help of your supreme perfection, here begins the *Ars generalis ultima*. Since we have written many Arts which were general, we would like to explain them in clearer fashion by means of this work, which we call "last" (*ultima*) because we do not intend to write any other.¹

The companion work to the *AGU* is the *Ars brevis* (*AB*), which is a résumé, a kind of skeleton or outline of the larger work, with none of its explanatory material. Lull was quite clear about this relation, to the point of beginning the shorter work quoting the beginning of the longer one:

God, with the help of your grace, wisdom, and love, here begins the *Ars brevis*, which is a replica (*imago*) of the *Ars generalis*, that is of the work beginning "God, with the help of your supreme perfection, here begins the *Ars generalis ultima*." (*SW I*, 579; *DI* 297)

A bit further on in the same Prologue he tells us that "This book is divided into thirteen parts, just like the *Ars magna*." And indeed the shorter work follows the longer one section by section, frequently adding remarks such as:

¹ *ORL XIV*, 5. Many of my translations of the *AGU* are either taken or adapted from Yanis Damberg's translation at <http://lullianarts.net/Ars-Magna/ars-magna.htm>.

If, however, the reader would like to know more about applying terms, he should turn to [the equivalent section of] the *Ars magna*, where we discuss the matter at length.²

One can surmise that he did this doubling of works—the only time he did it—because the length, complexity, and definitive nature of the *AGU*'s presentation of Lull's system made him feel he needed an outline for mnemonic reasons, one which would make the structure of the larger work clearer for students, and would therefore be usable as a text for lecturing in Paris, where he went shortly after completing both works.³

For a clearer understanding of the ternary Art, rather than studying these two works section by section, I thought it might be best to analyze them thematically. This involves separating their material into three themes: (1) foundations—the groups of concepts on which the Art is based; (2) combinatorics—the different ways in which they can be combined; (3) use and practice—how Lull uses the previous two categories to set up his demonstrative and inventive discourse (under this last heading we include the questions with which every version of the Art ends). The following little chart will show how these themes are related to the various parts—represented by Roman numerals—into which the *AGU* and *AB* are divided.

Foundations	Combinatorics	Use and practice
I. Alphabet		VI. Evacuation of Third Figure
II.1 First Figure (A)	II.3 Third Figure	VII. Multiplication of Fourth Figure
II.2 Second Figure (T)	II.4 Fourth Figure	VIII. Mixture of principles & rules
III. Definitions		IX. Nine Subjects
IV. Questions & Rules	V. Table	X. Application (Hundred Forms)
		XI. Questions
		XII. Habituation
		XIII. Methods of teaching the Art

² *SW* I, 616; *DI* 336. Notice that after the first sentence quoted above, in the *AB* Lull always refers to the *AGU* as the *Ars magna*.

³ We know in fact that he used the *AB* for this purpose, because of the letter he received from forty members of the University of Paris in February, 1310, giving their approval to his lectures on the work. See Hillgarth 2001, 80–82. The similar *Ars compendiosa* (or *Brevis practica Tabulae generalis*, to give it its more correct name) of 1299 might equally well have been used on his previous visit to Paris as an abbreviated text for lecturing on the *TG*.

As can be seen from the chart, the figures have been reduced from 12 (or sometimes 16) to 4. Of these—in strong contrast with the quaternary phase—only the first two present concepts; the last two are abstract figures used to combine these and other concepts. Moreover, as will become clear when we explain the first figure, graph theory now plays almost no role at all.

The Alphabet

First Figure	Second Figure	Questions and Rules	Subjects	Virtues	Vices	
B	goodness	difference	whether?	God	justice	avarice
C	greatness	concordance	what?	angel	prudence	gluttony
D	eternity or duration	contrariety	of what?	heaven	fortitude	lust
E	power	beginning	why?	man	temperance	pride
F	wisdom	middle	how much?	imaginative	faith	accidie
G	will	end	of what kind?	sensitive	hope	envy
H	virtue	majority	when?	vegetative	charity	ire
I	truth	equality	where?	elementative	patience	lying
K	glory	minority	how? and with what?	artifice ⁴	pity	inconstancy

The first two columns, as we will see in a moment, represent reworkings of Figures A and T of the quaternary Art. The third column introduces a fundamental investigative tool of the Art, and the last three refer to applications of the Art, or rather, topics to which the investigative mechanisms of the Art can be applied. With the *AGU* and the *AB* they appear for the first time in the Alphabet of the Art.⁵ Earlier works of the ternary Art, if they had any alphabet at all, only had the first three columns, which present the foundations of his system.⁶ We will

⁴ Thus in the *AGU*; in the *AB* it is given as “instrumentative”. See below, p. 163, for a discussion of the two terms.

⁵ With the exception of the *Liber de praedicatione* of 1304, but there in a different order, and with an added column, all more appropriate for the making of sermons. They are Subjects, Virtues, Vices, Rhetoric, First Figure, Second Figure, Questions and Rules. It is curious that this order, in a work on sermons and thus ultimately on the art of rhetoric, is more or less similar to that of Agrippa von Nettesheim’s attempt to turn Llull’s Art into a Renaissance Art of discourse. See Lullus 1996, II, 790ff.

⁶ Aside from the work mentioned in the previous note, the only works of the ternary Art which presented an Alphabet were the *Art of Proposing and Solving Questions*, and the *Ars compendiosa*. Some minor works, such as the *Liber de lumine*, *Liber de intellectu*, *Liber*

see later how the discourse of the *AGU* and the *AB* is in fact based on the mixture of Principles (of the first two columns) and Rules, which are used to examine the Subjects, virtues and vices.

Three aspects of this Alphabet will probably strike the reader. The first is that, as opposed to the *AD*, where a few of the letters represented entire figures and all the rest referred to the components of a single figure (that of S), here they represent only individual concepts, and most usually those of the first two columns. The second is its polysemy, the fact that each letter can stand for anything up to six concepts. In discussing the functioning of the Art, we will see how Lull deals with this problem, and makes clear to the reader which references are being used at any particular moment. The third is that everything has now been made to fit into a scheme of nine. The first two figures have been reduced from sixteen and fifteen components respectively. With the third column Lull has done it by squeezing two components into the last compartment, glossing over the fact that there are in fact ten Questions and Rules. Finally, he fills out the traditional seven virtues by adding two to each list. This generalization of the number nine—the square of three—is why this period of the Art has been referred to as the ternary phase.⁷

Finally it should be remarked, with regard to this Alphabet, that in the ternary phase Lull no longer uses letters in the actual discourse of the Art, nor does he draw boxes around pairs of concepts, and he almost never refers to the colors of the triangles of Figure T. This means that we no longer find ourselves confronting statements like:

de voluntate and *Liber de memoria* (all edited in *ROL* XX) have an Alphabet of the same three sets of terms, with the addition of a fourth term specific to each work, *lumen*, *intellectus*, *voluntas* and *memoria* repeated nine times. The Alphabet appearing along the top edge of the *Breviculum* miniature reproduced above on p. 20 is the one that might have appeared in the *AIV* if it had presented an Alphabet, but Le Myésier's point here is not so much textual accuracy as to symbolize the change to this new work of the ternary phase. (It should be stated that the Alphabet of that work printed in *MOG* V, 13, as well as the Table-like triads of letters on p. 14ff, seem to be inventions of the eighteenth-century editor; they are in none of the earliest manuscripts.)

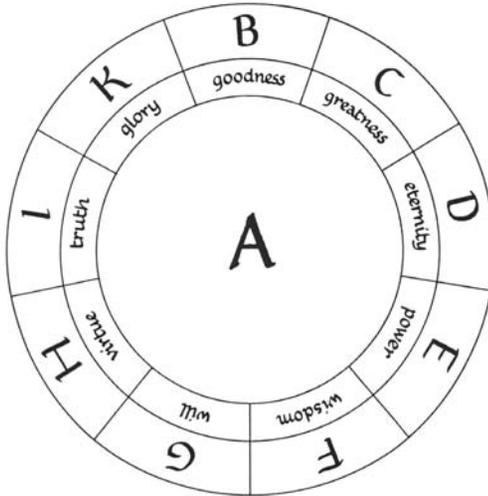
⁷ For the Latin (or Catalan) of the terms of the Alphabet, those of the first column can be found in the first nine terms of the list of p. 33 above; of the second column in the central three triangles of Ch. 2, n. 30, above; of the third column in our discussion of the Questions and Rules below; of the fourth column in that of the Nine Subjects below; of the last two columns on p. 36 above, taking into account the two extra virtues of *patientia* and *pietas*, and the vices of *mendacium* and *inconstantia*.

And since all of A cannot fit into S, E is transformed into N, and N into E by means of the black triangle and the compartment of predestination free will.⁸

As a result the actual discourse of the Art is now much less alarmingly algebraic in appearance.

Foundations

The First Figure, denoted by A



In the *AB*, the figure is described as follows:

The First Figure is that of A, and it contains nine principles, to wit, goodness, greatness, etc., and nine letters, to wit, B, C, D, E, etc. This figure is circular to show that any subject can become a predicate, and

⁸ See above, p. 70 for this passage. At the very beginning of the first work of the ternary phase, the *AIV*, he explains how from now on he will avoid the Alphabet of the *AD* and use the letters only in the figures (which includes the Table, and its articulation in structuring series of questions), and how in the text he will only use “the terms or principles of this Art under their own proper meanings”; see the prologue of that work (*MOG V*, 1).

vice versa, as when one says, “goodness is great”, “greatness is good”, and so on.⁹

Notice that, as opposed to the version of this figure in the quaternary phase, here there is no mention of God, nor even of what the letter A refers to.¹⁰ One in fact gets the impression that Llull prefers the more neutral designation “First Figure”, and has just kept the letter A as a kind of vestigial reference to the previous phase.

Nor is there any mention of “dignities”. When Llull does give the components of this figure a name, they are called “principles”, a term previously reserved for the components of Figure T, but now applied, as we will see when we come to the definitions below, to those of both of the first two figures. The word “dignities” is now reserved for these principles when referred solely to the first subject, that of God, and even then it is not always limited to the terms of the First Figure, but can include those of both figures, with the exception of “contrariety” and “minority”, and often “majority” as well, which are not applicable to the Divinity.¹¹

⁹ *SW I*, 582; *DI*, 300–1. The Figure A of the *AB* and the *AGU* drawn in *ROL XII*, 197, and *ROL XIV*, facing p. 10, with an extra circle listing the adjectives corresponding to the nouns (“good” corresponding to “goodness”, “great” to “greatness”, etc.), as Viola Tenge-Wolf explains in *ROL XXVII*, 114*, n. 247, is in none of the older manuscripts; this circle seems to be a 16th-century addition (it is already present in Lavinheta’s *Explanatio* of 1523; see Lavinheta 1977, 44). As she also points out, this extra circle has given some scholars the idea that Figure A is rotary, when in fact only the Fourth Figure is rotary.

¹⁰ Certainly not to God, who in the ternary Art (as we just saw in the Alphabet) is the first of the nine Subjects and is assigned the letter B.

¹¹ Giving a list of fifteen dignities. Llull explains this clearly in the *Investigatio generalium mixtionum secundum Artem generalem* (*ROL XVII*, 415): “We say that the dignities or attributes of God are goodness, greatness, along with the other principles of the First as well as of the Second Figure, except for contrariety, majority and minority.” In the *AB*, however (*SW I*, 606–7; *DI*, 326), as well as the *TG* (*ROL XXVII*, 60; *ORL XVI*, 344), Llull gives sixteen, saying that “In God there is no contrariety or minority, for they are principles having to do with privation and defect. Nevertheless, in God there is majority with respect to other beings...” The first set of fifteen dignities also constitute the roots of the Divine Tree of the *Tree of Science* (note that “contrariety” is also lacking in the roots of the Celestial and Angelic Trees). In the *AGU*, however, under the subject of God, Llull only lists the nine dignities of the First Figure.

He sometimes refers to the Principles as “similitudes” or “semblances” of the divine dignities, which is indeed how they began their later career in the later quaternary phase, but in the ternary he does so seldom and never in a work of the Art, because there he has exchanged a top-down approach, for a general one *also* applicable to the top of the ladder of being. See Ch. 6, n. 81 below for more information.

This point cannot be sufficiently over-emphasized. Not only because many recent studies of the *AB* still discuss the dignities, when the word does not even appear in that work, but also because this is much more than a mere matter of terminology. Ever since the final transitional phase of the quaternary Art, Llull has aimed at making his system more and more general, for reasons given in the prologue of the *AGU*:

The [human] intellect requires and desires one science which is general to all sciences, one with its own general principles in which the principles of the other individual (*particularium*) sciences are implicit and contained, like any particular in a universal. (*ROL XIV*, 5)

And this generality began with the Principles, a fact of which we will see further evidence when we study their definitions. With the *TG*, when other components of the Art take on the same nature, it is now finally that he can call his Art “general” and that the term begins appearing in the titles of works, as well as with almost obsessive frequency in the principal texts of the Art.¹² It was by this generality that Llull hoped to make the Art a science of sciences, and this was one of the aspects that most fascinated his followers in succeeding centuries.

Notice finally that Figure A has none of the crisscrossed lines of the quaternary version.¹³ The reason is that we are no longer dealing with a graph whose lines were mainly used to join concepts that were “concordant”: the dignities were all concordant with one another, each virtue with another virtue, each vice with another vice, etc.¹⁴ In the ternary phase the word “concordance” appears nowhere in the description of Figure A. The vaguely related (but not synonymous) concept of convertibility now describes only one of their possible relations. In the *AGU* this is expressed in terms of the Subjects:

By this circular motion the artist can know which things convert and which not, as for instance God and good, which can convert, but not

¹² Word searches in the three principal works of the ternary Art show that in the *TG* it appears 107 times, in the *AGU* 185 times, and in the far shorter *AB* 28 times. Before the *TG*, the other components of the first works of the ternary phase, as pointed out above on p. 119, are not yet really general.

¹³ As often represented in modern editions of the *AGU* and *AB*, including my own in *DI*, 300. This would seem to be another fashion (see n. 9 above) that began with Lavinheta.

¹⁴ The only exception to this rule was with the disconnected graphs of Figures S and T, whose squares and triangles represented quaternary and ternary relations respectively.

Comparing this Figure T and the quaternary version among the color illustrations, the most obvious difference is the loss of its first and last triangles, with the corresponding reduction of its principal components from 15 to 9.¹⁷ The first triangle of God/creature/operation always seemed somewhat out of place among the other more abstract, relational concepts of Figure T.¹⁸ It had, in fact, been little used, and then mainly in works proving the Articles of Faith. Thus, as part of Lull's program to make the Art appear more religiously neutral and more general, God and his creatures would now be dealt with in the applications of the Art, among the Nine Subjects which we will discuss below. The last triangle of doubt/affirmation/negation has been directly relegated to the first of the Ten Questions or Rules, for which also see below. The other noticeable change in the figure itself is that Lull now either dispenses with the color of the triangles, or if he does refer to it, he does so in completely residual fashion.¹⁹

Lesser, but nonetheless interesting, differences include a greater precision in the use of some of the terms. For instance, in describing the triangle of beginning/middle/end, he says:

The word "cause" written in the angle of "beginning" stands for the efficient, material, formal, and final cause. The words "quantity" and "time" refer to the other nine predicaments, and to those things that can be reduced to them.²⁰

This explicit reference to Aristotle's four causes and to his predicaments (or categories), contrasts with something only implicit in the vaguer language of the *AD*.²¹

¹⁷ In the first work of the ternary Art, the *AIV*, the comparison is made explicit: "The Second Figure is made up of three triangles taken from the *Ars demonstrativa*." *MOG* V, 6 (quoted in *ROL* XXVII, 118*, n. 260).

¹⁸ Cf. Platzeck 1962–4, I, 208, as well as Lulle 1991, 58 (both quoted in *ROL* XXVII, 119*, n. 261).

¹⁹ In the *AGU*, his initial description of the Second Figure says nothing about the colors of the triangles; he only mentions them where commenting on the triangles in the second half of the section on the figure (*ROL* XIV, 14–17). More important, with the exception of two almost passing references (*ROL* XIV, 62 and 74), he never uses the colors in the text, as he had continually in the *AD*. In the *AB* color is never even mentioned.

²⁰ *SW* I, 584; *DI*, 303, references which also apply to the next two quotations. For a list of the ten Aristotelian categories or predicaments, see n. 108 below.

²¹ See p. 42 above. Curiously enough, he gives more explanations of some of the secondary terms of Figure T in the much briefer *AB* than in the much longer *AD*.

Then in describing the next angle of the same second triangle, he says:

The angle of “middle” contains three species of middle. First there is the conjunctive middle, which exists between subject and predicate, as when we say, “man is an animal.” For between man and animal there are middle terms, such as their life and body, without which man would not be an animal. Then there is the middle of mensuration, which refers to the act existing between the doer (*agens*) and the doable (*agibile*), like loving (*amare*) between the lover (*amans*) and the lovable (*amabile*). And then there is the middle between extremes, like a line between two points.

The first species of “middle”, that between subject and predicate, is another introduction of logic into the *AGU*, and curiously, even more insistently in the *AB*, one which again we will discuss it in the next chapter.²²

The second species of “middle”, that of mensuration, involves another innovation in connecting Figure T with the correlatives, a connection continued in the next angle of this triangle, that of “end”, of which Lull says:

The second species is the end of termination, which refers to the extremities, like the two points that terminate a line, or like the lover (*amans*) and beloved (*amatus*) in relation to loving (*amare*).²³

“Absolute” and “Relative” Principles

The terms “absolute” and “relative” principles which for centuries now have been applied respectively to Figures A and T of the ternary Art²⁴ seem to have been an invention of the sixteenth-century commentator, Bernard de Lavinheta, whence it passed to Agrippa von Nettesheim, and from there into the modern literature.²⁵ Lull never uses these terms

²² See p. 224 below.

²³ The wording here is strongly reminiscent of the earlier *Book of the Lover and the Beloved*; see *DI*, 179.

²⁴ Curiously enough, critics have almost never used them for the quaternary Art, where they would not be so incorrect. Perhaps this is because, among the welter of figures of that period, these two have not been singled out with special descriptive terms.

²⁵ Lavinheta 1977, 45 and 54, and for Agrippa, Lullus 1996, II, 799 and 805. Although he might have gleaned them from some earlier source, as far as I have been able to discover, these terms are Lavinheta’s invention. He calls them *principia absoluta* and *principia respectiva*, whereas Agrippa changed the *principia* to *praedicata*, which is how

in this connection, nor does he make any terminological distinction in the ternary Art between the components of the two figures; they are all just “principles”. This post-Lullian innovation is part of what one might call the loosening of the very specific structures and contents of the Lullian Art to adapt them to broader Renaissance techniques. Nor is it just a minor quibble over words; calling the principles “absolute” and “relative”, in the first case implies things Llull was careful not to imply, and in the second omits things he wished to imply.

When Llull uses the word “absolute” he means it in a superlative sense, as when he says:

We call these principles “primary” simply and absolutely, not because others descend from them but because they themselves descend from no other.²⁶

Since God is an absolute being, and since he is consubstantial with his dignities, they too are absolute.²⁷ When, however, “goodness” or “greatness” are predicated of anything created, they are not absolute since they do descend from a higher “goodness” or “greatness”, of which they are only the likeness (*similitudo*, *semblança*). Nor must one confuse the concepts of “general” and “absolute”. The principles of the Art are general because, as we said before, they are applicable to all levels of being; they are absolute only when they are applied to the highest level, to God, in which case they can be called dignities.

As for Figure T, from which, as we have said, the word “principles” derived, Llull in the first place wanted to put all eighteen from both figures A and T on an equal footing as foundations of the Art. But more than that, the role of the concepts of this Second Figure have changed since the quaternary Art. In the former phase he “conditioned”

they appeared in the much reproduced Alphabet of the Art printed in Lullus 1996, I, facing pp. 1, 44, and 218. At some point in the intervening centuries these terms were further modified to “absolute and relative principles”. Whatever the actual terms used, it was probably this distinction which led some critics to call the principles of Figure A “substantial” and those of T “accidental”, a confusion nicely undone by Tenge-Wolf in *ROL* XXVII, 141*–143*, where she discusses at length a question just touched on in n. 41 below. See also the definition preceding n. 29 here below.

²⁶ *ROL* IX, 217; see p. 213 below for the entire passage from which this sentence is extracted.

²⁷ See, for instance, the works entitled *Liber de ente absoluto* and *De ente simpliciter absoluto*, as well as p. 225 below and the section of the next chapter on the superlative degree.

his Art by introducing T into the compartments,²⁸ that is, he used its concepts, such as “difference”, “concordance”, or “contrariety”, as tools for studying the consequences of relationships between other concepts. In the ternary phase, however, the components of Figure T take on a higher ontological status. Compare, for instance, from the beginning of the next section below, the remarkably similar definitions of “greatness” from the First Figure with “difference” from the Second:

Greatness is that by reason of which goodness, duration, etc. are great.

Difference is that by reason of which goodness, etc. are clearly distinguishable from one another.

In the *TG* Llull goes on to explain about the second that:

Difference must be a universal and substantial principle in order that things may be substantially distinct and different one from the other.²⁹

After continuing in his lengthy explanation of why “difference is one of the principles of this science”, he says that for all these reasons:

Through difference one arrives at simple principles, for it is the light in which appear the real reasons (*rationes reales*), and by means of which appear their composition, and their works.³⁰

In the *Art of Proposing and Solving Questions* Llull is even more explicit. After explaining how “difference” is substantial and “has within itself substantial *differentiativum*, *differentiabile*, and *differentiare*”, he says that:

It is the river and font from which are born and derived all the differences which exist in things with which substantial difference is mixed and joined, since each of them receives its semblance and impression by reason of the composition and conjunction of all of them together.³¹

²⁸ See the passage in Ch. 2, preceding n. 89 above.

²⁹ *ORL* XVI, 323; *ROL* XXXVII, 38.

³⁰ *ORL* XVI, 324; *ROL* XXXVII, 39. By “works” (operationes), of course, he means the activity which unfolds in correlative structures. The slightly expanded definition in the *Tree of the Philosophy of Love* has a similar emphasis: “Difference is that by reason of which goodness, greatness, and the others are clear and real reasons” (*ORL* XVIII, 74). So it is now no longer solely an epistemological or logical instrument, but one incrustated in the structure of the reality it describes.

³¹ *MOG* V, v, 27: 385. For the title of this work, traditionally called *Lectura super Artem inventivam et Tabulam generalem*, see n. 120 below. The passage is quoted in Sala-Molins 1974, 82, n. 136, at the end of an excellent section on “difference”.

Or in other words, “difference”, along with all the other concepts of Figure T, is now an archetypal form from which all other differences derive, with an ontological status identical to the concepts of Figure A.³²

This can probably best be understood if we examine how Llull works out the ternary relations of the Table we will study a bit further on. If we symbolize the components of Figures A as x, y, z , and those of Figure T as Q, R, S , we can see that Llull no longer limits himself to studying relations of the form xRy , which is what he did with the quaternary testing of the compartments of the Demonstrative Figure by “introducing T” into each of them, but instead can now study patterns of the form xyz, xyQ, xQR , and QRS . All eighteen principles are now on an equal footing. The importance of this change can scarcely be overestimated. One historian of mathematics has explained this as an advance of Llull (and Leibniz) over Aristotle, in that “antique thinking was unable to [take] the final step in abstraction, and to admit logical relations not only between complete logical subjects but between relations as well.”³³

This does not mean, of course, that the components of Figure T cannot continue to carry out their function of judging relations between two other concepts; it is that now they can also be applied to studying the relationships within a single being (such as its constituent principles of “goodness”, “greatness”, etc.), within a principle itself (its correlatives), between the species of a genus, or between the individuals of a species,³⁴ or, as we have just seen, they can be studied in relation to other principles of the same figure. So, although not completely wrong

³² Note the similarity with Platonic usage, of which Cornford says “Difference is not a relation subsisting between the two things. Two different Forms are said to ‘partake of the character of Difference’”. Cornford 1935, 284. For the eighteen principles treated as forms, see the *Logica nova* (*ROL* XXIII, 85–90; *NEORL* IV, 68–72), or rather seventeen, because it is precisely “difference” that is omitted since it had been previously treated among the predicables (*ROL* XXIII, 46–49; *NEORL* IV, 29–32).

³³ Fleckenstein 1967, 171. See also Bonner 1994, 67. The only place I have been able to find the term “relative” applied to the components of Figure T is precisely in this context. In the *Lectura Artis quae intitulatur Brevis practica Tabulae generalis* (*ROL* XX, 347) Llull says this figure is composed of “ternary relatives”, giving the example of “beginning”, “middle”, and “end”, neither of which can exist without the other two. So the relativity is *internal* to the figure.

³⁴ This is explained in the parts of the definition of “difference” we have not cited from the *TG*.

as was the case in calling those of Figure A “absolute”, it is all the same quite misleading to refer to those of Figure T as “relative”.

Definitions

One of the most important changes from the quaternary to ternary phase was the introduction of definitions. This permitted Lull to alter his method of demonstration, going from one built on comparisons of concepts based on generally accepted Neoplatonic criteria, to one which relied on agreement or disagreement with definitions of basic principles. Or, as Lull put it, “so that one may use them, affirming or denying in such a way that the definitions remain unimpaired.”³⁵ In addition, he would from now on be basing his Art not on isolated concepts but on propositions, which would allow him to use predication and thus open the floodgates to all his later logical formulations. It was thus a change which, as we will see, deeply affected both the foundations and the structuring of his system.

In the ternary phase, Lull invariably gave his definitions of the eighteen “principles”—those of the first two figures, with, as we have just pointed out, no distinction between them—in the section immediately following the presentation of the figures. From the first work of the period, the *AIV*, till the last two, the *AGU* and the *AB*, these definitions only varied as to whether they were accompanied or not by explanations; their actual wording varied scarcely at all. Because of their importance, we will give them in their entirety.

1. Goodness is that thing by reason of which good does good.³⁶
2. Greatness is that by reason of which goodness, duration, etc., are great.
3. Eternity or duration is that by reason of which goodness, etc., endure.
4. Power is that by reason of which goodness, etc., can exist and act.
5. Wisdom is that by reason of which the wise man understands.
6. Will is that by reason of which goodness, greatness, etc., are lovable or desirable.

³⁵ *SW* I, 589; *DI*, 309. “Unimpaired” is *illaesae* in Latin, from the verb *laedere*, “to injure, hurt, harm”.

³⁶ To this definition is frequently added “and thus good is being and evil is nonbeing”.

7. Virtue is the origin of the union of goodness, greatness, and the other principles.
8. Truth is that which is true concerning goodness, greatness, etc.
9. Glory is that bliss in which goodness, greatness, etc., come to rest.
10. Difference is that by reason of which goodness, etc., are clearly distinguishable from one another.
11. Concordance is that by reason of which goodness, etc., accord in one or in several things.
12. Contrariety is the mutual opposition of certain things as a result of different goals.
13. Beginning is that which is found in everything where there is any question of priority.
14. Middle is the subject through which end influences beginning, and beginning reinfluences end, and thus it participates in the nature of both.
15. End is that in which beginning comes to rest.
16. Majority is the image of the immensity of goodness, greatness, etc.
17. Equality is the subject in which the end of concordance, goodness, etc., comes to rest.
18. Minority is the thing close to nothingness (*SW* I, 589–590; *DI*, 309–310).

Several things will immediately strike the reader. The first is perhaps the fact of these definitions being not by genus and species, as had been classic ever since Aristotle, but by what a thing does, *per agentiam*. The standard definition is extensional, in the form of the kind of proposition which, for reasons we will explain in the section on “Predication” in the next chapter, Llull rejects; it is the one where the subject is included in the predicate: “man is a rational animal”. Rather than this taxonomic approach, Llull prefers one which is intensional, which describes the essence of a thing; and with his dynamic ontology, this essence is its activity, what it does. So for him the best way to define “goodness” is through its ability to produce good.³⁷

³⁷ As with methods of proof discussed on p. 15 above, Llull’s novel definitional techniques did not involve a rejection of traditional definitions, which, as can be seen in some of the examples below, for instance under the Nine Subjects or the Hundred Forms, and above all in the many definitions in Bonner and Ripoll 2002.

It is also worth noting how these definitions basically follow a single model, or rather two almost identical models. For the first definition above one could write “x-ness (or x-ity) is that by reason of which x does x”, and for the others “x-ness is that by reason of which y-ness is x”. The second model, of course, is little more than the logical application of the first to the other Principles. The reader will probably have been struck by the circular nature of the definitions of the second model as applied to the concepts of the First Figure, with each one referring to the others, but this ties in directly with what Lull says of the circularity of that figure.³⁸ In the *AGU*, however, he explains that concepts of this second set can also be defined according to the first non-circular model: “Just as it was said that goodness is that thing by reason of which good does good, so one can say that greatness is that thing by reason of which great does [something] great, and that eternity produces the eternal, and so on.”³⁹

Of perhaps even greater significance for Lull’s project is the fact that with this technique, each Principle has a single definition equally valid for whatever rung of the ladder of being to which it might be applied, whether it be the “goodness”, “greatness”, “difference”, etc. in God, the heavens, the elements, or whatever. To use the Aristotelian term, in this way the Art establishes the univocal nature of the Principles, which, to emphasize a point we have made before, prevents their being necessarily identifiable with any specific set of concepts, such as the dignities or even their semblances.⁴⁰ Moreover, with this new technique, Lull can do something normal Aristotelian definitions cannot do, that is, to define God or most things concerned with him, for the simple reason that, to do so with the standard Aristotelian model would have involved an impossible limitation, i.e., God as a species within some larger genus.

³⁸ In an expansion of this technique, the definitions can be combined with one another; see p. 157 below under “The mixing of Principles and Rules.”

³⁹ *ROL* XIV, 24. In the *AA* (*ROL* XXIX, 142–3; *ORL* XVII, 35) he distinguishes the two sorts of definition, that like “goodness” in terms of itself, which he calls “essential”, and that like “greatness” in terms of the other principles, which he calls “accidental”. See Bonner and Ripoll 2002, 29.

⁴⁰ “Things are univocally named, when not only they bear the same name but the name means the same in each case”, Aristotle, *Categories* I. Most of the above paragraph is a translation of a private communication of Ruiz Simon, in which he emphasized the importance of this question and remarked on the little attention it has received in the literature.

In the shorter works of the ternary art, Llull just gives the definitions printed here. In the longer works before the *AGU*, however, he accompanies them with extended explanations, very often in terms of what he chooses to emphasize at that moment. In the *AIV*, the last nine definitions of the components of Figure T, for instance, are very much based on the figure itself, and therefore make extensive use of the ancillary concepts of its three outer rings. In the *AA* he tends to explain in terms of *amància*. In the *TG* his interest is more philosophical: he discusses, for instance, the universal, substantial, and accidental nature of each principle.⁴¹ By the time of the *AGU*, however, he has found a way to incorporate these explanations into the fabric of the Art, under the heading of “The Mixture of Principles and Rules” which we will study further on.⁴²

What the *AGU* does have, is an appendix to the definitions in which Llull explains and defends his definitional technique, ending with the often cited complaint about “People with a dog’s fang and a serpent’s tongue who reject and disparage my principles and their definitions” (*ROL XIV*, 25). This was probably due to the fact that his definitions *per agentiam* led him to such apparent tautologies as *homo est ens* (or *animal homificans*, which could have been easy bait for the parodist. But Llull was clearly aware of the problem, and also aware of the importance of this definitional technique for his Art.⁴³

Questions and Rules

With the ten Questions and Rules, we come to the third column of the Alphabet of the ternary Art, and the last component of the foundations of the Art. In listing them we will include the Latin original of the question (even in Catalan works, the name of the question is usually given in Latin), along with its name when considered as a Rule. Each

⁴¹ The exception is “contrariety”, which can only be accidental. Or as he puts it in the *AGU*, “Of these principles some are substantial and some accidental. Contrariety, however, is always an accident.” (*ROL XIV*, 22). Cf. *ROL XXVII*, 141*, n. 343.

⁴² These explanations, in whatever form Llull presents them, have not received the attention they deserve; they are a remarkably rich source for Llull’s thought. The only scholar I know who has discussed them is Platzek 1962–4, I, 129ff.

⁴³ For more on Llull’s definitional techniques, see Bonner and Ripoll 2002, 22–29 and 36–41.

of these Questions or Rules is divided into “species”, as can be seen from the explanations below:

B. Whether? – Utrum? – Possibility

“Whether” has three species, to wit, dubitative, affirmative and negative, so that, from the outset, the intellect assumes either side to be possible, and does not bind itself to believing, which is not as natural to it as is understanding.⁴⁴

This is clearly where the first black triangle of Figure T of the quaternary phase has come to roost. In the *AGU* he precedes this little classification with an explanation, saying that this Rule of B

has to do with possibility, namely whether the thing inquired about exists or not. And such possibility is the cause of faith or belief or supposition, since it supposes that both sides can be possible, that is to say whether they are affirmative or negative.⁴⁵

This notion of possibility is essential for Llull; without it there can be no beginning to any sort of rational discussion or disputation. It was a point on which he insisted from the very beginning of his career with the *Compendium logicae Algazelis*.⁴⁶

This question of *Utrum*, the commonest way of articulating philosophical and theological texts of the time,⁴⁷ is one that could be used to discuss a wide variety of subjects. Most prominent is its use in initiating demonstrative arguments: “Whether God exists?”, “Whether the world was created?”, and so on. But it could be put to many other uses; one only has to see its use in investigating the predicaments in the *Logica nova (LN)* to see how varied it could be. For instance, he asks: “Whether created substance, if it could be deprived of its accidents, would have extra-mental being?” “Whether quantity can be defined?” and “Whether quality is more a property than a disposition?”⁴⁸ So it is the place where

⁴⁴ This and the following Questions and Rules from the *AB* are taken from *SW I*, 591–5; *DI*, 311–315. See Bonner and Ripoll 2002, 30–37 for an analysis of those (principally) of the *LN*. For an overview see Ruiz Simon 2005, 191–5.

⁴⁵ *ROL XIV*, 26–27. Here some of the functions of the former Figure S are taken over.

⁴⁶ See Lohr 2001 and Fidora 2003.

⁴⁷ One only has to open Aquinas’s *Summa theologica* at any page to see long series of questions beginning with “Whether...”. Even commentaries, such as Boethius of Dacia’s on Aristotle’s *Topics*, could be formulated as a series of questions concerning the text, each beginning with “Concerning this point it is asked whether...”.

⁴⁸ See the references in n. 57 below.

he can ask a variety of general questions about the concept being investigated, and it is thus the freest of all the Questions.

It should be remarked, however, that this is the one question which can be omitted. In texts such as the *Liber de homine*, *Liber de natura*, or *Rhetorica nova*, for instance, where he is just explaining or investigating some subject, without any need to defend its existence or to prove anything about it, it is absent.⁴⁹

The second one is explained at greater length:

C. What? – Quid? – Quiddity

“What?” has four species:

1. The first is definitional, as when one asks: What is the intellect? To which one must reply that it is that power whose function it is to understand (*intelligere*).
2. The second species is when one asks: What does the intellect have coessentially in itself? To which one must reply that it has its correlatives, that is to say, intellective, intelligible, and understanding (*intelligere*), without which it could not exist, and would, moreover, be idle and lack nature, purpose, and repose.
3. The third species is when one asks: What is the intellect in something other than itself? To which one must reply that it is good when understanding in goodness, great when understanding in greatness, etc.; and grammatical in grammar, logical in logic, rhetorical in rhetoric, etc.
4. The fourth species is when one asks: What does a thing have in another thing? As when one asks: What does the intellect have in another thing? To which one must reply, In knowledge, understanding, and in faith, belief.

The first species is where the definitions discussed in the previous section fit into this scheme. The second, as Lull also makes clear, is the slot for his doctrine of the correlatives.

D. Of what? – De quo? – Materiality

“Of what?” has three species, which ask what a thing is derived from, what is it composed of, and whose it is.

⁴⁹ For the *Liber de natura*, see Pring-Mill 1966, 569, and 1991, 205–6, where he notes that the anonymous eighteenth-century editor of the work pointed out that the work begins with the question of *Quid?*, and that “the reason is that Nature is supposed to exist in something, and it is therefore unnecessary to investigate it by the first question, which is *Utrum?*”

E. Why? – Quare? – Formality

“Why?” can inquire about formal or final cause, or as Lull puts it in the *TG*, “why things exist, and why they do what they do”, which in the *AGU* he explains by saying that one is *per existentiam* and the other *per agentiam*.⁵⁰

F. How much? – Quantum? – Quantity

Under this question Lull adopts the standard medieval division between continuous and discrete quantities (traditionally the domains respectively of geometry and arithmetic).

The next Rule is important in Lull, so we give it in full:

G. Which, of what kind? – Quale? – Quality

The sixth rule concerns quality, and it has two species:

1. The first is when one asks: What is the proper and primary quality of intellect? To which one must reply, The intelligibility with which it is clothed. Extrinsic understanding is a secondary and more distant property, with which the intellect understands a man, a lion, etc. It is with this that the intrinsic and substantial understanding of the intellect is clothed, and similarly for what is extrinsically intelligible.
2. The second species is when one inquires about appropriated quality, as when one asks: What is an appropriated quality of the intellect? To which one must reply, Believing, doubting, or supposing. For these acts are not those of the intellect proper, which is rather that of understanding.

This contrast between proper and appropriated qualities is sometimes explained in terms of man, whose proper qualities are goodness or risibility, and who can appropriate justice as a result of good habits.⁵¹ Lull’s usual example, however, comes from elemental theory where the proper quality of fire is heat, while it appropriates dryness from earth. This was one of several aspects of elemental theory which made it continue to have exemplary value, even in a work such as the *LN*, where it acts as a kind of substitute for the classical square of opposition.⁵²

⁵⁰ For the *TG*, *ROL* XXXVII, 57; *ORL* XVI, 341. For the *AGU*, *ROL* XIV, 33.

⁵¹ As for instance, in the *LN* (*ROL* XXIII, 35; *NEORL* IV, 18).

⁵² The appropriated qualities are represented by the arrows, the contrary qualities by the diagonal lines of the Elemental Figure on p. 59 above. For other exemplary

The next two Rules are relatively straightforward, and Lull says that each has the fifteen species of the Rules of C, D, K.⁵³

H. When?	–	<i>Quando?</i>	–	Time
I. Where?	–	<i>Ubi?</i>	–	Place

Of the last two rules squeezed together under the last letters of the Alphabet, we have:

K¹. How?	–	<i>Quo modo?</i>	–	Modality
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This question asks how a thing is, as for instance “How is the intellect [constituted]?”⁵⁴

K². With what?	–	<i>Cum quo?</i>	–	Instrumentality
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This question asks with what a thing exists or is constituted.

In spite of the similarity between this list of Questions and Rules and that of the Aristotelian categories, or predicaments, as they were called in the Middle Ages, the reader should understand that for Lull they were quite different.⁵⁵ For him the predicaments are real, substantial entities,⁵⁶ whereas the Questions and Rules are modes of inquiry. He treats the former ontologically, the latter as an epistemological tool, and this to the point that he can use, as we have just seen, the latter to investigate the former.⁵⁷

aspects of Lull’s elemental theory, see the explanation following the square of opposition on p. 196 below.

⁵³ 4 for B, 3 for C, 4 for K¹, and 4 for K² makes 15.

⁵⁴ *Quomodo est intellectus?* in the *AB* (*ROL* XII, 216). In the *LN* he asks *quomodo est homo?* (*ROL* XXIII, 37).

⁵⁵ For the usual list of predicaments, see n. 108 below.

⁵⁶ See for instance the *LN*, where he explains that “relation” is substantial (*ROL* XXIII, 65; *NEORL* IV, 48). See also their development from primal chaos in the *Liber chaos* (*MOG* III, v, 26ff: 274ff).

⁵⁷ As in the passages preceding n. 48 above, taken from complete investigation of the predicaments by this technique in the *LN* (*ROL* XXIII, 56–81; *NEORL* IV, 39–63). For the possible origin of the Questions and Rules in Avicenna, see Bonner 1995a, where the reader will find previous bibliography on the subject.

Before leaving this topic, one point should be emphasized. Since the definitions are included in the Questions and Rules (under the first species of Rule C), the latter can be seen as an instrument which amplifies the scope of the former, making them into a more general investigative tool. One can see how this works with the Hundred Forms, which we will meet later in this chapter, where Lull defines all sorts of concepts mostly (but not always) exterior to the Art. In the *LN* the first two of these Hundred Forms are not only defined, but treated extensively by the ten Questions and Rules, as a model of how the other ninety-eight should be studied, and the second of these is precisely “Goodness”, in which we find embedded the same definition of the first of the eighteen Principles above.⁵⁸ In the *AGU* the last of the Hundred Forms, “Memory” is studied by the same technique, again expressly as a model for the previous ninety-nine.⁵⁹

Combinatorics

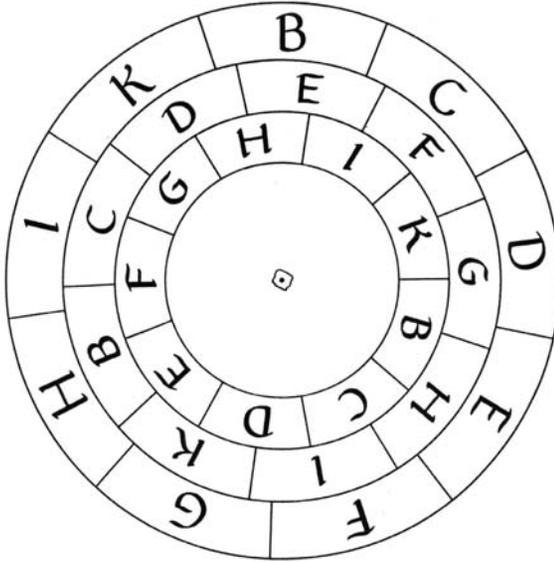
The working out and displaying of the new combinatorial mechanisms begins with the two remaining figures of the ternary Art:

The Third Figure

BC	CD	DE	EF	FG	GH	HI	IK
BD	CE	DF	EG	FH	GI	HK	
BE	CF	DG	EH	FI	GK		
BF	CG	DH	EI	FK			
BG	CH	DI	EK				
BH	CI	DK					
BI	CK						
BK							

⁵⁸ *ROL* XXIII, 85–88; *NEORL* IV, 68–70. This is, of course, identical to the mixing of Principles and Rules which we will study in a subsequent section.

⁵⁹ *ROL* XIV, 393–5. See p. 166 below for how this is worked out.

The Fourth Figure

These two figures represent respectively binary and ternary combinations of 9 letters. Even though the Third Figure is a half-matrix similar to the second figures of the *AD*, it is different in that it is no longer simply another way to display the graph of a particular (first) figure. Both it and the Fourth Figure are more general, in that their letters can now be substituted for any of the concepts of the first two figures, or even of the Questions and Rules from the third column of the Alphabet. Moreover their different forms have no deep significance; they merely have to do with problems of graphical representation. The Third Figure could as easily have been displayed as two revolving circles; Lull probably chose the half-matrix form as less complicated to manage (and explain), as more familiar (from the quaternary phase), and as giving a more explicit image of all the possible binary combinations without repetitions.⁶⁰ With the Fourth Figure, and Lull's venture into one of

⁶⁰ In spite of the half-matrix form, it is no longer "adjacent" to any "first figure", since none of the latter is a graph; it is now an independent way of displaying binary combinations.

the major innovations of the Art of this period—that of ternary combinations—,⁶¹ he was forced to choose the method of rotating disks or volvelles, since matrices can only represent binary relations. And this indeed involved him in an explanation for contemporary readers:

The middle circle revolves on top of the outer fixed circle, so that, for instance, C can be put opposite B. The innermost circle revolves on the middle circle, so that, for instance, D can be put opposite C. And in this way nine compartments are formed at a time, one being B C D, another C D E, and so on. After that... (*SW I*, 587; *DI*, 306)

He also explains that the Third Figure has 36 binary compartments⁶² and the Fourth 252 ternary compartments, which in practice—by treating differences of ordering as identical (that is, by not differentiating between B C D, C D B and D B C)—are reduced to 84.⁶³

The Table

In the *AGU*, Llull says that “the reason this [Fourth] Figure has been put in this Art is so that it can be used to lay out a Table from it” (*ROL XIV*, 20). He does this by setting out each of the 84 compartments as a first component of a column of 20 new ternary combinations. In the version given here from the *AB* he uses only “7 columns, in which the 84 columns of the *Ars magna* are implicit.”⁶⁴ He goes on to explain:

⁶¹ Or rather a generalization of an earlier experiment, for which see pp. 103–4 above.

⁶² The first formula on p. 29, n. 6, gives $9!/7! \cdot 2! = 9 \cdot 8/21 = 36$.

⁶³ Llull is saying that with the outer wheel remaining fixed, we can place opposite the B on that wheel all the possible two-letter permutations of the remaining eight letters of his Alphabet, which gives $8!/6! \cdot 2! = 8 \cdot 7/2 \cdot 1 = 28$ usable portions of the two inner wheels (i.e., without repetitions of letters). Now since in any one position we can read off nine compartments between the linked-up spokes, as it were, this gives a total of $28 \cdot 9 = 252$ compartments. This does, however, involve repetitions; in fact, each compartment is repeated three times, since B C D = C D B (which appears when C K are lined up under B) = D B C (which appears when I K are lined up under B). For Llull’s combinatorial purposes here, however, this is unimportant. In the Table of the *AGU* he has eliminated the superfluous combinations, giving $252 / 3 = 84$. This ends up the same as calculating $9 \cdot 8 \cdot 7/3 \cdot 2 \cdot 1 = 84$.

⁶⁴ *SW I*, 596; *DI*, 316. He chooses them by taking the first of the 28 starting with B, the first of the 21 starting with C, etc., ending with the one starting with H. Note that these are the same 7 columns as are used in the “Questions of the Table” in the *AGU*, as examples of how to pose questions for all 84 (see *ROL XIV*, 395ff.). The 20 combinations of each column can be calculated as follows: if instead of differentiating the concepts of Figures A and T by an interposed “T”, we do so by using upper case and lower case letters respectively, it will become clear that the first column, for

“In this Table T means that the letters which come before it belong to the First Figure, and those after it to the Second Figure.”

B C D	C D E	D E F	E F G	F G H	G H I	H I K
B C T B	C D T C	D E T D	E F T E	F G T F	G H T G	H I T H
B C T C	C D T D	D E T E	E F T F	F G T G	G H T H	H I T I
B C T D	C D T E	D E T F	E F T G	F G T H	G H T I	H I T K
B D T B	C E T C	D F T D	E G T E	F H T F	G I T G	H K T H
B D T C	C E T D	D F T E	E G T F	F H T G	G I T H	H K T I
B D T D	C E T E	D F T F	E G T G	F H T H	G I T I	H K T K
B T B C	C T C D	D T D E	E T E F	F T F G	G T G H	H T H I
B T B D	C T C E	D T D F	E T E G	F T F H	G T G I	H T H K
B T C D	C T D E	D T E F	E T F G	F T G H	G T H I	H T I K
C D T B	D E T C	E F T D	F G T E	G H T F	H I T G	I K T H
C D T C	D E T D	E F T E	F G T F	G H T G	H I T H	I K T I
C D T D	D E T E	E F T F	F G T G	G H T H	H I T I	I K T K
C T B C	D T C D	E T D E	F T E F	G T F G	H T G H	I T H I
C T B D	D T C E	E T D F	F T E G	G T F H	H T G I	I T H K
C T C D	D T D E	E T E F	F T F G	G T G H	H T H I	I T I K
D T B C	E T C D	F T D E	G T E F	H T F G	I T G H	K T H I
D T B D	E T C E	F T D F	G T E G	H T F H	I T G I	K T H K
D T C D	E T D E	F T E F	G T F G	H T G H	I T H I	K T I K
T B C D	T C D E	T D E F	T E F G	T F G H	T G H I	T H I K

This Table (*Tabula, Taula*), presented in a separate chapter of the *AGU* and the *AB*, does several important things with the representation of the ternary relations of the Fourth Figure.⁶⁵ In the first place, and most obviously, it expands the possible combinations of the Fourth Figure; or, more accurately, it multiplies them by twenty. And as we will see, this concept of “multiplication” is fundamental to the way Lull treats the figure. Secondly, it is an equivalent of the matrix notation, in that it displays explicitly all the possible ternary combinations implicit in the Fourth Figure. Thirdly, by using the letter “T” syntactically as a separator, the problem of polysemy is solved, showing us clearly to which of the first two figures the letters refer. Lastly, this same method can show explicitly which repetitions are permissible. In other words it can ban a repeated letter, such as B B, from representing a repetition of terms from the same figure (“goodness goodness”) while allowing that from different figures (“goodness difference”). The system Lull adopts is not

instance, consists of all the permutations of the six letters B C D b c d taken three at a time, or $6 \cdot 5 \cdot 4/3 \cdot 2 \cdot 1 = 20$.

⁶⁵ The complete 84-column version appears in three works: the *TG*, the *Ars compendiosa*, and the *AGU*. The 7-column version only appears in two: the *AB* and the *Liber de experientia realitatis Artis ipsius generalis*.

only simple and ingenious, but mathematically sound (and perhaps even innovative). He has, in fact, found a way of displaying all the possible ternary combinations of the components of the first two figures without the repetition of any single component, and thus has created a precise ternary equivalent of the binary matrix of the Third Figure.

Thus, with the combinatory mechanisms of the Third and Fourth Figures and the Table, as ways of studying the conjoint presence of the Principles of the Art (along with their definitions) and the Questions and Rules, Llull has provided the foundations of the ternary Art. It is from these foundations that he can construct his arguments and answer questions, according to the methods we will now study.

Use and Practice of the Art

For the person tackling the working out of the Art for the first time, it could well be terminally confusing to present all the possibilities Llull presents in any sort of detailed manner. I will therefore mostly limit myself to an outline, with enough examples to show how the mechanisms work. Llull begins working them out in three chapters of the *AGU* and *AB*:

- Part V. The Table (which not only presents this mechanism, as we have seen above, but also gives extensive explanations and samples of its use)
- Part VI. Evacuation of the Third Figure
- Part VII. Multiplication of the Fourth Figure.

Instead of discussing these three sections in order, we will start with the binary combinations of VI, and then go on to the ternary combinations of the closely related Parts V and VII.

Binary combinations

In Part VI, dealing with the Evacuation of the Third Figure, Llull presents his binary relations with the example of the first compartment of B C, from which, he says, 12 propositions can be extracted. To show how the mechanism works, we add the combinatory letters omitted in the text, using the same notation explained in n. 64 above,

that is upper case letters to refer to the First Figure, and lower case to the Second, giving us:

B C	goodness is great	b B	difference is good
B b	goodness is different	b C	difference is great
B c	goodness is concordant	b c	difference is concordant
C B	greatness is good	c B	concordance is good
C b	greatness is different	c C	concordance is great
C c	greatness is concordant	c b	concordance is different ⁶⁶

With a calculation similar to that above concerning the Table, we have a permutation of four letters, B C b c, taken two at a time, giving $4 \cdot 3 \cdot 2 / 1 \cdot 2 = 12$, which is effectively what Lull offers. Again he does it by permitting the repetition of letters only when they are not from the same figure.⁶⁷

To each of these 12 propositions one can further add the two Questions corresponding to the same letters of B C, which are “whether” and “what”. Thus, for the first proposition above, we could ask “Whether goodness is great?” and “What is great goodness?”, giving a total of 36 possibilities extractable from the initial compartment of B C. What Part VI does is to go through all 36 compartments, taking only the first three propositions of each, along with their corresponding two questions. So, for example, for the first compartment we have:

Goodness is great.	Goodness is different.	Goodness is concordant.
Whether goodness is great?	Whether goodness is different?	Whether goodness is concordant?
What is great goodness?	What is different goodness?	What is concordant goodness?

⁶⁶ ROL XIV, 76; *SW* I, 598; *DI*, 318. In the *AB* (*SW* I, 586–7; *DI*, 306), when describing the Third Figure itself, he gives a different list, one which corresponds to the first column of the half-matrix.

B C	goodness is great	B b	goodness is different
B D	goodness is enduring	B c	goodness is concordant
B E	goodness is powerful	B d	goodness is contrary
B F	goodness is knowable	B e	goodness is beginning
B G	goodness is lovable	B f	goodness is mediating
B H	goodness is virtuous	B g	goodness is ending
B I	goodness is true	B h	goodness is magnifying
B K	goodness is glorious	B i	goodness is equalizing
		B k	goodness is lessening

⁶⁷ Something not explicitly stated in the Third Figure.

To give an example, or rather extracts, of how he works the second of these columns, we see that he starts with the proposition, and then goes on to show how to answer the questions:

Goodness is different.

It is asked whether goodness is different? To which one must answer “yes” as is clear from the angle of the Second Figure,⁶⁸ and also by the definition of goodness. For goodness would not be the reason for good producing good unless it were different, or had natural difference, nor could difference cause natural relations with its own natural passive qualities.

It is asked: what is differing goodness? To which one must answer by the first species of rule C that differing goodness is the reason for which the bonifier (*bonificans*) distinguishes itself from the bonified (*bonificatum*) and the act of bonifying (*bonificare*).

Moreover by the third species (of Rule C) differing goodness is diffused in many habits which differ in species, for one is in gold, another in a ruby.⁶⁹

Llull continues thus, through all the compartments of the Third Figure, adapting the questions asked to the letters of the compartments. So, for instance, with the compartment of D E, he asks of duration and power, “Of what does powerful duration consist?” and “Why does power last?” So this part of the *AGU* gives a sampling of how to study all the propositions one can form from the binary combinations of concepts of the first two figures.

Ternary combinations

Curiously enough, in the *AGU* the use of ternary combinations is explained in Part V, after the presentation of the Table, and thus before the equivalent explanations of binary ones from Part VI which we have just described.⁷⁰ Here, in Part V, Llull devotes two sections to explaining how to use the Table: one general one, similar to the Evacuation of

⁶⁸ A reference, of course, to Figure T, of which the angle of “Difference” must necessarily apply to the concepts of Figure A.

⁶⁹ *ROL* XIV, 78–79, from which the above quotations are just a sample. For the last word, the printed text has *rubrico*, but it is difficult to know if this is an editorial interpretation; all the older manuscripts have *robis(s)o* or *rubis(s)o*, which would appear to be a Catalanism in the Latin text.

⁷⁰ This might be due to an accident of organization. In the *TG* he had a brief description of the use of binary combinations in the description of the Third Figure, before that of the ternary combinations of the Fourth Figure and the Table. Having

the Third Figure we have just seen, and another dedicated to solving a single problem, that of the eternity of the world.

In the first one he shows us how we can extract twenty questions from the first column of the Table, listing them as follows.⁷¹

- B C D Whether goodness is so great that it is eternal?
- B C b Whether there is some goodness so great that it contains within itself different things coessential with itself?
- B C c Whether goodness is so great that it contains within itself concordant things coessential with itself?
- B C d Whether goodness which contains in itself contrary things is great?
- B D b Whether eternal goodness is different?
- B D c Whether eternal goodness is concordant?
- B D d Whether eternal goodness harbors contrariety within itself?
- B b c Whether goodness contains within itself difference and concordance?
- B b d Whether goodness contains within itself difference and contrariety?
- B c d Whether goodness contains within itself concordance and contrariety?
- C D b What is great difference of eternity?
- C D c What is great and eternal concordance?⁷²
- C D d What is great and eternal contrariety?
- C b c What is great difference and concordance?
- C b d What is great difference and contrariety?
- C c d What is great concordance and contrariety?
- D b c Of what does difference of concordance and eternity consist?
- D b d Of what does difference of eternity and contrariety consist?
- D c d Of what does concordance of contrariety and eternity consist?
- b c d Of what does difference of concordance and contrariety consist?

now added Parts VI and VII, and, as we shall see in a moment, using the latter for more general considerations, the previous order has been changed.

⁷¹ *ROL XIV*, 49–50. Once again we have added the combinatorial letters to show how the mechanism works. In the text they are just numbered “First”, “Second”, up to “Twentieth”. This section is discussed in Carreras y Artau 1939–43, I, 441–3.

⁷² The “contrariety” for “concordance” is an error of *ROL XIV*.

To give a sample of Llull's explanations of how this functions, we will give part of his answer to the question under C b d above:

To the fifteenth question, "What is great difference and contrariety?"; one must answer that it is the cause causing one contrary in another, like an elemented thing in which are mixed fire and water, or air and earth, and like that between natural innate good and moral evil, that is to say sin, and so on. And this is clear from the definitions of greatness, difference and contrariety. And indeed, difference proclaims and distinguishes these contraries, and contrariety deprives a subject of concordance and generates contrary appetites and ends. And this is signified by... (*ROL XIV*, 59)

This first thing to be remarked about this expansion of the previous binary mechanism to a ternary one is that here Llull admits only questions, with no propositions such as a possible "Great goodness is eternal". The second is that the choice of questions is governed by the first letter of each triad. I think if Llull had been asked why, here and in the previously described section, he chose to limit his questions this way, I think he would have answered that it was simply to give short, easily ordered examples of these mechanisms, but if the "artist" wished to choose another, not only would that be perfectly acceptable, but he might even have added that this was the whole point in an Art of possibilities, in an "open" system, as we said before.

Also the reader might have noticed that with the addition of the questions, he is turning his binary and ternary combinations into ternary and quaternary respectively. In other words—writing the letter corresponding to the questions in italics—for the first two questions under the Evacuation of the Third Figure, "Whether goodness is great?" and "What is great goodness?", we would get *b* B C and *c* B C,⁷³ and under the first two we have just listed above, we would get *b* B C D and *b* B C b.

A single question

As we said above, after this first section on the general use of the Table, Part V of the *AGU* presents a second section in which Llull gives an example of how to apply one column to a single question, "Whether

⁷³ Note that here the questions are not governed by the first letter, but successively from the two letters of the compartment.

the world is eternal?”⁷⁴ For this question “whether” requires B and “eternal” D, and the problem is “to apply the question to C, by which application, we will acquire whatever C contains”. His first answer, which uses C = “greatness”, says:

In answer to the question “Whether the world is eternal?” we say by B C D that it is not, because if it were eternal, its foundation (*ratio*) would be eternal and it would produce eternal good throughout eternity while greatness, by its definition, would magnify this good foundation from eternity and in eternity; and eternity would make this production last from eternity and in eternity, so that there could be no evil in the world, because good and evil are contrary. But there is evil in the world, as we know by experience. We therefore conclude that the world is not eternal. (*ROL XIV*, 63–64)

In the next paragraph Lull uses the various species of Rules C and D explained above on p. 139f.:

Moreover, Rule B says that one must answer this question negatively, according to the definitions we have given and according to what we propose to say by Rules C and D, which is: if the world is eternal, its eternity is that which causes evil to endure as much as good, as shown by the first species of Rule C. By the first species of Rule D, evil and good are equally primordial. No day is first or last. By the second species of Rules C and D the world is composed of good and evil in eternity and from eternity. By the third species of Rule C the world is infinite in eternity but finite in good and evil. By the fourth species of C the world has repose in things subjected to generation and decay, where generation is due to good and decay is due to evil. And by the second species of Rule D God’s eternity and goodness need evil and repose in causing the world’s eternity. And since all these things are impossible, the answer to the question is clearly negative. (*Ibid.*, 64)

This proof is similar to those of the *AD* in that here he is also testing a hypothesis, demonstrating that it cannot be true by showing how its consequences would lead to an impossibility. But where the former proofs used the comparison of compartments with their understood onto-theological “truth values”, here we are dealing with what is implied by the definitions of the eighteen Principles and by the Rules, and how these implications are what lead to an impossibility. Instead of the long chains of compartments whose selection and interpretation seemed so

⁷⁴ Or “how to multiply twenty reasons for the solving of a single question”, as Lull says in the *TG* (*ROL XXVII*, 72, 142; *ORL XVI*, 357, 424), in a module on the same technique applied to the same question that he has taken over with only a few changes for the *AGU*.

difficult, we now have a table from which we can choose the column that most closely coincides with the question to be answered, applying to it techniques we have learned in previous parts of the work.

As for the problem of the eternity of the world, it was a question much debated in university circles during his time not only because of the disagreement between the authority of Aristotle, who said it had to be eternal, and Christian doctrine, which said it had been created by God,⁷⁵ but even more vehemently between thinkers like St. Thomas who said that its creation was an article of faith and could not be proved, and the Augustinian line of the University of Paris (followed by St. Bonaventure) who defended its demonstrability.⁷⁶ It was a question on which Lull, not unlike St. Thomas, expended a considerable amount of intellectual energy.⁷⁷ In this he was probably not only trying to disprove “Averroist” positions, but also showing his allegiance to the Augustinian stance, crucial for his programmatic need to prove the Articles of faith. Thus if his position might seem to us a foregone conclusion (making some modern critics wonder why all the fuss), it was a matter not at all so clear to his contemporaries.

Specifically in this place in the Art, however, perhaps more than his position on the question, he is using it to show how to use this technique of the Art. He explains himself clearly at the end of this Part:

⁷⁵ The so-called Parisian “Averroists”, such as Siger of Brabant or Boethius of Dacia, said that the world was eternal according to the principles of philosophy (or physics), but its creation in time had to be accepted as a simple truth of faith, something they were careful not to deny. Cambi 2002, 65 (with a good bibliography), suggests that Lull was perhaps arguing more against a second generation of Averroists like John of Jandun and Thadeus of Parma.

⁷⁶ See the various excellent introductions in Aquinas et al. 1984, where (p. 14) one can see how St. Thomas treated the question almost obsessively. He devoted one work, the *De aeternitate mundi*, to the question, and discussed it at length in six other works. There one can also see how he directed his criticisms not so much against the position of Aristotle and the Parisian “Averroists”, as against the Augustinians, whom he called “murmurers” (see p. 17), and how his position on its indemonstrability ran counter to most scholastics of the thirteenth century, such as Alexander of Hales and Albert the Great (p. 12).

⁷⁷ Lull had already proved it similarly with the Table in two previous works of the Art: in the *TG* (*ROL* XXVII, 142–151; *ORL* XVI, 424–432) and in the *Art of Proposing and Solving Questions*, under the topic of Creation (*MOG* V, v, 169: 527), where he doesn’t go into it, but just refers the reader back to the *TG*. In the *Declaratio Raimundi*, in chs. 87–91 and 98 (*ROL* XVII, 332f; Lulle 2006, 95f) Lull treats the subject at length, including, in ch. 89, an answer to St. Thomas’s position on its indemonstrability. For the degree of emphasis on this topic in the *Declaratio*, see Lulle 2006, xxv–xxvi and 179. For other places in which Lull treats the question, see Giletti 2004.

Thus has been given a practical doctrine of this Art, that is to say, how the intellect descends to particulars by multiplying twenty reasons to arrive at one and the same conclusion through the mixing of the Principles and the species of the Rules, by which mixing and “invention” are produced the solutions to the questions. And this has been exemplified by means of the first column, but it could similarly be exemplified by the second and third and the other columns in proper order. At this point it becomes apparent how the intellect has a general subject in the Table of this Art for finding a means for arriving at conclusions on any matter proposed.⁷⁸

Llull is here making several points. The first is that this whole section has been presented so as to “give doctrine”, to show how, to “exemplify” the use of the columns of the Table. And this is to be done by “multiplying twenty reasons to arrive at one and the same conclusion”, a technique on which Llull placed great emphasis.⁷⁹ In a previous work of the ternary phase, the *Art of Proposing and Solving Questions*, for instance, he dedicates pages and pages to the solving of a hundred and ten questions, each of which is accorded twenty proofs based on a single column of the Table.⁸⁰ Here he ends explaining that the other columns could equally well have been used. These possible multiplicities are points Llull emphasizes again and again, and which he sees as one of the chief benefits of the Art, which can not only answer a multiplicity of questions, but also use multiple means to answer any single question.

Finally, the degree to which this section is intended as an example is clear from the questions at the end of the *AGU*, where he shows how the Table can be used to deal with other problems, such as “Whether

⁷⁸ *ROL* XIV, 75. I have translated *media*, “middles” as “means” and curtailed and simplified the rest of the sentence to avoid broaching a complicated subject that will be discussed in the next chapter.

⁷⁹ In addition to the works cited in the previous and the following notes, and here in the *AGU* where Llull repeats with great insistence the idea of multiplying many reasons to arrive at one and the same conclusion (*ROL* XIV, 19, 20, 63, 75, 103, 323), it is also discussed in the *Lectura Artis quae intitulatur Brevis practica Tabulae generalis* (*ROL* XX, 425), the *Liber de experientia realitatis Artis ipsius generalis* (*ROL* XI, 179, 185), and in the *Ars compendiosa Dei* (*ROL* XIII, 225, 299), where he says that “this multiplication of propositions is an abyss of material for the intellect to find (*inveniendum*) God as well as those things which can be rationally said about him”.

⁸⁰ See p. 174 below for how this is worked out. See the note itself for the title, instead of the more usual *Lectura super Artem inventivam et Tabulam generalem*.

a single angel is greater than heaven itself?” or “When is there lesser charity, patience, and pity?”⁸¹

Part VII on the Multiplication of the Fourth Figure, is, in the *AGU*, rather surprising. It is divided into five sections, under the first of which, on the “multiplying of many arguments towards a single conclusion,” one might expect a disquisition on the elaboration of ternary combinations corresponding to the binary ones in Part VI. Instead, Llull dispatches the matter in less than a page, referring the reader back to Part V, merely adding a few extra possibilities to the “multiplication” he presented there.⁸² Then comes a series of four essays in which, Llull, as Carreras y Artau put it, “attempts the mechanization of the art of syllogistics”.⁸³ They are on (1) the finding of the middle, (2) the three proofs (*propter quid*, *quia*, and *per aequiparantiam*), (3) fallacies, and (4) how to use the Art to teach other sciences. The second we have already met, while the first and third will be discussed in the next chapter on Logic (along with a new presentation of the second). The last shows how the Art can be used to confirm the statements of authorities, by taking the example of three well-known tropes of theology or philosophy: “God is pure act”, “Nothing can come from nothing”, and “Being and one convert”. In the last chapter we will see the consequences of this approach, which Llull often calls by the provocative name of “reducing authorities to necessary reasons”.⁸⁴

I would like to end this presentation of the use of the combinatory mechanisms of Parts V–VII with two general remarks. The first is that in these parts Llull surprisingly poses and solves questions, something he normally reserves for the last section of a work of the Art. Then in Part XI of the *AGU* devoted to questions, which is where one would expect to find such material, with one exception which we will discuss when we come to that part, he just gives brief remarks on how to use the mechanisms of these earlier parts. The reason for this, it seems to me, is that the questions presented in these sections prior to the central

⁸¹ See the two passages preceding and following n. 114 below.

⁸² This is undoubtedly why he uses the word “multiplication” in the title of Part VII, as opposed to the “evacuation” in that of Part VI. Notice, however, that in the *AB* (*SW* I, 602; *DI*, 322) he uses “evacuate” as a synonym for “multiply”. In the *Lectura Artis quae intitulatur Brevis practica Tabulae generalis* (*ROL* XX, 425) the term *multiplicatio* is used for both figures. The distinction, therefore, does not seem very important.

⁸³ Carreras y Artau 1939–43, I, 447, where the reader can find a good exposition of the Multiplication of the Fourth Figure.

⁸⁴ See pp. 280–281 below.

Part VIII on “mixing” are there more to demonstrate techniques of formulation than to provide answers which would be of any real interest to the reader. In this way, the rest of these questions are like the one about the eternity of the world: only interesting as models, a fact which, by the same token, would exonerate Llull from asking similar ones in Part XI. The placement of questions here might also be due to the fact that Llull is summarizing techniques amply presented in previous works of the ternary phase, before going on to the “mixing” which is so fundamental to the last works of the phase.

The Mixing of Principles and Rules

With Part VIII, “Which Treats of the Mixture of Principles and Rules”, we come to a new formulation in the presentation of the Art, or rather a reformulation in new terms of a function we studied with the *AD*, namely the “Conditions”. As we saw in the passage from the *Taula d’esta Art* quoted on p. 67 above, Llull defines “condition” as “the mixing of principles, with some conditioned by others according to their definitions and properties”.⁸⁵ But now, instead of the “conditioning” of each binary compartment by means of Figure T to investigate all the possible relations between its two concepts, it is done simply by the “mixture” of the principles “according to their definitions”. As we have also seen, the notion of mixture or mixing (*mixtio*, *mescla*), was fundamental as a cosmological concept, applied first to the four elements, and then to the semblances of the dignities in the constitution of the world. As a result, these combinations trace back along the paths by which the furniture of the world is constituted, and therefore are yet another component in Llull’s grafting of his epistemology onto his ontology.⁸⁶ This technique now becomes absolutely central to the functioning of the Art, so much so that in the *AGU* Llull says flatly that “This mixing is the center and subject of this Art” (*ROL XIV*, 120), to which the *AB* adds that it “is

⁸⁵ Other interesting definitions can be found in the *AIV* (*MOG V*, 13), and the *AA* (*ROL XXIX*, 230; *ORL XVII*, 151), discussed by Perarnau in *ATCA* 16 (1997), 89, 96. See also the “conditioning” of memory on p. 166 below.

⁸⁶ See moreover what is said about the logical use of “mixture” in the next chapter, p. 204.

the center and foundation for the finding of all sorts of propositions, questions, middles, conditions, solutions, and even objections.”⁸⁷

After the previously quoted passage of the *AGU*, Llull continues:

Although the mixture of Principles and Rules has already been dealt with in the Third Figure, in this part we will clarify it sequentially, by studying each Principle with all the Principles and then with all the Rules, in a continuous process, so that the intellect can investigate each Principle in turn. (*ROL XIV*, 120)

So although it is similar to the binary combinatory methods applied to the Third Figure, it is organized differently, in a way permitting a much more continuous, detailed, and better organized study of each Principle.⁸⁸ It is also different since, instead of producing simple propositions in which the two concepts combined form the subject and predicate, it now, as we will see from the examples below, forms more extended arguments, directly involving the definitions of the principles.

Finally, as we said above, this mixing can substitute for the lengthy explanations which often accompanied the definitions of the Principles in previous works of the Art.⁸⁹ Now he can just give the standard, brief definitions of “goodness”, “greatness”, etc., which we saw above, and the student who wants more information can go to this section on “mixing”, where “goodness” is studied with the other seventeen Principles and with the ten Rules, followed by the same thing done with “greatness”, etc.

This technique of studying each Principle with all the other Principles, which might seem uncomfortably inward-looking, is in fact the beginning of a long outward movement, like the expanding ripples from a stone dropped in a pond. If this is the first circle of ripples, the second is the studying of each Principle by means of the Rules and Questions. These first circles constitute the two sections of Part VIII. Then in Part IX he uses the Principles and Rules to study the Nine Subjects,

⁸⁷ *SW I*, 603; *DI* 323. The *AB* merely gives a one-and-a-half-page introduction, referring the reader to “this type of mixing [which] is explained and exemplified in the *Ars magna*”, in a text of nearly 70 pages! A similar process of mixing is described in the *Ars compendiosa Dei* (*ROL XIII*, 28–30), but there it is called *De modo connectendi*.

⁸⁸ The first section, which studies the Principles using the other Principles, is taken verbatim from the *Liber de praedicatione* of 1304 (*ROL III*, 174ff.), where it first appeared as a fundamental tool of the Art. There Llull said (p. 175) that it was not only useful for the preacher, but also as a general technique for the acquisition of knowledge (*scientia*).

⁸⁹ See the passage corresponding to n. 42 above.

which constitute the Lullian outline map of the real world. In Part X the Principles and Rules are finally employed (less systematically and often more implicitly) to study the Hundred Forms, an *ad hoc* list of specific topics from the fields of philosophy, theology, natural science, the liberal arts, etc., which we will study in a moment.

This “mixing” also involves an important change in how Lull constructs his arguments and demonstrations. Instead of the former comparative techniques, combinations of concepts are now studied, as we said above, by means of the definitions given of each term. Some examples will show how this works. If we look at the first two definitions given on p. 134, we will see how he has merely combined them for this first pair of goodness-greatness (B C).

Goodness is by itself the reason that good does and produces good. And since it is great by greatness, this is a double reason (*ratio duplicata*)⁹⁰ for good to produce great good.

For the next combination of goodness-duration (B D), he uses an additive process which combines the third definition with the first two:

Goodness by duration is durable, and thus by duration it is the reason that good produces durable good. And since it is great by greatness, as we have just said, it is a triple reason (*ratio triplicata*) for good to do great, durable good.

After continuing the process for “power” as a “quadruple reason”, he says that:

one could go on multiplying mixtures, according to the method we have explained, excluding, however, contrariety and minority, with which there can be no multiplicity. But since we want to avoid excessive length we do not want to give examples of quintuple, sextuple, etc. We will, however, give examples and show how [this principle of goodness] is combined with all the [other] principles. (*ROL XIV*, 120)

All of which confirms the circular nature of the definitions which we discussed on p. 136 above. In fact, it becomes clear that the extraordinarily endogamic business of studying one principle with another is little more than an application of this circularity.

As with many—if not almost all—aspects of the Art, Lull uses not only what is explicit in his formulations, but also what else they could

⁹⁰ The *ratio supplicata* of the *ROL* text seems to be, according the manuscripts I have been able to consult, a mistake.

imply, what other possibilities they hold. And this is equally true of the definitions, as we can see in his combinations of Figure A with T, when he comes to “contrariety” (which he precede by “concordance” with which he contrasts it):

Goodness by concordance is concordable. And thus it is well (*bene*, the adverb from *bonus*, “good”) and concordantly the reason why good concordantly does concord good.

Goodness is contrary to evil, which is its contrary. And thus concordance is contrary to contrariety because they are contraries. And thus goodness is well (*bene*) and concordantly contrary to evil and contrariety. (*Ibid.*, 121)

The definitions say nothing about evil, but Lull would say that the conclusion here is implicit in his definitions of “goodness” and “contrariety”.⁹¹

With the second section of Part VIII of the *AGU*, we begin, as I said before, the expanding progress of the mixing, in which we study the Principles not with one another, but by means of the Questions and Rules. Lull begins this section by pointing out that:

Although the Principles were discussed by means of the compartments in the Third Figure, here we will present a continuous discourse with the Rules, so the intellect can get to know each Principle combined with them in a linear sequence. (*Ibid.*, 149)

So, in other words, instead of the earlier discussion with its combinatorial limitations,⁹² now he studies each Principle systematically with all of the Rules/Questions. This allows a more extended and varied discussion, as for example under the eighth Principle of Figure A, “truth”, where he explains:

We ask whether created truth converts with its unity? And we answer that it does not, because if it did, it would be too similar to the infinite truth of God which does convert with its unity. Furthermore, it would convert with goodness, greatness, duration, etc. and thus it would extend itself beyond time and place, which is impossible. And this is shown by rule B. (*Ibid.*, 165)

⁹¹ For reasons explained in Ch. 2, n. 128, for Lull, evil in itself is non-existent and can only be defined negatively, as lack of goodness.

⁹² See p. 146ff. above.

Notice how under the first rule of *Utrum* (rule B) he asks about a possible conversion of the principle being studied. With others he asks whether they are general, whether they exist, whether they are infinite. This is a clear example of what we said above about this rule allowing for many possible types of question.⁹³

Under the second rule of C, he starts with a curious assertion not directly connected with the definition of the concept.

By the first species of rule C we ask: what is truth? And we answer that it is an incorruptible being; for if it were corruptible, this would imply a contradiction, namely, that it would exist and not exist, which contradiction is impossible. (Ibid., 166)

And then he goes on to study it with the other three species of this rule:

By the second species of rule C we ask: what does it contain in itself essentially and naturally? And we answer that it has its correlatives without which it cannot exist; and in which and with which all things are verifiable, and without its essence no being can be true, as is seen by the first species of rule D.

Further, by the third species we ask: what is truth in another thing? And we answer that it is the cause whereby things are verifiable.

By the fourth species of rule C we ask: what does truth have in another thing? And we answer that it has a habit for verifying the subject in which it exists. (Ibid.)

And so on, for the species of the other rules. Notice that even here he is still giving very general information, explaining aspects of the foundations of the Art that might not be immediately apparent. But mainly he is offering a model of how to study these foundations. If once he asks whether a Principle is general, another time whether it is infinite, these are just meant as samples, any one of which (along with many others) could be applied to all the principles. One must keep in mind that Llull is seldom saying “this is the way the Art *must* be used”, but rather “this is one of the many ways in which the Art *can* be used.”

⁹³ See the text following n. 47 above.

The Nine Subjects

With the nine Subjects we come to the next circle of expanding ripples, one in which for the first time we leave the circumscribed world of the Art and start examining what lies beyond its own foundations. To do so, Llull has set up a kind of general ontological taxonomy, dividing all being into the categories we saw listed in the fourth column of the Alphabet: God, angels, heaven, man, imaginative, sensitive, vegetative, elementative, and instrumentality or artifice, categories which, as Llull says, “include everything that exists, for outside of them there is nothing” (*SW I*, 605; *DI*, 325).

In previous works of the Art, the nine Subjects were used as one of the possible ways of organizing the questions with which such works invariably ended.⁹⁴ Only with the *Liber de praedicatione* of 1304 were they incorporated into the structure and Alphabet of the work,⁹⁵ and it is with the *AGU* and *AB* that they are finally given a central role in a work of the Art. And here, what Llull does is to continue the “mixing” process, studying each Subject first with the eighteen Principles one after another, and then with each of the ten Rules. As a result, Llull has turned a simple *scala naturae* into a veritable treatise on ontology, systematized by the mechanisms of the Art. To see how Llull does this, we will pick out a few examples.

The top step of the ladder, that of God, is preceded in the *AGU* by four sections which investigate the divine dignities, their acts, their infinity, and the infinity of their acts. Then he begins, with the first Principle:

God is good, and he is his own goodness; from which it follows that his goodness is a reason for him to produce good necessarily. This is because goodness and God convert in unity of nature... (*ROL XIV*, 195–6)

⁹⁴ With a minor role in the identical texts of *TG* (*ORL XVI*, 505–514; *ROL XXVII*, 235–243) and *Ars compendiosa* (Ottaviano 1930, 155–161), and a major role in the questions of the *Art of Proposing and Solving Questions* (*MOG V*, v, 311ff.: 669ff.). They’re listed as “objects to be remembered” in the *Arbre de filosofia desiderat*, and a similar list is used to organize the questions in one of the last works of the quaternary phase, the *Questiones per Artem demonstrativam seu inventivam solubiles*, as well as in works of the post-Art phase, the *Liber de experientia realitatis Artis ipsius generalis*, the *Liber de refugio intellectus*, and the questions (frequently listed as a separate work) at the end of *Liber facilis scientiae*.

⁹⁵ In the *Liber de praedicatione* they occupy the first column of the Alphabet, and are treated at the very beginning of the work, as the essential tool for the preacher seeking subject matter for his sermons. It is interesting to note that they hold the same position in Agrippa von Nettesheim’s Renaissance commentary on the *AB*, which of course makes sense in a work also trying to adapt the Art to rhetoric, albeit of a different kind.

The text continues at some length, and under all the Principles the treatment is similar. Some, however, require special explanations, as when he treats God under the heading “difference”, in which he has to make clear that with the Divinity, it cannot be a question of difference between sensual and sensual (see the secondary concepts of the green triangle of Figure T on p. 41 above), or between sensual and intellectual, but only between intellectual and intellectual. And he adds that in God there are not only differences between his dignities, but between the correlatives of each one. This, however, could lead to a problem:

Here the intellect asks whether the said difference posits many essences in God? And it considers the matter and recognizes that it does not; just as there are not several goodnesses in him. And this is because the bonifier (*bonificans*), which is goodness (*bonitas*) in its entirety, produces from itself the entire infinite and eternal bonified (*bonificatum*), and bonifying (*bonificare*) is produced from the entirety of both. And here the intellect realizes that there is clear difference without confusion in this infinite goodness, as it posits that the bonifier is one thing, distinct from the bonified and bonifying; and the bonified is another, distinct from the bonifier and bonifying; and likewise, bonifying is another, distinct from the two others... (Ibid., 201)

Under the second rule of *quid*, we are given fourteen definitions of God “in which”, as Llull says, “subject and predicate necessarily convert”, of which a few samples are:

God is the being whose reasons [i.e., dignities] convert [with one another]; indeed, the being whose reasons convert is God.

God is the being in which the divine reasons have infinite acts, such as infinite goodness which has infinite bonifying, and greatness (*magnitudo*) infinite magnifying. And the being in which there is goodness having an infinite act is God.

God is substance free from all accidents; indeed, substance free from all accidents is God.

God is the being that needs nothing outside itself: indeed, the being that needs nothing outside itself, is God.

God is the being for whom it is impossible not to be: indeed, the being for whom it is impossible not to be is God.⁹⁶

⁹⁶ Ibid., 209–210. Most of the sources end by saying “We have defined God by twenty reasons”, in spite of which only fourteen have been given!

Under the fourth Subject of man “mixed” with the Rule of C, Llull gives thirty definitions, of which the second to last is his famous “Man is a manifesting being”.⁹⁷

Under the sixth Subject, in the mixing of the sensitive power mixed with the Principles, we can see a certain variability concerning the names of some of the concepts of the First Figure depending on their referents. Just as is in the Alphabet on p. 123 we saw “eternity or duration” which depend on whether one is speaking of the uncreated or created world,⁹⁸ so in the following excerpts the Principles of “wisdom” and “will” are treated as “instinct” and “appetite” respectively, because we are here dealing with levels below that of the rational soul.⁹⁹ Under the first of these two, Llull says:

The sensitive power has innate instinct, as is seen in man who is afraid of a snake which he sees, or a goat of a wolf; and similarly with a kitten whose eyes are not yet open but can, by means of instinct and appetite and touch, find the mother’s nipple which it imagines. And with this, the intellect knows what principles dispose the imagination to imagine.

The sensitive power, in an animal, causes appetite for sensing things, so that, through the animal in which it exists, it can exercise its act. (*ROL* XIV, 246)

Then when the sensitive power is mixed with the Principle of “middle”, Llull takes the opportunity to present a sixth sense, that of “affatus”, which in effect is the power of communication:

⁹⁷ *Homo est ens homificans* (Ibid., 237) Sometimes that last word is *hominificans*, and instead of *ens*, we sometimes find *animal* (see Bonner and Ripoll 2002, s.v., for these variations).

⁹⁸ In *Felix* (*SW* II, 897) a hermit explains to Felix, “Dear son, God’s wisdom accords with great greatness of goodness, infinity, eternity, power and will, which is why he wants man’s wisdom to be great in goodness, quantity, duration, power and will,” with “infinity”/“eternity” corresponding to “quantity”/“duration”. One could consider “duration” the general term of which “infinity” is the highest instance, or conversely of “duration” being the created semblance of uncreated “infinity”.

⁹⁹ Or as Llull puts it in forms 27 and 28 of the Hundred Forms of the *AB*: “Instinct is the figure and likeness of the intellect”, and “Appetite is the figure and likeness of the will.” (*SW* I, 618; *DI*, 338). In the *Tree of Science*, after giving the Principles “Wisdom” and “Will” as roots of the Elemental Tree, and after definitions identical to those given on pp. 134–5 above, He says of the first, “In the elemental tree [it manifests itself as] natural instincts . . .”, and of the second, “In the elemental tree, according to its nature, there is no spiritual will, but in it [the elemental tree] are sown the wills of irrational animals and the appetites of plants and elements,” so with animals “wisdom” is replaced by “instinct”, but, as opposed to the levels below them, animals still possess “will”.

...And here the intellect realizes that the common sense is a cause of sense objects inasmuch as they are sensed, as the common sense uses its particular senses as instruments: for instance, an apple is sensed by the common sense using taste, smell, sight, touch and the affatus¹⁰⁰ that calls it an apple, and hearing by which the word is heard. (Ibid., 247)

Under the eighth Subject of the elemental power, in the mixing with the Principles, we find a curious example under the Principle of “truth”:

The elementative power has true conditions whereby one species does not transmute itself into another species. And this is how we know that alchemists¹⁰¹ have reason to weep. (Ibid., 257)

The ninth Subject, in the *AGU* is called “artifice”, and in the *AB*, “instrumentative”. The latter includes the instruments used to carry out acts, which can, as Lull explains, be considered in two ways, “that is, naturally, like the eyes, which are the instrument for seeing, and morally, like justice, which is the instrument for judging.”¹⁰² “Artifice”, on the other hand, “is that which man does *extra naturam*, such as the liberal and mechanical arts, and the moral virtues and the vices.”¹⁰³ Both include the virtues and vices, or in other words, the last two columns of the Alphabet, either as the instruments for doing good or evil, or simply as part of man’s ability to act not just naturally, as when he eats and sleeps, but “artificially”, that is with his free will allowing him to make moral choices. As an example of Lull’s treatment of this theme, we give his mixing of the virtue of “faith” with the Principle of “middle” (or “medium”).

¹⁰⁰ Lull introduced this extra sense of *affatus* in a work of that name written in 1293, and made extensive use of it in his principle epistemological work, the *Liber de ascensu et descensu intellectus*. The best way to see the fairly large modern bibliography on the theme is to go to the Lull DB, Bibliography, and in the first title field type “fatus” (thus truncated, to catch variant spellings). As for “common sense”, the older meaning current in Lull’s time was, according to the *OED*, “an ‘internal’ sense which was regarded as the common bond or centre of the five senses, in which the various impressions received were reduced to the unity of a common consciousness.”

¹⁰¹ The *aliqui mystae* of the *ROL* edition is a mysterious error. The manuscripts I have been able to consult have mere variant spellings of “alchemists”: *alkimistae*, *alquimistae*, or *alquimistae*. This quote is clear proof that Lull was not an alchemist, as some followers pretended.

¹⁰² *SW* I, 612, and *DI*, 332. The same term is used in the *Ars compendiosa* (Ottaviano 1930, 155); and the *Liber de experientia realitatis Artis ipsius generalis* (*ROL* XI, 178).

¹⁰³ *Liber de praedicatione*, *ROL* III, 153. The same term is also used in the *TG* (*ORL* XVI, 380; *ROL* XXVII, 85). In the *AGU* (*ROL* XIV, 263) he adds that the liberal and mechanical arts will be treated under the Hundred Forms in Part X.

Faith is the medium through which the intellect earns merit and rises to its prime object, which instills faith into the intellect, so that it may be as a foot the intellect uses in its ascent. And since the intellect has another foot of its own nature, namely understanding, it climbs upward like a man climbing a ladder with both feet, who begins by putting the foot of faith on the first rung, with the foot of the intellect coming next, and likewise with the second rung, where the foot of faith comes first as the intellect continues to ascend step by step with faith coming first and understanding second, just as in a disputation, doubt comes first, followed by affirmation or negation. (*ROL XIV, 278*)

So, for Llull faith is clearly prior, and as such it is the instrument permitting the intellect to carry out its task. Notice the reference at the end to the three species of the Rule of “whether”, for the explanation here ties in with the connection between possibility and faith explained under that rule.¹⁰⁴

Among the vices, in the mixing of that of lust with the two Principles of virtue and truth, Llull shows a nice understanding of people’s capacity for self-deception.

A man deeply ensnared by lust believes that he is highly virtuous, and at such credulity the intellect is greatly astonished.

Lust is a mendacious habit because it begins with beauty and ends in turpitude. And it treats that which is evil as if it were good. (*Ibid.*, 298)

The Hundred Forms

With the Hundred Forms we come to the last circle of expanding ripples, one in which Llull shows how the Art can be applied to an ordered set of sample subjects. Llull had already displayed such lists in the *Tree of Science*, the *Proverbs of Ramon* (Part II), the *LN*, and after the *AGU* and *AB*, in the *Ars consilii*. They are all different, answering to the needs of what concepts Llull wanted to give as examples for a particular work involved.¹⁰⁵ The only exceptions are the *AGU* and *AB*, where the lists are, as might be expected, identical. Here they appear in Part X, “Which Treats of Application”, where they are preceded by brief explanations of other manners of applying the Art (such as

¹⁰⁴ See p. 138 above.

¹⁰⁵ See Bonner and Ripoll 2002, 88–98 for the various lists of Hundred Forms, along with others which present a lesser number of forms.

applying terms implicit in the Art to those that are there explicitly; applying abstract terms to concrete ones of the Art; by using the First Figure, etc.). Afterwards we get the Hundred Forms themselves:

1. entity	21. habit	41. potentiality	61. fortune	81. philosophy
2. essence	22. position	42. punctuality	62. order	82. geometry
3. unity	23. time	43. line	63. counsel	83. astronomy
4. plurality	24. place	44. triangle	64. grace	84. arithmetic
5. nature	25. motion	45. square	65. perfection	85. music
6. genus	26. immobility	46. circle	66. explanation	86. rhetoric
7. species	27. instinct	47. body	67. transubstantiation	87. logic
8. individuality	28. appetite	48. figure	68. alteration	88. grammar
9. property	29. attraction	49. directions	69. infinity	89. morality
10. simplicity	30. reception	50. monstrosity	70. deception	90. politics
11. composition	31. fantasy	51. derivation	71. honor	91. law
12. form	32. fullness	52. shade	72. capacity & incapacity	92. medicine
13. matter	33. diffusion	53. mirror	73. existence & agency	93. government
14. substance	34. digestion	54. color	74. comprehension & apprehension	94. chivalry
15. accident	35. expulsion	55. proportion	75. discovery	95. commerce
16. quantity	36. signification	56. disposition	76. semblance	96. navigation
17. quality	37. beauty	57. creation	77. antecedent & consequent	97. conscience
18. relation	38. newness	58. predestination	78. power, object, & act	98. preaching
19. action	39. idea	59. mercy	79. generation, corruption, & privation	99. prayer
20. passion	40. metaphysics	60. necessity	80. theology	100. memory

The first thing to be noticed about the concepts listed here,¹⁰⁶ is that not only are some explicitly grouped in twos or in threes (among numbers 72–79), but many others presented separately could also be so grouped (unity/plurality, genus/species/individuality, simplicity/composition, etc.).¹⁰⁷ One can also see larger groupings: 14–24 form the categories or

¹⁰⁶ Some of the terms require some further explanation. The full version of 41 is “being existing potentially”. The word *situs* of 22 could also be translated as “situation”. The “punctuality” of 42 does not have its modern meaning, but is simply the abstract of *punctus* “point”. For 49 the *AB* has “The general directions (*generales rectitudines*) are six, with the body at the center of diametrical lines”, obtained by dividing each Cartesian coordinate of three-dimensional space into its positive and negative components. 66 is *declaratio*, which could also be translated as “clarification”.

¹⁰⁷ In the *Tree of Science*, after presenting each of the Hundred Forms separately, they are all then explicitly grouped in twos and threes in a later section (See Bonner and Ripoll 2002, 73–74).

predicaments;¹⁰⁸ 42–49 concern geometry, which gets a separate entry among the liberal arts of 82–88; and finally 89–97 embrace themes of a moral, social and practical nature.

The *AB* has only a one-sentence definition of each Form, whereas the *AGU* has an explanation based on the Art, sometimes of a short paragraph of 8 to 10 lines, and other times forming an essay of several pages.¹⁰⁹ At the end, as we explained above on p. 142, with the hundredth form of “memory”, he offers a detailed model for how to do this, studying it, just as he had done for the Nine Subjects, with the Principles and Rules, to show how “any of the Hundred Forms can be studied by the Principles and Rules”.¹¹⁰

To see how this works, we can take as an example how Lull mixes “memory” with the principles of “wisdom” (here under the heading of “intellect”) and “will”, with a result reminiscent of Figure S of the quaternary Art.

The memory has one condition with the intellect and another with the will. For through the intellect, by remembering, it receives species which are understood, and in this way it recovers understood species; and through the will it receives species which are either loved or hated; and in the same way that it received species from the will, it returns them to it. (*ROL XIV*, 391)

It is only with the first species of Rule C that we get the definition:

By the first species of rule C we ask: what is memory? And we say it is a power whose specific function is to remember. (*Ibid.*, 393)

With the second to last Rule of “how” Lull explains how to make the memory function:

With the first rule of K we ask: how can one dispose the memory toward remembering an object that one wants to, but cannot remember? And the answer is that the method consists in applying the definitions of the

¹⁰⁸ Consisting of 14. “substance” and 15. “accident”, which accidents include 16–24. The predicables are spread among 6, 7, 9, and 15 (omitting “difference”, which was presented under the Second Figure).

¹⁰⁹ Usually, each Form of the *AGU* starts off with the definition of the *AB*; occasionally, however, the definition of the *AB* is omitted from the *AGU*, which starts right off with its explanation. Examples are forms 17–22; in 17–19 Lull even refers to the definition, as if he expects the student to have the *AB* at hand.

¹¹⁰ *ROL XIV*, 391. In the *LN* he does the same for the first two of the Hundred Forms (“individual” and “goodness”), but there studying them only with the ten Rules (see *ROL XXIII*, 82–88; *NEORL IV*, 65–70).

principles and the species of the rules to what is similar to the thing one wants to remember. For just as the intellect attains the thing which is similar through similarity (*per similitudinem attingit similitatum*), so does the memory remember its desired object through similarity. For if someone sees a man similar to his son, he immediately remembers his son by that similarity. (Ibid., 394–5)

With the previous 99 Forms he gives more general, schematic advice on how the Art should be used to study each subject, picking out those components which seem most appropriate. We will give three examples, of which the first is philosophical.

73. Existence and Agency

Existence is the form with which a thing existing is what it is; and agency is a form which moves an existing thing toward its intended goal. This is shown by the second species of rule C, by rule E,¹¹¹ and by the definitions of goodness, power and middle. And if the definitions of goodness, greatness and eternity—or duration—are added, it necessarily follows that existence is as great as agency. And here the intellect realizes that God acts [*est agens*] as much by the second species of rule C as he exists [*est existens*] by the first species of rule E. Of which things, that is existence and agency, in the first Subject, one can find more information in the chapter in which God is defined. (ROL XIV, 353)

Our next example shows the spiritual or contemplative use of the Art, something which remained one of the central goals of his entire enterprise ever since the *Book of Contemplation*.

99. Prayer

Prayer is a form with which the person praying speaks to God in a holy way. Whoever wants to pray well should acquire a good disposition by the ninth subject, because God deserves to be honored with the virtues and not with the vices.

The person praying or contemplating should pray or contemplate God by similitude with the evacuation of the Third Figure, as the prayer is signified by evacuation in this figure, saying, for instance: “O good and great Lord God, since your innate and supreme goodness and greatness have supreme concordance in their natural distinction and concordance; and since goodness and greatness are in you reasons for naturally

¹¹¹ The printed text kaleidoscopes *Et hoc patet per secundam speciem regulae C et per regulam E* to *Et hoc patet per secundam speciem regulae E*, but the crucial *C et per regulam* is in the best manuscripts.

producing great good, may your supreme goodness and greatness produce morally and effectively in this world many good and great things whereby your people can be good and great, in blessing you, magnifying you and being in concordance with you. Because you, Lord, are the essence and substance in which your goodness and greatness have the good, infinite and mutually distinct correlatives which we see in you. And because you, Lord, are a good and great creator, may it please you that your people become good and great in praising and serving you, so that you have a good and great people with different and concordant customs.”

After evacuating compartment B C, the person praying or contemplating will then evacuate compartment B D, etc. [saying, for instance]: “O sweetest goodness existing in distinct diffusion in an eternal subject, removed from all contrariety, you who are primordial and sole eternalizer subject to nothing else, with your kindness and eternity, please defend your people from everlasting torment.”

The person contemplating or praying, after having evacuated the entire Third Figure during successive days, then goes on to multiply the Fourth Figure by contemplating or praying according to the method of this figure, namely by the column B C D... (*ROL XIV*, 388)

Notice how in the paragraphs quoted—as well as in much of the rest of three-page explanation of this form—Lull refers to “prayer or contemplation”, thereby equating the two. Notice too how, in spite of the apparently “drier” more combinatorial approach here, many of the formulations of this section could be found in the *Book of the Lover and the Beloved*, the *Art of Contemplation* following it, and the *Tree of Philosophy of Love*. One thing Lull never does is to oppose the Art and spirituality; for him, if the first could not serve the second, it would be useless.

Our last example is from quite a different realm. In addition to the Principles and Rules of the Art along with the Subjects, it makes ample use of the three powers of the soul, and, towards the end, of the virtues and vices.

95. Commerce

Commerce is an acquired practice with which the merchant¹¹² knows how to buy and sell so as to increase his wealth... In commerce, the buyer relates to the seller with his principles and rules, and the seller likewise relates to the buyer with his principles and rules, so that each can acquire something more in exchange for something less. Hence it follows that the

¹¹² In Latin “merchant” and “commerce” have the same root: *mercator* and *mercatura*, from *mercatus*, “trade; market”.

merchant who knows how to discourse with his intellect, imagination and senses by means of the principles and rules of this Art has greater knowledge in buying and selling than another merchant ignorant of this discourse. Whatever a merchant does in buying or selling, he does with his intellect, will and memory; and so he must use these powers in their natural order in the soul, whereby the intellect first of all understands, followed by the will which makes a choice, then followed by the memory which preserves the species with which buying and selling was done in the marketplace...

Subject E deals with man, and therefore with the human body, which is a part of man. And since buying and selling involves things needed by the body, a good merchant should discourse with his intellect through subject E so that he can become aware of the body's needs. The third and fourth species of rule C are great topics for the merchant, by means of rules H I, because the same merchandise or goods have lesser or greater value in different places or at different times..., and more in one man, for questions of need and utility, and in another because of matters of well-being and honor...

In a faithful and virtuous man, commerce is a perfect practice; but in a deceitful and sinful man, it is an imperfect practice. And therefore, anyone who wants to be a good merchant should become practiced in the virtues according to the way in which they are discussed under the Ninth Subject. For well-formed commercial practice fosters a common and social approach to goods, whereas an ill-formed practice destroys this common and social approach... (*ROL XIV*, 378)

With these Hundred Forms we can see not only the variety of topics, but also the variety of methods with which each one can be studied, both contributing to the remarkable adaptability and generality of the Art.¹¹³

Questions

In the ternary phase the Questions fulfill the same purpose as that explained above with the quaternary Art (see p. 77 above), but as might be expected, they are now structured very differently. To explain this, we will start by seeing how they are distributed in the two last works of the Art.

¹¹³ For brief notes on some of these forms, the reader can consult *SW I*, 616–625; *DI*, 336–343.

In the *AB* the Questions are applied to all of the previous sections of the work, whereas the *AGU* omits the first six, and starts directly with the Table. The distribution is as follows.

<i>AB</i>	<i>AGU</i>	sections	<i>AB</i>	<i>AGU</i>	sections
1	—	1st Fig.	7	1	Table
2	—	2nd Fig.	8	2	Evac. 3rd Fig.
3	—	3rd Fig.	9	3	Mult. 4th Fig.
4	—	4th Fig.	10	4	Mixt. Pr. & Rules
5	—	Definitions	11	5	Nine Subjects
6	—	Rules	12	6	Hundred Forms

In the *AB*, since he is merely giving a sample of how to use them, there is a roughly equal space allotted to each of the twelve sections, whereas in the *AGU* the differences are notable. In the latter work the Questions relating to the Table, the Nine Subjects and the Hundred Forms are presented at great length, whereas the other three sections are dispatched in one page (or less) each. We will therefore follow Llull in studying the sections he chose to emphasize. It should be understood, however, that he does not solve all the Questions in the same way, as he explains in the introduction to this part:

Some of the questions we solve here, and others we refer to the places of this book in which their solution is implied and signified. (*ROL XIV*, 395)

The questions “we solve here” are those using the Table, but now instead of solving a single question with one column, he takes the seven-column version of the *AB* shown on p. 145 above, to show the many different questions that it can solve by not restricting the interpretation of the letters to the First and Second Figures, but including also the other four columns of the Alphabet. So for instance, from the first column of the Alphabet he has:

BCTD Question: Whether a single angel is greater than heaven itself?
The answer is yes. The reason is that an angel has correlatives with which it objectifies God, whereas heaven does not.¹¹⁴

¹¹⁴ *Ibid.*, 396. *Objectare*, which I have translated as “objectifies” meant to convert something into an object of the senses or of the powers of the soul, “like the truth which is captured by the understanding when it understands, or a color which is perceived by the sight when it sees;” see Colom Mateu 1982–5, s.v. *object*.

Looking at the Alphabet on p. 123 above, we can see that “whether” = B, “angel” and “greatness” = C, and “heaven” = D.

In an example from the last column, he writes:

HTIK Question: When is there lesser charity, patience, and pity? The answer is: Whenever there is greater ire, lying, and inconstancy. (Ibid., 416)

Here “when”, “greater” (“majority”), “charity”, and “ire” = H; “patience” and “lying” = I; while “lesser” (“minority”), “pity”, and “inconstancy” = K, thus using four different columns of the Alphabet. So even though the interpretations of the letters are still limited to concepts displayed in the Alphabet, their use is much freer (the letter T, for instance, no longer has its separating role), and the generation of possibilities is almost limitless, a factor which for Llull was important and which was of great interest to his followers in the Renaissance and after.

As for the other questions, for which Llull refers us “to the places of this book in which their solution is implied and signified”, we come to a new departure in the structuring of the Art. In previous works even of the ternary phase, he had always answered questions by means of the re-use of combinatory mechanisms presented before in the same work,¹¹⁵ but here he merely answers them by referring the reader back to the place in the *AGU* where he will find the answer.

It is striking that in a passage immediately following the one just quoted about “the places of this book”, he repeats twice more the word “places”. Now, to a contemporary reader, the word *locus*, which was the translation of the Greek *topos*, referred to the fundamental component of dialectic, heir to Aristotle’s *Topics*.¹¹⁶ Usually, as in the standard medieval treatise on logic by Peter of Spain, it referred to what was called the “the seat of an argument”, that is, a kind of maxim or general rule to which an argument could be referred to test its validity.¹¹⁷ In the *AGU*,

¹¹⁵ With the exception of the *Tree of Science*, where this referring back to the main text is one of the three methods used to solve questions. See the prologue to the last tree, the “Tree of questions”. This is not, however, a work of the Art, nor does it deal in a general way with combinatory mechanisms.

¹¹⁶ Llull, instead of the more usual *locus* (pl. *loci*), uses here—at least in the *AGU*—the older neuter *locum* (pl. *loca*).

¹¹⁷ See the definition of a locus as a *sedes argumenti* in Peter of Spain 1972, 58. Of the three methods in the *Tree of Science* mentioned two notes ago, the first is that of finding the “place” among previous trees, the second is precisely that which is

however, “the seat of an argument” is not a closed list of several dozen aphorisms, but quite literally the “place” in the text of the work where the doubt could be resolved or the solution found.

As an example of how this works, for the Nine Subjects, the first question under the heading of God is:

Question: Is it in any way necessary for God to produce good? Solution: go to the subject of God in the paragraph of “goodness”.¹¹⁸

Where the reader will find the argument reproduced on p. 160 above. This is the way it is done with the Nine Subjects, with each question referring to the “place” of the text where the solution can be found.

With the Hundred Forms he gives no specific references, but since the questions are grouped by forms, this instruction is understood. Another difference is that here the solution is often more implicit than explicit. Sometimes this relation is quite clear, as for instance under Form 40, Mathematics, we are asked “Whether mathematics is a sign of ideas?” If we look under that form, we will find the assertion that “Mathematical form is exemplified and caused by means of an idea, stripped of all created beings, so that the idea may be known by the mathematical form.” Sometimes however we have to work out the answer by analogy. Under Form 52, Shade (or Shadow), for instance, we are asked “If a mirror is diaphanous, why is it shady?” Under the corresponding form he explains how air and ice are both diaphanous and take on the color of the thing being perceived through that medium.

As a result, the second and third sets of questions—those on the Nine Subjects and on the Hundred Forms—now act as a kind of encyclopedic index to the contents of the Art. If the reader is interested in grammar, for example, he can go to the questions to Form 88, Grammar, where, among others, he will find:

Where is the adverb implied?
 Where is the preposition implied?
 Where is the interjection implied?
 Where is the nominative case implied?
 Where is the genitive case implied? (*ROL XIV*, 513)

provided by “maxims conditioned according to the nature of the trees”, and the third is a combination of the first two.

¹¹⁸ *ROL XIV*, 422. In the 1645 edition of the *AGU*, the reader is referred back to page, article, and number. In Yanis Damberg’s web translation of the work (see n. 1 above), the textual reference is replaced by a hyperlink.

with the knowledge that the answers are to be found in the corresponding Form 88 of the Hundred Forms.¹¹⁹

At the same time, this method implies that the answer to just about everything one would like to know is found explicitly or implicitly in the text of the *AGU*, a text based—in the case of the Table—on an extreme flexibility of interpretation of the nine-lettered Alphabet, and—in the case of the Nine Subjects and the Hundred Forms—on the mixing of Principles and Rules.

Should the reader still harbor any doubt as to the importance of these questions, Llull says at the end of the immense long chapter presenting them:

We are done with the questions from the eleventh part of this book, and we have provided a doctrine whereby the artist will know how to make questions and refer them to their places. The practice of this Art consists mainly in the solving of questions. (*ROL XIV*, 523)

The Application of the Ternary Art in Other Works

A good example of the use to which the ternary Art can be put is the immense *Art of Proposing and Solving Questions*, written in 1294–5, shortly after the *TG*.¹²⁰ This work, in addition to treating an extraordinarily wide variety of subjects, is “principally dedicated to demonstrating the truth of the Catholic Faith with necessary reasons”, and thus deals with subjects, as we have seen, avoided in central expositions of his system.¹²¹ It is divided into two parts. The first, containing Distinctions I and II (totaling 134 pages of double columns of the *MOG*), presents extended explanations of the Rules, the Definitions, and the binary combinations of the Third Figure, along with two exemplifications of the working out of the Table and the Fourth Figure, different from

¹¹⁹ The manuscripts I have consulted are clearly laid out so the reader can find his way. The questions on grammar, for instance, are preceded by a heading “De quaestionibus grammaticae”, and in the margin we find the number “lxxxviii” corresponding to the Form on that subject.

¹²⁰ The title of *Ars ad faciendum et solvendum quaestiones* (*Art de fer e solve qüestions* in Catalan), is, I am told by Joan Carles Simó who is editing the work for *ROL*, the only one found in the manuscripts; that of *Lectura super Artem inventivam et Tabulam generalem*, under which it appears in all modern bibliographies, including that in the catalogue of *SW*, is apparently an invention of the Renaissance editor of the work, Alonso de Proaza (in his edition of Valencia, 1515).

¹²¹ *MOG V*, v, 1–2: 359–360.

those in the *TG* and *AGU*.¹²² The rest of the work—Distinction III (224 pages) devoted to the answering of questions—is itself divided into two sections. The first takes ten subjects, Production in God,¹²³ Incarnation, Creation, Angels, Eucharist, Original Sin, Predestination, Resurrection, Heaven, and Hell, and treats each one by the Ten Questions, to each of which he assigns a ternary group of concepts heading a column of the Table, so that he can answer the question by the twenty compartments of that column. To see how this works, under Predestination, for example, the first question, “Whether a predestined man can be damned?”¹²⁴ uses the combination B D E to find twenty answers in the compartments of that column of the Table, the second question “What is predestination?” uses C E I to be answered with the compartments of that column, the third, “Of what is predestination [composed]?” by D E F, the fourth question of “Why” asks “What is the reason for predestination?” and is answered by B E F, and so on. The Table used is the full one of the *TG* (identical to that of the later *AGU*), but the columns are chosen more or less at convenience for the matter under discussion, with the only restriction being that each must contain the letter of the question being asked.¹²⁵

To see how the actual arguments are generated, we can look more closely at the treatment here of predestination, and in the process see how it differs from that of the quaternary phase we saw before. The first question of “Whether” takes the example of an individual (which

¹²² The definitions of Principles and the working out of the implications of their binary combinations, are, I think, by far the longest and most detailed of any work of the ternary Art.

¹²³ “‘Production’ (*productio*) is a technical term of trinitarian theology. Divine ‘productions’ (to be well distinguished from ‘causations’) concern, in the inner life of God, the eternal generation of the Son by the Father, and the eternal spiration of the Holy Spirit by the Father and the Son.” (Rijk 1990b, 67 n. 13).

¹²⁴ Note that in Lull “predestined” means, as is frequent in theological use, “predestined to salvation”. See *OED*, s.v.

¹²⁵ Quite by chance, the column of D E F mentioned above also appears in the briefer Table of the *AB* (the third column of the Table on p. 145 above). The *ad hoc* choice of columns is important for showing the freedom with which the techniques of the Art can be manipulated, as opposed to the systematic use we saw above under “A Single Question” where the Table is ordered to instruct the reader in its use, as well as to show the validity of ALL the combinations in answering said single question. In the work we are studying here Lull even admits ternary combinations not represented in the Table, such as that of D B G on *MOG* V, v, 139: 497, but if the reader takes the trouble of checking, it proves identical with the column of B D G with the first two letters systematically interchanged.

we will cite and analyze using, as before, superscript indices to indicate the letter of the Alphabet being used):

Whether a predestined man can be damned?

Solution: Compartment B D E.

Supposing that Martin is predestined, we ask whether^B he can be damned after having been predestined, given that Martin's predestination is good^B, eternal^D, and powerful^E, insofar as God's wisdom knows eternally that Martin is predestined. (*MOG V*, v, 237: 595)

He then goes on to argue that no power^E would be against this good^B and eternal^D knowing because of the equality of these qualities in God. By the same token, Martin could not be damned, for if he were, then God's power^E would be contrary^D to that which it could do (as just proved), making for contrariety^D in God, which is impossible. These two arguments, one affirmative and the other negative, are mirrored in two similar arguments involving God's justice (as opposed to his wisdom), one affirmative proving that Martin could be damned through the exercise of his free will, and a negative one stating that in this case he could not be predestined.

Llull's solution is curious.

Since according to wisdom we must affirm that Martin can be predestined and by justice that he can be damned, we do not know whether he is predestined or damned,¹²⁶ and therefore our initial position of saying that he was predestined, and asking if he could be damned, caused our intellect to fall into a contrariety between the two preceding negations and the two affirmations. As a result we should affirm and hold that Martin can be predestined and damned, and therefore there is no contrariety of the affirmation against the negation, nor of the negation against the affirmation. (*Ibid.*)

He goes on to say that the perfection of god's wisdom and justice permit each to act without contrariety, thus falsifying the two previous negative arguments.

And this falsity enters our reasoning because of the false position from which the question was posed, when we asked whether a predestined Martin could be damned. (*MOG V*, v, 238: 596)

¹²⁶ Here Llull uses the word *praescitus*, "foreordained to perdition".

Notice the similarity of structure between the opposing parts of this argument and of the one given on p. 49ff. above. There is even a vague parallel between the conclusion here that the question was incorrectly proposed, and the complaints about the “false significations which S received” of the earlier proof. Here, however, except for the three Principles of goodness, eternity and power (B D E), and the two divine attributes of wisdom and justice, there are no references to structures of the Art, none of the complex mechanisms of the earlier proof, with its progress from one Figure of X to another and its alternation between various species of Figure S, nor is there any algebraic notation in the text of the proof, nor rectangles drawn around compartments. As a result the newer argument is easier to follow; the reader can now read it without a crib. Based as it is on fewer and more general elements of the Art (the five just listed), one now could read it much more like a normal theological text, without necessarily being aware to which structures of the Art these five elements refer. The result is that, in this case at least, the difference of form is greater than that of content.

In the next compartment of the same column, B D T B, the last letter refers to “difference” and the answer revolves around the idea that the difference between God’s wisdom and justice which Martin perceives cannot be true (because of the equality of all his attributes), thus producing the conclusion that

Martin can be predestined [to salvation] and he can be damned, and in this conclusion the human understanding transcends over and above its human reason by understanding predestination and free will according to God. (Ibid.)¹²⁷

These are just outlines of the first two of the twenty compartments of the column B D E used to answer the question of “Whether”. The second question of “What”, uses the column of C E I to define “predestination”, the first compartment of which uses those letters to define it as

Predestination is the act of God’s wisdom which, with greatness^C of power^E and truth^I, men understand that they are predestined to salvation before they are born; and predestination is the same act of wisdom which a predestined man does not understand when he is in mortal sin. (*MOG* V, v, 241: 599)

¹²⁷ This, again, is a reference to the transcendent points; see Ch. 2, n. 96 above.

In the same column, under the eighth compartment of C T C E, he uses the concepts of greatness, concordance and beginning to define it as with its correlatives:

Predestination is that which has a great^C beginning^E in the concordance^C of predestinative, predestinable, and predestinating; and sin is that which destroys this concordance. (Ibid.)

And so it goes, with each of the 10 questions answered by 20 compartments, making for 200 solutions, or what amounts to a veritable treatise on the subject of predestination, and with each of the other nine subjects treated in the same manner.

The second section, with its almost a thousand “small” questions, answers them basically in two different ways. With the first 176 on God and the Incarnation, on angels, on the heavens, and on the rational soul, he answers then by a variety of means from the Art, and sometimes merely with general arguments, or with references to places in this work or to other works where the answer will be found.¹²⁸ This part is chiefly interesting because of specific theological problems Lull is willing to tackle such as “Whether God could make the world be eternal without beginning and end?”, “Why the divine persons are three and not fewer or more?”, “How can God the Father generate God the Son without time, quantity, place, motion and composition?”, or “Since Jesus Christ’s human nature was produced from non-being and has natural imperfections, we ask: how can it be that the divine nature joined with it does not, through the human nature, have some imperfection from the joining of both natures?”, etc.¹²⁹

Almost all the rest of the subjects, which, along with the previous five, form a kind of lesser list of a Hundred Forms and similarly cover all manner of topics,¹³⁰ are systematically solved with the ten Questions, thereby carrying out the program adumbrated in the equivalent sections of the *LN* and the *AGU*.¹³¹

¹²⁸ “De mille minutis quaestionibus”, *MOG V*, v, 310–321: 668–679.

¹²⁹ The answer to the last question (*MOG V*, v, 313: 671) reminds one of the more metaphorical techniques of the quaternary phase: “Just as a drop of red wine thrown into the sea can not retain its red coloring, so in Christ’s human nature joined to the Deity there remains no imperfection.”

¹³⁰ *MOG V*, v, 321–358: 679–716. See the list in Bonner and Ripoll, 86–87.

¹³¹ See n. 110 above.

Testing the Art

Now that the ternary Art had been established, one big question remained: would it prove valid in university circles? If not, it ran the risk of remaining in a kind of intellectual backwater, and of never being accepted by others as a valid missionary tool. Llull had already written two works in 1298 during his second visit to Paris to try to storm this particular citadel: the *Declaratio Raimundi per modum dialogi edita contra aliquorum philosophorum opiniones* and the *Disputatio eremitaie et Raymundi super aliquibus dubiis quaestionibus Sententiarum Magistri Petri Lombardi*.¹³² The first is a defense of the condemnation in 1277 by the bishop of Paris, Étienne Tempier, of 219 articles considered heterodox, articles attributed largely to the more radical Aristotelians of the Arts Faculty,¹³³ in which Llull analyzes each article, explaining why he thinks it erroneous, and in the course of his defense of the condemnation, treating many topics still very much under discussion in Paris. The second enters more squarely into the curriculum of the Faculty of Theology, one of whose fundamental texts (along with the Bible) were the *Four Books of Sentences* of Peter Lombard, a work on which every student of that faculty had to produce a commentary in order to complete his course. Llull does the same, offering a solution to a representative sample of articles of the *Sentences*, but instead of using contemporary commentary techniques, he uses those of his Art, thereby presenting it as an alternative to one of the benchmarks of contemporary scholasticism.¹³⁴

But probably the most persuasive tactic would be to get an outsider, conversant with Parisian academic circles, to put to him a series of questions on all kinds of subjects—of a kind which scholastics called *quodlibeta*—to see if his Art could answer them as effectively as a Parisian master. And who better for the task than the canon of Arras, Thomas Le Myésier, who was a *socius* of the Sorbonne, where he had studied theology.¹³⁵ From whom came the initiative, or whether they thought it up together, we don't know, but the fact is that Le Myésier drew up

¹³² The first edited in *ROL* XVII, 219–402, and in a French translation with helpful notes in Lulle 2006; and the second in *MOG* IV, iv: 225–346.

¹³³ The condemnation itself has been edited and commented on in Hissette 1977 and Piché 1999. For more information, see Bianchi 1990, Bianchi 1999, and Aertsen et al. 2001. See also Ch. 1, n. 23 above.

¹³⁴ See Bonner 2002b.

¹³⁵ Also associated with the Sorbonne were the two leading masters of theology of the moment, Henry of Ghent (who, interestingly enough, had been on the 1277

a series of questions as a test, which Llull answered in the *Quaestiones Attrebatensis*, or more properly, *Quaestiones Magistri Thomae Attrebatensis* (*Questions of Master Thomas of Arras*) in the following year of 1299.¹³⁶ In the little prologue Llull explains to his disciple:

I received your letter containing some questions for me to solve for you, but when I received it I was occupied with a *New Book of Geometry* which I was writing, and then with another book on the *Principles of Theology*. Having finished this last, I began this treatise, hoping you have not taken it ill that I have waited so long. You should know, Sir, that in some I answer you explicitly using the Principles of the Art, as you may find in the *Practica Artis generalis*, applying the terms of the Art to the questions.¹³⁷

He then adds instructions for readers not as familiar with the Art as Le Myésier:

But in case some, unfamiliar with the Art, will not be able to understand, I pray you, just as I labor for the public good, that you too may work for the same end, and present these questions in a form that may be understood by those who do not know the Art. And this will be good, because the questions are very good.

As promised in the prologue, Llull not only answers questions using the Art, but often offers brief explanations of the components he is applying, as for example in Question II:

Whether God punishes some people in this world for their offenses?

Solution: It is said in Rule C that a being can be considered in itself and in another thing, that is, What is it in itself? And, What is it in another thing? And God is a being who in himself is just from and in eternity;

commission studying the 219 articles) and Godefroy of Fontaines. See Hillgarth 1971, 159 and 189–190, for more details.

¹³⁶ Le Myésier gave the work a certain prominence in the *Electorium* (see Hillgarth 1971, 159–161 and 405). It had been preceded some years before by the *Quaestiones quas quaesivit quidam frater minor* of ca. 1290, a work of a completely different, non-quodlibetal nature (with questions mostly having to do with the Creation; see *ROL XXIX*, 439–501). As Ruiz Simon has pointed out to me, this practice of writing works to answer questions of contemporaries was not uncommon at the time. In the catalogue of Aquinas' works found in Weisheipl 1974, 389ff., for instance, one can find some seventeen such works (nos. 64–65, 69–83). Llull's innovation is to use the technique to show how the Art can be used to provide the answers.

¹³⁷ Lullus 1746, 1–2. The first two works cited are the *Liber de geometria nova* and the *De quadratura et triangulatura circuli* (whose alternate title is, curiously enough, *Principia theologiae*). The third work is the *Ars compendiosa* (with an alternate title of *Brevis practica Artis generalis*), a shortened version of the Art also written in Paris just six months before these *Quaestiones* addressed to Thomas Le Myésier.

and consequently, in his effect, wherever this effect may take place, and in all time. We can therefore conclude that in this world God will punish sinners for their offenses. (*Ibid.*, 3–4)

Or on a completely different topic, we have Question XXVII:

Whether a man's life can be prolonged by nature and art?

Solution: This question must be answered in the affirmative, as is signified by Rules D and G in the following way. Radical humidity in man lives by means of the nutritional humidity proper to it, for without the matter coming from nutritional humidity, the radical humidity would not have anything on which to live, and of this we have experience from our food and drink, and from the oil in a lamp by which the flame lives. But by the additional appropriated matter a doctor can help nature by properly disposing that nutritional humidity which it is nature's task to provide.¹³⁸

As might be expected of such a test, the work touches a wide variety of subjects, not only theology and medicine as in the two examples just cited, but also philosophy, astrology, alchemy, etc. and even tackles politically charged questions such as "Is a Christian obliged to obey the Pope's command against the dictates of his conscience?"¹³⁹ The questions end with five about the Art itself, containing, for instance, the passage quoted on p. 101 above.¹⁴⁰

After answering the fifty questions Le Myésier had sent him, Llull steers off into his definitional doctrine, saying:

Master Thomas, you also sent me the following words so that I might define them for you according to the Art, and as you know, the second Rule is that of definitions, insofar as it asks "What?" And since it has been said in the Art that the mixture of Principles and Rules of the Art should be done in resolving doubts and to uncover secrets, therefore we will give doctrine for making definitions of things by mixing a Principle with a Principle, a Rule with a Rule, or a Principle with a Rule, so that

¹³⁸ *Ibid.*, 23–24. Radical humidity, humor, or moisture was, in medieval natural philosophy, the vital humor to which was attributed the conservation of animal life. See *SW* II, 848, and the references there.

¹³⁹ This problem, as Hillgarth has pointed out, was of considerable interest in 1299 given Boniface VIII's opposition to Phillip IV of France. See Hillgarth 1971, 159–161, for an excellent analysis of the work as well as other examples of questions answered.

¹⁴⁰ Questions XLVI–L (*Ibid.*, 41–46).

the definition will appear, and this we will exemplify concerning those things about which you asked.¹⁴¹

His first example begins:

We first ask, what is God?

Solution: God is that goodness which eternally and infinitely is the good reason which does eternal and infinite good. This definition is done by the definition of “goodness”, and applied to the “greatness” and “eternity” of God, and in this way is shown the mixture of Principles by means of the first species of the second Rule.

For the second species of Rule C he mixes a Principle and another Rule:

We then ask: What does God have in himself coessentially, substantially, and naturally?

Solution: To what is asked by the second species of the second Rule, I answer by the definition of “power” and by the second species of the third Rule.¹⁴² God has in himself coessential, substantial, and natural deifying, deifiable and deify, that is to say, the blessed Trinity. If not, there would be no power (*potestas*) in God by which deity could be (*potest esse*) sustained, nor...

After going through the last two species of Rule C, and performing the same four-part operation to define “angel” and “heaven”, he says the same technique can be applied to the other words he was asked to define. He then proceeds to give short definitions of twenty-seven other concepts, in a kind of abbreviated version of the Hundred Forms.¹⁴³ The only difference is that here, as with the previous questions, he usually cites the component of the Art he is using, as, for instance, in his definition of “faith”.

Faith is an act of the intellect which, over and above its own forces, understands truths about God, obeying the will which loves those truths about God. And this is signified by the third species of the second Rule, by considering that the intellect is in truth true through God. (Ibid., 53)

¹⁴¹ Ibid., 47. Notice how the notion of “mixing” which we saw before, is important in this work.

¹⁴² “Of what is something made or constituted?”

¹⁴³ See Bonner and Ripoll, 91–92 for a complete list, as well the individual entries for each in the text of the dictionary.

A Demonstrative Art?

The reader expecting to find in the ternary Art an updated replica of the *AD*, the point of which was to offer proofs, might be puzzled by the lack of anything here explicitly referred to as a demonstration. In the quaternary phase, for instance, one finds a total of ten works with the word “demonstrative” or “demonstration” in their title,¹⁴⁴ whereas in the Ternary, if we discount works using Aristotelian logic with its accompanying syllogistics, and which are therefore part of the overlap between that and the post-Art phase, it appears in none. In works using such logic the word reappears in only one title, that of the important *Liber de demonstratione per aequiparantiam*, and then again in three of the post-Art phase.¹⁴⁵ Moreover, in the *AD*, the three kinds of proof are treated, together with the Alphabet of the Art, at the very beginning of the work, and are thus part of the foundations on which the entire fabric of the work is built. In the *AGU*, on the other hand, the subject of demonstration is not broached until Part VII, “On the Multiplication of the Fourth Figure”, in which Llull shows how the ternary mechanisms of that Figure are applied to logic, and specifically to the syllogism, with its major premise, minor premise and middle (term).¹⁴⁶ In the third section of this chapter he takes up the three kinds of demonstration, *propter quid*, *quia*, and *per aequiparantiam*, but instead of giving examples vaguely similar to those of Stoic logic, he now begins his arguments with the classic syllogism of “All animals are substance, but all men are animals, therefore all men are substance”.¹⁴⁷

Equally symptomatic of the new climate of the ternary Art is the fact that in the questions at the end of the work, instead of demonstrations,

¹⁴⁴ *Llibre de demostracions*, *Ars demonstrativa*, *Liber propositionum secundum Artem demonstrativam compilatus*, *Introductoria Artis demonstrativae*, *Liber de quaestionibus per quem modus Artis demonstrativae patefit*, *Lectura super figuras Artis demonstrativae*, *Regles introductòries a la pràctica de l'Art demostrativa*, *Liber exponens figuram elementalem Artis demonstrativae*, *Compendium seu commentum Artis demonstrativae*, *Quaestiones per Artem demonstrativam seu inventivam solubiles*. Even though these titles (all except the first) use the word in their title because they are commentaries on the *AD*, they collectively clearly bespeak—especially in view of the alarm over the word expressed in the note in the Venice manuscript—a strong orientation in this phase of the Art.

¹⁴⁵ *Liber de conversione syllogismi opinativi in demonstrativum cum vicesima fallacia*, *Liber de novo modo demonstrandi*, and *De ostensione per quam fides catholica est probabilis atque demonstrabilis*. For the *Liber de demonstratione per aequiparantiam* see p. 212 below.

¹⁴⁶ P. 154 above. The word “demonstration” does not even appear in the *AB*.

¹⁴⁷ *ROL XIV*, 107. Compare p. 65 above for the *AD* with p. 211 below for the *LN*.

as was pointed out before, what we find are directions as to where to locate them in the previous body of the *AGU*, which means that they can be spread about among different parts of the work. For instance, looking at Part XI of Questions for a proof of God's existence, we will find none asked directly, but instead two related questions: "Whether God is a necessary being?" and "How is God's existence proved?", which are answered with instructions as to where to find them. For the first he says "Solution: Go to the first paragraph of the Rules [applied to] the first Subject", and for the second, "The solution is in the same paragraph and in others" (*ROL XIV*, 423). We therefore turn back to Part IX, under the subject of God treated by Rule B, where indeed the question is asked directly:

It is asked whether God exists. To prove God's existence we will use the method of the Art, that is mixing Principles with Rules to investigate and prove God's existence by making Eternity the subject and applying it to all the Principles.

To do so he must carry out a prior proof:

Eternity exists, for if it did not exist, the world would give rise to itself and thus it would exist before existing, which is impossible, for no being can give rise to itself. And thus eternity exists, and consequently, so does its concrete, namely eternal being, as no essence can exist without its concrete being, and vice versa. (*Ibid.*, 206)

It should be remarked that this point was already made at the end of the section of Part V on the eternity of the world answered by twenty compartments from the Table, where he says:

We have proved that the world is new, and this proof necessarily proves the existence of God, since the world cannot create itself or bring itself into being from non being; for if it could do so, it would have existed even before existing, which is a contradiction. We have therefore found, with his grace and blessing, that God exists. (*Ibid.*, 75)

To get back to our proof from Part IX, having taken care of that point, Lull can now "mix" eternity with the eighteen Principles, among which we can take two examples, the first based on "goodness":

Infinite duration exists, and its infinity is good; hence it follows that goodness is eternally a reason for doing good from eternity and in eternity by means of infinite and good duration. And this being is infinitely good, whose goodness is the reason for it to do good infinitely and eternally. And this is the being whom we call God; therefore it is obvious from the

above that God exists. And the human intellect following the mode of this general Art cannot deny this fact. (*ibid.*, 206)

Our second example concerns the Principle of “middle” (from Figure T):

Eternity (*aeternitas*) exists, and since it is an absolutely infinite essence, it then follows that in it there is *aeternare*, that is the middle between *aeternans* and *aeternatum*; otherwise, eternity would be infinite in existence but finite in action, by reason of which, its absolute infinity would be destroyed, which is impossible. And this eternity in which there is coessential *aeternare* is what we call God. (*ibid.*, 208)

both of which explain the equation between God and eternity. Llull can thus end this section saying:

We have proved God’s existence, and solved the question that asks whether God exists. And as we proved it with the Principles and Rule B, we can learn about his existence and intrinsic activity (*agentia*), with the help of his grace. However, I say this with respect to apprehension not to comprehension, because God is infinite and our intellect is finite. (*Ibid.*, 209)

So, instead of the self-contained proof from the *AD* analyzed above on p. 80ff., we have a chain of similar demonstrations (like the “multiplication” of the proof about the eternity of the world), with the same point even made in different places in the work.¹⁴⁸ And instead of the comparisons of concepts of the quaternary Art, we have the mixing of Principles and Rules (in this case the Principle of “eternity” mixed with Rule B and the other seventeen Principles, including in the first or our two examples the definition of “goodness”).

The fact that this demonstration is found under the Nine Subjects and not at the questions at the end of the work, and there under Rule B of “Whether”, finds a curious but significant confirmation in another work where it is connected with what we said just before about the avoidance of the word “demonstration” in the ternary Art. In the *TG* the word is only used once in the entire work, and this precisely under the rule of Whether.¹⁴⁹ If we look at the presentation of that

¹⁴⁸ One should also keep in mind the definition quoted in the text preceding n. 96 above, which says that “God is the being for whom it is impossible not to be”.

¹⁴⁹ *ORL XVI*, 339; *ROL XXVII*, 54. The verb “to demonstrate” is used three times in the Latin text, and then only in a more general, non-technical sense. The first time it is present in both versions (*ORL* 297; *ROL* 1), but then of the two places on *ROL* 52 of the Latin text, the Catalan of *ORL* 337 in one place has *mostrar*, and in the other simply omits it.

rule on p. 138 above, we will see that its species are doubt, affirmation, and negation—those three former components of Figure T—which necessarily form the beginning and end of any demonstration, in that to ask the question is to present a doubt concerning a problem, one which can only be solved by giving some sort of proof for its negative or positive solution.¹⁵⁰ This is also why, as we pointed out on p. 139 above, he feels free to omit this Rule or Question in works where no demonstration is involved.

This in turn ties in with the equivalence between a proof and the answering of a question. As a modern text on mechanical theorem proving puts it, “The set of facts necessary for question answering (or problem solving) can be viewed as axioms of a theorem, and the question (or the problem) can be viewed as the conclusion of the theorem”. It then goes on to differentiate various classes of questions to which such an “answering system” could be applied, of which the first two are “the kind of question that requires a ‘yes’ or ‘no’ answer,” and “the kind that requires a ‘where is,’ ‘who is,’ or ‘under what condition’ as an answer.”¹⁵¹ Now with Llull’s ternary Art the “set of facts necessary for question answering” are basically the definitions of the Principles. Moreover, since definitions are prior to demonstration, they need none. Or, to put it another way, they are on a level with postulates and axioms, as things the reader (or the opponent in an argument) must accept as true.¹⁵² This is what permits Llull with the ternary Art to reverse, in many cases, his manner of proceeding with respect to the quaternary phase. Instead of testing hypotheses by mean of comparisons among the components of the Art, he can now build a structure *on* these definitions, along with the ten Questions and Rules, in a progression that now has more in common with an axiomatic system.¹⁵³

To all this one must add the fact that the ternary Art is geared not only to answering the question of “Whether” (which, as Chang and

¹⁵⁰ See the text corresponding to n. 44 above in this chapter, as well as that corresponding to Ch. 2, n. 114.

¹⁵¹ Chang and Lee 1973, 234–5, citing Green and Raphael 1968. See also Hintikka 1985; his characterization (p. 9) of scientific investigation as “the logic of information-seeking by questioning” is not far from Llull’s method in the questions at the end of all his works of the Art as an art of investigating or finding (*ars inveniendi*) the solutions to all manner of problems.

¹⁵² Notice that in Euclid the definitions come *before* the postulates and axioms; they are what permit the whole mechanism to be set in motion.

¹⁵³ More will be said about this in Ch. 6, where I will try to compare the different proof methods Llull used.

Lee phrase it in the above quotation, requires a “yes” or “no” answer, that is the above-mentioned affirmation or negation), but also the other nine questions (corresponding to the other category of questions mentioned by those authors), and it was thus conceived as a much more general investigative tool. This is particularly evident in the *Art of Proposing and Solving Questions*, analyzed above, where each of its ten subjects (all theological) is answered by the Ten Questions and Rules. Thus, for example, under the heading of Incarnation, Llull not only asks “Whether God was incarnated?”, but also “What is God’s Incarnation?”, “Of what is God’s Incarnation?”, etc., with each question given twenty answers based on a specific column of the Table. So while it is only in such applications of the Art that he feels free to prove the Articles of Faith, he can there apply the same general treatment as in more central presentations of the Art, offering exhaustive explanations of their nature and meaning.

Combining this information with things pointed out before, we can see that the Questions and Rules are in fact a generalization of many of the strategies of Llull’s Art; not only under the first Question do we find his demonstrations, but under the second Question his definitions and his correlatives (under its first and second species respectively), and under the fifth Question his doctrine of proper and appropriated qualities. The Art has thus incorporated a broad panoply of demonstrative, descriptive, and explanatory techniques, to give a much wider, more structured view of the subject being investigated.¹⁵⁴ We could therefore follow Chang and Lee and consider the ternary Art as one geared more generally to question-answering or problem-solving, one in which the first Question of “Whether” which expects a “yes” or “no” answer is only marginally different from one which expects an answer explaining “What” or “Why”.

To sum up: in the ternary Art, demonstration has become a byproduct of a more general method, or rather a way of answering just one of many types of questions. In the process, any more overt use of demonstration became assigned to the field of logic, which is what we will study in the next chapter.

¹⁵⁴ This, for instance, is the technique used in the *Liber de anima rationali* and in large parts of the *Liber de homine*. Llull also uses the Principles and Questions as investigative tools in works from such widely different fields as his *Tractatus novus de astronomia* (in Part I, 2, 2–3) and his *Rhetorica nova* (Part III).

One wonders how much the very reduced role of demonstration in the ternary Art, or at least in the first fifteen years of the that period,¹⁵⁵ might be related to the *cri de coeur* at the beginning of the manuscript Llull sent to the doge of Venice, in which he complains rather bitterly about the scandal caused (undoubtedly in Parisian university circles) by the title of *Ars demonstrativa*, and the mistaken assumption that it was a work which set out to prove “the Articles of Faith and the Sacraments of the Church, and other things which exceed the capacity of the human intellect” by means of the Aristotelian demonstrations of *propter quid* and *quia*. It is for precisely this reason, he says, that he never pretends to deal with such matters with anything more than “persuasive reasons”.¹⁵⁶ That last bit is probably a tactical move, but the temporary change in emphasis could be related to a feeling of having his fingers burnt by repeating that theologically (i.e., politically) incorrect word in so many titles.

It might also be that, at a certain point he decided to separate the more general argumentative strategies of the Art from the purely logical techniques he would now put in motion, and, as we will see in the next chapter, precisely (and surprisingly) to prove the Articles of the Faith.

¹⁵⁵ That is, from *AIV* of 1290 to the *Liber de demonstratione per aequiparantiam* of 1305.

¹⁵⁶ See Soler 1994, 50–51, where (in n. 4) he quotes Ruiz Simon as saying that “The text confirms the hypothesis according to which for his contemporaries . . . *ars demonstrativa* was synonymous with the scientific model described by Aristotle in his *Posterior analytics*, where he speaks of *propter quid* and *quia* as the two species of scientific demonstration.” (cf. Ruiz Simon 1993, 97) Indeed, this work of Aristotle was known to the Arabs as *The Book of Demonstration* (*Kitāb al-Burhān*), Gerard of Cremona’s translation was called *De demonstratione* (Dronke 1988, 156), and Boethius of Dacia’s lost commentary, *Ars demonstrativa* (Boethius of Dacia, 1976, xxxiv and 129). Llull’s complaints could also be related to that in the *Vita coetanea* about the “the attitude of the students” and “the weakness of human intellect he had witnessed in Paris”; see *SW I*, 29; *DI*, 23–24.

CHAPTER FIVE

THE POST-ART PHASE: LOGIC

Preliminaries

Before setting out on this new phase of Llull's production, we must explain a few things about the changes involved. In the first place, when we speak of 'logic', we don't mean to imply that previous phases had nothing to do with logic, or that the Art itself is not a logical structure. We mean that now we find Llull entering the world of classical Aristotelian logic, and basing his arguments on the categorical syllogism. This implied a major change in Llull's approach, going from a combinatorial and generative system of his own invention to one grounded much more directly on the standard academic formulations of his time.

Even from a purely logical point of view, the change was a major one. As we have seen with the *AD* and even with the Table of the *AGU*, he starts with a hypothesis (such as an Article of Faith) and shows how its negation leads to a contradiction among the various 'universals' of the Art.¹ One only has to compare this with the syllogisms presented below to see how different is the new technique, in which Llull follows standard practice in arguing the truth of his premises to establish that of the conclusion, and in which he defends the *nature* of his premises in purely Aristotelian terms. As we will explain in the last chapter, this involved going from an 'upside-down' system (starting with the thing to be proved) to a 'right-side-up' one (arriving at the thing to be proved).

As for the label 'post-Art', it should not be taken to mean a total abandonment of components of the Art, but rather that Llull, with rare exceptions, no longer uses the Art as a totalizing structure, one consisting of Figures, Principles, the Table, Definitions, Rules and Questions, the Nine Subjects, the Hundred Forms—interconnected

¹ Which means that if he did, in the course of 'Artistic' demonstrations, use abbreviated syllogisms in the form of enthymemes, they were hypothetical and not categorical. Nor did he ever (discounting two pages in the early *Compendium logicae Algazelis*) before the *LN* indulge in any theoretical discussion of the syllogism.

by various combinatory devices—and above all in the *AGU* and *AB* by that of ‘mixture’—in order to produce demonstrative mechanisms for answering the very general questions at the end of a work. With the possible exception of the Nine Subjects and the Hundred Forms, which are more ways of channelling the applications of his system, all the other components are *absolutely essential* for the kind of demonstrative mechanisms used in the Art. On the other hand, since the Art is the sum of its parts, any one of which is necessary but not sufficient for its functioning, this permits him to use individual components of the Art in other works in which he proposes demonstrative techniques of a completely different nature.

The most glaring example comes with the Dignities/Principles which are present in almost every work Lull wrote. He uses, for instance, the eighteen Principles as well as the Rules and Questions in both the *LN* and the *Rhetorica nova*, but this does not make them works of the Art; they are works in which he has used components of the Art to rework medieval logic and rhetoric.² If he says at the end of a post-Art work such as the *Liber de syllogismis contradictoriis* that “This book or art is very useful and is derived from the *Ars generalis*”, he is referring to the use of ten dignities (the nine usual ones plus Perfection), but the way he uses them in arguments such as

Every optimal and eternal being exists and acts optimally and eternally; God is an optimal and eternal being; therefore God exists and acts optimally and eternally. As a consequence of this syllogism, it follows that God intrinsically acts optimally and eternally, and thus is Trinity. From this it also follows that . . . (*ROL VII*, 194)

where superlatives are used as premises of a syllogism—all things we will be discussing in this chapter—bears little resemblance to anything we have witnessed in previous chapters.

As a by-product of the reuse of the Dignities/Principles as an isolated component, separate from the other mechanisms of the Art, Lull can now vary their number. To the usual nine of Figure A—or sometimes all eighteen from the first two figures—he now very frequently adds Unity and Perfection, and occasionally a whole list of others, such as Operation, Infinity, Simplicity, etc.

² If he refers to the *LN* and *Rhetorica nova* as “arts”, it is in the medieval sense of logic and rhetoric being two of the arts of the trivium.

The new orientation also caused an important change in the form of Llull's works. The reader is no longer asked to absorb all those complicated mechanisms of the Art, which take up a good half of a work, before he can tackle the demonstrations presented in the last half; instead he can now use the Aristotelian techniques, which, in the Middle Ages, formed the basis of his training, to absorb works that, after the briefest of prologues, enter almost at once into the principal arguments proposed.

As for the chronological division between the two stages, it is not a clean break but rather a gradual change-over,³ with a five year overlap between his first introduction of logical techniques with the *LN* in 1303 until the last work of the Art, the *AGU* finished in 1308, a period which we will explain in detail below. It was a transitional stage somewhat similar to the one preceding the ternary phase, except that instead of modifications gradually leading up to the next period, here we have two formulations running parallel, starting with an attempt to show how the Art can be used to renovate logic and ending with the incorporation of logic *into* the Art. After 1308, moreover, his abandonment of techniques (as opposed to components) of the Art is gradual. In the *Liber de experientia realitatis Artis ipsius generalis*, for example, written less than eight months after finishing the *AGU*, we find an almost intact version of the Art, including the Alphabet and the four figures. In the *Liber de divina unitate et pluralitate* of 1311 we find combinatorial arguments very like those of the Art. Later on he even writes two new applications of the Art, the *Ars abbreviata praedicandi* of 1313 and the *Ars consilii* of 1315. For the most part, however, it is a matter of his abandonment of the structure the Art, as well as a reorientation of his argumentation in the direction of a much more openly Aristotelian logic.

The Transition from the Art to Logic

We have seen at the end of the last chapter how Llull, in central works of the ternary Art, played down the idea of demonstration, leaving it as only one of the many functions of an Art now presented as much

³ Ruiz Simon, 1999, 274, speaks of the progressive introduction of the categorical syllogism into Llull's arguments, this at the beginning of a section discussing much of the material of this chapter, although from a different point of view.

more general. Since for Llull the primary task of demonstration was to prove the truth of the two doctrines most distinctively Christian and most controversial for Muslims and Jews—those of the Trinity and Incarnation—, this separation was an almost natural extension of his desire to remove from the Art any overt expression of its religious aims. As we have also seen, it was in collateral works applying the Art that he carried out such demonstrations, of which the principal ones prior to the change of tactics discussed in this chapter, are the *Art of Proposing and Solving Questions* of 1294–5, the *Apostrophe* of 1296, and the *Dictat de Ramon* (*Tractatus compendiosus de articulis fidei catholicae* in Latin) of 1299.⁴ And these works—with one exception—use arguments either structured on the Art or with no particular structure. The exception is interesting: in the contemporary (free) Latin translation of the *Apostrophe* we find a timid and unprogrammatically use of the syllogism,⁵ but that is, as far as I have been able to determine, its only appearance before the *LN* (finished in May of 1303), in which Llull starts trying to make his arguments conform to the stricter contemporary idea of what constituted a demonstrative science, or, as was said at the end of the last chapter, to turn his attention to the theory of scientific demonstration. This meant adapting his arguments to the criteria of Aristotle's *Organon*,⁶ but in such a way that might circumvent the strictures of the theologians of Paris while at the same time persuading them of the validity of his aims.

⁴ He also offers proofs in a section of the poem, *Medicina de Pecat* of 1300 (*ORL* XX, 74–111).

⁵ The text(s) of this work have a complicated history. In the original Catalan version, Llull says (*NEORL* III, 70) that “I, unworthy Ramon, have written this book and I have had it put into Latin, not literally (word for word = *letra a letra*) but freely (meaning to meaning = *sen a sen*).” It was this translation which was to be presented to Pope Boniface VIII, and, as the wording of the passage quoted would seem to indicate, Llull had somebody else do the job, perhaps in the process ‘improving’ his simple Latin to make it more elegant and persuasive for the imperious Roman nobleman who had succeeded Celestine V. That the translator—probably working with Llull—adapted the form of the arguments to contemporary theological discourse could explain the recasting in syllogistic form of some of the looser arguments of the original Catalan. For the history of the text see the Llull DB under Works, catalog number III.24, with the Notes at the end of the page.

⁶ The *LN* covers most of the subjects which constituted the study of the *Organon* in the Middle Ages, including not only the *Categories*, *Prior Analytics*, *Posterior Analytics*, *Topics*, and *Sophistical Refutations*, but also Porphyry's *Isagoge*. The only one which Llull hardly mentions is *On Interpretation*, which has to do with matters of language.

His manner of setting out on the path of Aristotelian logic, however, is surprising for the momentum it gathers after a slow start and for its almost total domination in the post-Art phase. After a translation of al-Ghazali's logic written, as one critic has said, almost as a school exercise at the very beginning of his career,⁷ the subject only receives a few desultory mentions in his works until the *Aplicació de l'Art general* of 1301, one section of which he devotes to showing how the Art can be used as a foundation for logic.⁸ This brief (five pages) and very general presentation, is fleshed out in the *LN* of 1303, his first mature incursion into the subject. Along with the many other novelties which contribute to make this logic *nova*, was the appearance of the syllogism. After the first thirty years of writing, in which he seems to have avoided it like the plague,⁹ suddenly we find the central portion of the *LN* devoted to the subject. Not only is the syllogism explained in terms of the Ten Questions,¹⁰ but it is itself used to explain many other subjects such as Lull's three types of demonstrations, the loci of dialectics, and the fallacies, including a new fallacy of apparent contradiction which Lull introduces and which will assume greater importance later on.¹¹

⁷ Ed. Lohr 1967, where the debts to al-Ghazali and Peter of Spain are carefully explained. Johnston 1987 also has a good chapter on the work. The phrase is from Cruz Hernández 1977, 67. For an earlier overview of some of the developments explained in this chapter, see Bonner 1995b.

⁸ Part III, *ORL* XX, 225–9. It is divided into six distinctions treating: (1) how the five universals (i.e., the predicables) can be studied with the Principles of the Art; (2) how the same can be done with the ten predicaments (the categories); (3) how the universals and predicaments can be defined according to the techniques used to define the Principles; (4) how to mix logical conditions with those of the Art; (5) how to use the Table in logic; (6) how to use techniques of the Art to answer questions of logic. We are still far from any of the mechanisms of the *Prior Analytics*. Mention should be made of his interesting use of consequences in his *Principia philosophiae* of 1299–1300 (*ROL* XIX, in which see pp. 46–56 of the introduction; in the Catalan edition in *NEORL* VI, xxviii–xxx). Except for a certain use in the *Dictat de Ramon* written right afterwards, and possibly in the *De quadratura et triangulatura circuli* written just before, it is a technique which he seemed not to have explicitly used again. For a brief mention in an earlier work, see the *Compendium seu commentum Artis demonstrativae* (*MOG* III, vi, 156: 446), cited in Ruiz Simon 1999, 184–5.

⁹ With the exception of the use explained in n. 5 above, where he might have been helped by someone at this point more knowledgeable. His avoidance was undoubtedly due to the little use he could find for a technique more concerned with formal consistency or validity than truth, as explained in n. 24 below.

¹⁰ *ROL* XXIII, 108–111; *NEORL* IV, 91–92.

¹¹ For the new presentation of the three types of demonstration in the *LN*, see p. 211 below. For the fallacy of apparent contradiction in the *LN*, see *ROL* XXIII, 128–130, and *NEORL* IV, 111–2.

This sudden and insistent introduction of the syllogism is followed in Llull's production by its actual use in four works written shortly after the *LN*, along with one written a few years later:

Disputatio fidei et intellectus (October, 1303; *ROL* XXIII, 224–279)

Liber ad probandum aliquos articulos fidei catholicae per syllogisticas rationes (February, 1304; *ROL* XX, 445–485)

Liber de demonstratione per aequiparantiam (March, 1305; *ROL* IX, 216–231)

Liber de Trinitate et Incarnatione (September, 1305; *ROL* XII, 90–137)

Disputatio Raimundi christiani et Homeri saraceni (April, 1308; *ROL* XXII, 172–264)

This last year of 1308 is pivotal for the relations between the Art and logic. Aside from finishing the *AGU* and the *AB*, it was then that Llull began branching out into other logical fields beside that of the syllogism. In the *AGU* the role of the 'middle' suddenly assumes primary importance,¹² and in the *Ars compendiosa Dei*, finished in May of that same year, he not only uses this new-found middle, but also introduces the technique of contradictory syllogisms and the use of the superlative degree. With the initial work of the post-Art phase, the *Liber de novis fallaciis* finished in October, 1308, we get the first full presentation of the fallacy of apparent contradiction,¹³ as well as of his system of *demonstratio per hypothesim*. This leads him naturally into the idea of contradictory suppositions, which appears in the *Liber de modo naturali intelligendi* of two years later (October, 1310). These are all techniques that will be discussed in the course of this chapter,¹⁴ techniques which form the basis of most of the post-Art phase in the stricter sense, in which Llull produced some 115 works, almost half of his entire production.¹⁵

Here we find Llull once again taking up proofs of the Trinity, the Incarnation—and in some of them also the Creation and the

¹² Between writing the *AB* and finishing the *AGU*, he treated the question of the middle extensively in the *Liber de venatione substantiae, accidentis et compositi* (February, 1308; *ROL* XXII, 83–91).

¹³ It had appeared briefly in the *LN*, as explained in n. 11 above, and in the *AGU* (*ROL* XIV, 116–7).

¹⁴ Most of these topics were discussed in Johnston 1987, but more from the point of view of *logica theorica* than *logica utens*.

¹⁵ Since many of these 115 are short, the comparison is not quite fair; but the volume of output during this period still outweighs the attention it has received from modern scholarship. The most notable exception to this use of logical techniques is the spate of almost 300 sermons written in Majorca in 1311–1312.

Resurrection¹⁶—but now insistently using syllogisms instead of the purely Artistic arguments of former works. Using Aristotelian techniques, however, to demonstrate central Articles of faith would seem to be courting the very criticisms about which he expressed such anguish in the manuscript sent to Venice, including, as we have just seen, the reappearance of the word *demonstratio* in one of his titles!

As to possible reasons for this tactical reorientation, it seems clear that either by his own observations of the Parisian academic scene or on the advice of friends immersed in it, he realized that, as effective as the Art might be, its potential audience of scholastic theologians would never consider its methods as constituting real proofs. Or as Charles Lohr put it, “Llull’s encounter with the position of the theologians of the university of Paris . . . forced him to turn his attention to the theory of scientific demonstration”.¹⁷ This meant trying to make his arguments conform to the existing canon, whose cornerstone was Aristotle’s statement in the *Posterior Analytics*: “By demonstration I mean a syllogism which produces scientific knowledge.”¹⁸ Theologians of the time—particularly the Dominicans—who were interested in showing that their subject was a science in the Aristotelian sense, adapted their discourse to this canon. The *Summa theologica* of Aquinas, for instance, is a chain of syllogisms, on which are hung arguments proving or disproving the premises.

Any attempt on Llull’s part to adapt his system to the basic form of Aristotelian logic, however, entailed two problems that were more substantial than merely formal. The first was that, according to the same *Posterior Analytics*, such a demonstration involved the knowledge of causes, either directly, as in the *demonstratio propter quid*, or by their effect, as in the *demonstratio per quia*. The second was the finding of the explanatory middle term—or better yet, just middle¹⁹—, which explanation according to Aristotle also had to be causal. As we already know, ever since the *AD* Llull had been at pains to explain that causal demonstrations cannot, for the most part, apply to the Divinity, which was why he had

¹⁶ The second of the five works listed above also proves the existence of God, whereas the third work proves only the Trinity.

¹⁷ *ROL* XI, 3. See also Lohr 2004.

¹⁸ 71b18–19, trans. Hugh Tredennick, *Loeb Classical Library*. For the medieval attempt to adopt demonstrative discourse to the Aristotelian canon, see Kretzmann et al. 1982, 496ff.

¹⁹ See n. 82 below.

developed his new one *per aequiparantiam*. What he will do now is, as it were, syllogize this demonstrative procedure, and reformulate a new kind of middle. But first we must consider some general matters, and see how Lull approaches predication.

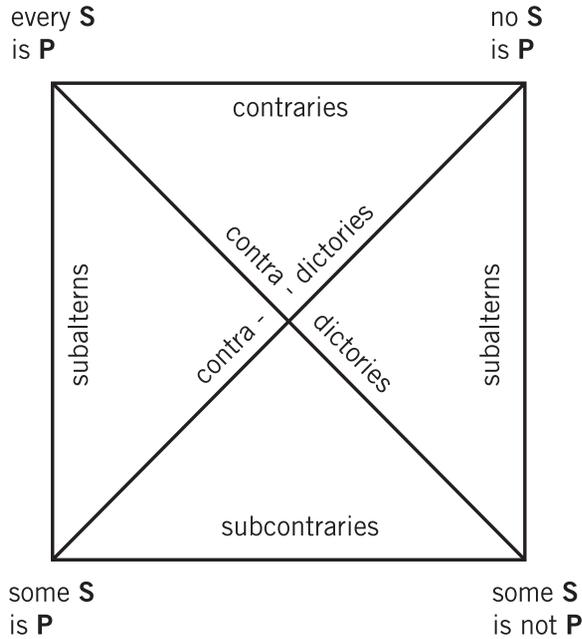
General Character of Lull's Logic

For decades now scholars have accepted that Lull's logic is not a formal logic, but rather one that tries to recycle techniques of classical logic to make them applicable to the structures of the Platonically real world. As Vittorio Hösle put it, "Lull tries to combine logic and ontology, that is to build the former on the latter—a program which naturally harmonizes with Lull's concept of the unity of being and thought."²⁰ What is involved in this change from a formal to an ontological logic, however, can still be surprising and even grating with respect to the reader's expectations of what logic 'should do'. We will therefore pick several examples where understanding this change might help reorient the reader.

The first has to do with the fact that formal logic—as the name implies—has to do with the form of propositions, and very little to do with their content. It is centered on the secondary modifying vocabulary—"all", "some", "not", "if", etc., what the medievals called the *syncategoremata*,—and so little on the chief bearers of meaning such as nouns and verbs—the *categoremata*—that most of the system can be explained using mere letter symbols or place holders for the latter, something which began with Aristotle and continues very much into modern logic. This permits most texts on classical logic to present very near the beginning a square of opposition in which the relations between the *syncatoremata* are thus schematized:²¹

²⁰ Lullus 1985, lxiv. This whole introduction is particularly good on the metaphysical foundations of Lull's logic.

²¹ The one here is adapted from Peter of Spain, where instead of S and P he has "man" and "animal", but they are merely examples functioning as place holders. Aristotle uses letters in the *Prior Analytics*.



Now Llull, who, as Charles Lohr has shown, was perfectly conversant with the logical tradition represented by Peter of Spain²² and must therefore have come across this figure countless times in his readings, never uses it in any of his logical works or elsewhere. What he uses instead—a fact which perhaps understandably has surprised and misled scholars—is his elemental theory which deals with the interaction of the qualities of the four elements. Now the curious thing is that in the ternary phase, not counting works dealing directly with natural science (two medical works, one on astronomy,²³ and the *Tree of Science*), the work in which elemental theory plays perhaps the greatest role is the *LN*.

It is used in this work not because of any physical resemblance between the square of opposition and the square of elements given above on p. 59; for one must not forget that the latter is a modern representation of Llull's theory—he himself never presents it that way.

²² See Lohr 1967.

²³ *De levitate et ponderositate elementorum*, the *Liber de regionibus sanitatis et infirmitatis*, and the *Tractatus novus de astronomia*.

Secondly, it is not a question of one replacing the other. The square of opposition shows the relationship between different extensions of the subjects of propositions, relationships fundamental to Aristotle's theory of the syllogism, and hence is the foundation-stone of his logic. Llull's elemental theory, on the other hand is not fundamental to his logic; it is rather used to offer analogical explanations of certain relations, explanations which, being based in ontology, are for him stronger than the purely formal ones of classical logic.²⁴ Two examples from the *LN* will bring out this point.

In the first example he uses it to prove that, as opposed to genus, species, and difference, accident is not a real universal. He starts by asking:

Whether every accident comes from a single real and general accident?
We say it does not, which we show by the following example.

Every animal is a substance; every man is an animal; therefore no man is a substance. Now just as this last proposition does not participate in any element of truth with the above two premises, in the same way truth and falsehood, good and evil, heat and cold, and similarly of other accidents contrary to one another without any intermediary, cannot, because of their number, derive from one general, real and simple accident. For if they could, every man is substance and no man is substance could derive from the same premises, which is not possible and is a contradiction. (*ROL* XXIII, 52–53; *NEORL* IV, 35–36)

Here Llull has used contrary qualities as analogies, saying that if they had, as it were, a common ancestor, then so would the contrariness of affirmation and negation, placing us in a world where the latter could also find being and hence justification. In general Llull is much more interested in the ontological opposition of contrariety (from the green triangle of Figure T), than the logical one of contradiction (except as a determinant in his proofs by *reductio ad absurdum*).²⁵

²⁴ A formal argument can only guarantee validity, but it does not necessarily give any information, and it can easily be absurd and arrive at a false conclusion. For instance, "All G are E; some P are G; therefore some P are E" is formally correct, i.e., produces a valid argument (a valid syllogism in Darii). Yet if we substitute G = goats, E = enjoy eating clothes off the line, and P = piano-tuners, we arrive at the conclusion that "some piano-tuners enjoy eating clothes off the line".

²⁵ See the passage referred to in Ch. 4, n. 77 above for an example of how Llull uses it, as well as the interesting explanation in the *AIV* (*MOG* V, 8) also using elemental theory to explain "contrariety". Aristotle, *Categories*, 11b17–22, gives four kinds of opposites: relatives (double and half), contraries (good and bad), privation and possession (blindness and sight), and affirmation and negation (he is sitting—he is not sitting). For

His main interest in elemental theory in the *LN*, however, derives from its explanations of proper and appropriated qualities. Under Question G “Which, of what kind?” he explains:

This rule or question is of quality, and it has two species, proper quality and appropriated quality, as in man his natural goodness is his proper quality, and similarly for his risibility; but if he is just, his justice is a good habit appropriated by the customs he has adopted. This is like fire, in which its heat is its proper quality, and its dryness is a quality it has appropriated by means of earth. This rule shows how the logician might know how to make necessary demonstrations by means of proper qualities, and to make proofs by means of appropriated qualities. (*ROL* XXIII, 35; *NEORL* IV, 18)²⁶

He also uses it to explain the predicament of “action”:

What is action? Action is the form with which the agent acts in the subject, just as fire acts with heat in the thing heated (*cum caliditate in calefacto*). (XXIII, 69; *NEORL* IV, 51)

In short, there are almost forty passages in the *LN* where Lull offers analogical explanations based on elemental theory.

Another manner in which Lull’s logic is unusual has to do with the fact that most medievals considered logic, along with grammar and rhetoric, as a *sermonicalis scientia*, which could be translated as “a linguistic science” or a “science of speech”.²⁷ As such, it could only formulate valid inferences within the domain of the speech utterance being analyzed; to jump from there to any concept of truth, needs some theory of reference. Medieval logicians tried to formalize, or rather to codify the use of language in predication, with a version of such a theory, which they called the theory of “supposition”. Thus, for example, they argued that the word “man” had a different reference or supposition in the sentences “Man is a monosyllable”, “Man is a

a definition of contrariety as a greatest difference, as a difference between extremes, see *Metaphysics*, 1055a4ff. This kind of contrariety—the one Lull is interested in—should also be distinguished from the logical one present in the square of opposition (the last of Aristotle’s four above), that is between “all S is P” and “no S is P”, which is again distributive or extensional, and which Aristotle refers to as “verbal”. See *Prior Analytics*, 63b23–30.

²⁶ Lull similarly explains the predicament (or category) of “quality” by analogy with elemental theory (*ROL* XXIII, 63–65; *NEORL* IV, 45–48).

²⁷ William of Sherwood 1983, 222.

species” or “Some man is not Greek”.²⁸ Modern logicians have had to make do—if the phrase is not too disrespectful—with Tarski’s famous dictum: “It is raining” is true if it is raining. For Llull, on the other hand, since he is dealing directly with a Platonically real world, there is no need for a system of reference. To put it another way, he is already working in the place to which standard logic must refer.

This we can see in how he uses the concept of signification. For medieval writers the word had its two modern senses, one of the reference between signifier and signified, between word and thing, and the other of how one thing can be a sign of another, such as fever of illness, or a gift of good will.²⁹ The second sense was dominant in medieval works on preaching and rhetoric, whereas logicians were really only interested in the first sense.³⁰ Llull, with the two exceptions discussed below, was only interested in the second sense, of how one thing can signify another. One has only to read his two longest tracts on signification, Distinction 35 (chs. 234–7) of the *Book of Contemplation*, and his *Liber de significatione* written shortly before the *AGU*, to see the total absence of any mention of linguistic reference, and, on the contrary, to see how, for instance, goodness signifies good, which in turn signifies the doing of good, etc., on through the relations of signification between the various entities of the Platonically real world the Art is studying.³¹ So important for Llull are the mechanisms by which the various portions of the divine and created worlds signify one another, that common synonyms for “signification” are “manifestation”, “revelation” and even “demonstration”.

One of the only two headings under which Llull discusses words as linguistic items as such is that of rhetoric, but even there he does it in a way that has been much commented upon precisely because of how

²⁸ See the excellent résumé in Paul Vincent Spade, “The semantics of terms” in Kretzmann et al. 1982, 188–196, at the end of which he gives a list of eight possible varieties of supposition.

²⁹ The medievals distinguished these two senses by saying that words could signify arbitrarily (*ad placitum*) and naturally. See Peter of Spain 1972, 2; William of Sherwood 1983, 223.

³⁰ It is noteworthy that Peter of Spain only discusses the *ad placitum* sense: “The signification of a term, as it will be used here, is of the arbitrary representation of a thing by a word.” Peter of Spain 1972 79.

³¹ *ROL* X, 16ff. For the use in the *Book of Contemplation*, see Gayà 1995. See also Vida i Roca 1990.

little it has to do with the usual approaches towards language.³² In the *AGU*, under Form 86 of Rhetoric, he says:

The rhetorician ornaments [his speech] with the *vox significativa*, as when he says “April” and “May”, which are more beautiful words than when one says “October” and “November”, because they signify flowers and leaves, and the song of birds, and seasonal renewal and regeneration, whereas this is not true of “October” and “November”. (*ROL XIV*, 364)

In the *Rhetorica nova* he explains how “queen” is a more beautiful word than “servant girl”.³³ In short, the beauty of a word is judged solely by the place which its referent occupies in the moral, social, spiritual or aesthetic order of the world. In Lull we find nothing about how the *vox* is *significativa*, only about what it signifies.

The other exception is more important for us since it comes under the heading of logic, in Lull’s treatment of the fallacy of apparent contradiction.³⁴ But, as we will explain on p. 239, once again the relationship is not between the sign and the referent, but now about possible confusions as to what it signifies. Here Lull is interested primarily in the truth or falsehood of propositions depending on whether they refer to the divine or created worlds. As with rhetoric, Lull is not dealing with how the sign functions, only with problems concerning its possible referents.

Predication

On pp. 125–6 above we quoted the description of the First Figure from the *AB*:

The First Figure is that of A, and it contains nine principles, to wit, goodness, greatness, etc., and nine letters, to wit, B, C, D, E, etc. This figure is circular to show that any subject can become a predicate, and vice versa, as when one says, “goodness is great”, “greatness is good”, and so on.

The equivalent description from the *AGU* says that:

³² Rubió i Balaguer 1985, 218, 220; Badia 1992, 79ff.

³³ Part I, 1.2.

³⁴ He also discusses linguistic problems under the first group of fallacies, those *intra dictionem*, but he attaches little importance to them, except for that of *ignorantia elenchi*, from which he derived his fallacy of apparent contradiction; see Ruiz Simon 1999, 170ff.

And [this figure] is referred to as circular because the subject can be changed into the predicate, and vice versa, as when one says: great goodness, good greatness, eternal greatness, great eternity, etc. (*ROL* XIV, 10)

It may also be remembered that he said the Evacuation of the Third Figure could be carried out by a series of propositions: “Goodness is great”, “Greatness is good”, “Goodness is different”, “Difference is good”, etc.³⁵

As I also said above, this use of predication is a distinct novelty in the presentation of the Art.³⁶ Lull’s use of it, however, will arouse understandable objections for anyone conversant with logic, because within the space of these brief introductory remarks he has already broken several of its fundamental rules. In the first place, he is treating propositions (“goodness is great”) and descriptive expressions (“great goodness”) as if they were more or less identical formulations of the relation between subject and predicate.³⁷ The second formulation is different not only because it asserts nothing that can be declared true or false, but also because normally a predicate has to be attached to a subject by some sort of verb form to make a complete sentence (“Peter walks” or “Peter is good”).

In the second place, when he does use the standard verbal form of predication, Lull does it without quantification (i.e., not phrasing it “all goodness is great”, etc.).

In the third place he treats the adjectival (“good”, “great”) and nominal (“goodness”, “greatness”) forms as if there were no essential difference between them, or, to put it more generally, as if there were no essential difference between the form of a subject and that of a predicate. This is because he is dealing with predicates of predicates. In normal logic, statements such as “Peter is good” or “some men are virtuous” belong to what is called first-order logic, in which a predicate (in this case, a quality) modifies a subject (usually consisting of an

³⁵ P. 147 above.

³⁶ Except for a brief mention in the *ACIV* (*MOG* I, vii, 2: 434), predication does not appear in a description of the First Figure or, in general, as we will explain later, in the Art, until the *Lectura Artis quae intitulatur Brevis practica Tabulae generalis* (*ROL* XX, 346), nor in any really systematic way until the time of the *AB* and *AGU*. See the text preceding n. 16 of Ch. 4 above for the technique of *mixtio* which preceded it.

³⁷ The medieval manuals of logic clearly distinguished the *oratio perfecta* (*homo est albus*) from the *oratio imperfecta* (*homo albus*). Cf. Peter of Spain 1972, 3, and William of Sherwood 1983, 224.

individual or a set of individuals).³⁸ When, however, we say “good (or goodness) is a virtue”, with a quality modifying a quality, we get into second- or higher-order logic, which is a more complicated matter, and normally out of bounds for classical logic, and certainly for medieval logic.³⁹ Notice too how we have had to phrase our example of “good (or goodness) is a virtue”, in addition to having changed “virtuous” to “virtue”. This is one of the standard complaints about higher-order logic, that it involves the already mentioned linguistic legerdemain of switching back and forth between the adjectival and nominal forms of a concept.⁴⁰

Lastly, the possibility of a simple conversion between subject and predicate as suggested in the description of Figure A above contravenes a basic rule of standard logic, in which conversion is dependent on quantification. It is, for instance, permissible for “no P is Q” or “some P is Q” (which can convert respectively to “no Q is P” or “some Q is P”), but not for “every P is Q” or “some P is not Q”.⁴¹

I would like to suggest that these anomalies are not due to the innocence of an untutored mind, but instead are the natural consequence of Lull’s attempt to formulate a new kind of predication. To explain this, however, we need to discuss some of the underpinnings of his approach.

First of all, one must also remember that the concepts of the first two figures, when used as foundation stones of the Art, are ‘principles’, but in a more general way they are ‘forms’, along with the many others in

³⁸ I am using the term “first-order logic” in an analogous sense; technically it applies only to modern mathematical logic. The analogy, however, is useful.

³⁹ See n. 97 below for Lull’s use of an almost modern text-book example of second-order logic.

⁴⁰ See Purtil 1971, 206, where he complains that to go from the first order statement “X is courageous” to the second-order one, you would technically have to say, not “courage is a good quality”, but “is courageous is a good quality”, which is not only an ill-formed proposition, but nonsensical.

⁴¹ *Prior Analytics*, 25a14–37. It is interesting to note that one of Leibniz’s reasons for wanting to mathematize logic was that mathematical operations could be commutative (like multiplication, where the order of terms is irrelevant, i.e. $2 \times 3 = 3 \times 2$), whereas in the inclusion relation of classical predication the order of terms could not be interchanged. See Couturat 1901, 324, where in n. 2 he also discusses Leibniz’s observation that the above-mentioned distinction between noun and adjective is not essential. For more on Lull’s use of convertibility, see nn. 49 and 53 below, as well as the convertibility of premises discussed under “The Finding of the Middle”. It should be pointed out that Aristotle also discusses conversion in the *Topics* (102a18–19) between a thing and its property, but this is not a general case between individual terms.

the various versions of the Hundred Forms.⁴² This means that they are substantial, real entities,⁴³ and Llull's interest in them is ontological not linguistic, intensional and not extensional (i.e., having to do with their meaning rather than the set of things to which they refer). This involves a very different kind of predication, one which no longer functions taxonomically, where one concept can be included in another. We are no longer dealing with the traditional axiom of *praedicatum inest subjectum* ("the subject is contained in the predicate"), in which the two can be mapped onto a Venn diagram, with "man", for instance, represented by a smaller circle inside a larger one of "animal", corresponding to the proposition "every man is an animal". As Ruiz Simon points out, Llull's predication "implies the immanence, the real and effective presence of the reality denoted by one concept in that denoted by the other."⁴⁴ This is why he frequently refers to the process as *mixtio* of one concept with another, or the "putting" or "entering" of one into the other, or of "applying" one to another.⁴⁵

Cornford's description of the equivalent process in Plato seems to me apposite. After explaining that Plato does not analyze "Man is rational" into two different things—subject and predicate—united by a copula, he says: "the Form Man and the Form Rational are combined or blended in reality. When two things—say, two colours—are blended, there is no link coupling them together; nor is there any suggestion that the two elements are of different sorts, one a 'subject', the other a 'predicate'. There is nothing but the mixture. [...] Plato's language seems to show that he did not imagine eternal truths as existing in the shape of 'propositions' with a structure answering to the shape of statements. He conceived them as 'mixtures' in which Forms are blended... and we must realize that Dialectic is not Formal Logic, but

⁴² In the *LN* (*ROL* XXIII, 85–90; *NEORL* IV, 68–72) the eighteen Principles of the Art are listed as Forms 2–19. Llull emphasizes the point by beginning many of the definitions of the Hundred Forms in the *AB* with the phrase "X is the form that..." (*SW* I, 616ff.; *DI* 336ff.).

⁴³ Cf. Platzeck 1962–4, I, 258. In the *LN* Llull even argues that the predicables of "genus", "species" and "difference" are real (see *ROL* XXIII, 38, 41, 46; *NEORL* IV, 21, 25, 29).

⁴⁴ Ruiz Simon 1986, 92.

⁴⁵ See again Ch. 4, n. 16 above for the technique of *mixtio*. For the "applying" see the *Lectura Artis quae intitularur Brevis practica Tabulae generalis* (*ROL* XX, 349).

the study of the structure of reality—in fact Ontology, for the Forms are the realities.”⁴⁶

Notice how this fits in with Ruiz Simon’s statements,⁴⁷ as well as with what we explained above about Llull’s earlier descriptions of the combinations of components of the first two figures as ‘mixtures’. It also helps us to understand what seemed like anomalies. In the first place, whether Llull uses the propositional form or the merely descriptive one, depends solely on the argumentative context in which he wants to use his ‘mixture’. In such a context, the difference between the adjective “good” and the corresponding noun “goodness” is likewise an accident of the language in which we express the ‘mixture’. One could even say that problems of higher-order logic disappear in a world of Platonic forms, or, better yet, what disappears (at least when not dealing with quantification) are the differences between first- and second-order logic.⁴⁸ Notice finally that the possible convertibility of the Principles of Figure A is quite unproblematic with the Platonic mixing of forms.⁴⁹

All this does not mean, however, that Llull should be considered a Platonist in any unqualified way; the differences are too great and too obvious. One only has to recall the definitions of the Principles given above, the dynamism—articulated by the correlatives—of Llull’s ontology, as well as his attempt to discover a formal structure to all of reality, as in the *Tree of Science*. What we are saying here is that certain aspects of Plato’s use of the forms (and of Cornford’s interpretation

⁴⁶ Cornford 1935, 266. For the matter of *sumplokē eidōn* (the interweaving of Forms), see also Ackrill 1971, and Allen 1971. Notice how all this is not without relevance to the considerations about Mixture. In general for predication with Platonic forms (or ‘ideas’) see the excellent Brunner 1997.

⁴⁷ For a general analysis of Llull’s Platonism and its possible sources, see Ruiz Simon 2005, 178ff.

⁴⁸ Cocchiarella 1986 is excellent on the modern logical implications of nominalistic, conceptualistic, and realistic predication. See p. 13, as well as pp. 17, 165–6, for the (non-)problem of nominalized predicates (“runs” → “running”, “wise” → “wisdom”). The problem of paronyms (also called denominatives or derivatives) was first discussed in Aristotle (*Categories* 1a12–15) and taken up in the Middle Ages by Anselm in his *De grammatico*, using Aristotle’s example of *grammatica* (grammar) → *grammaticus* (grammarian), but more from the point of view of semantics and what is denominated by these terms than of predication.

⁴⁹ The convertibility of “God is good” involves what we would call a symmetrical relation; the proposition “an angel is good” involves a non-symmetrical (anti-symmetrical or asymmetrical) relation. This is a matter of the hierarchy of forms. The fact that “goodness is great” is symmetrical in God but not in creation, is also a matter of hierarchy, but here of the ontological level in which the predication takes place. For the hierarchy of forms in Plato, see Cornford 1935, 263–4, and Ackrill 1971, 216.

of that use) can perhaps help us understand corresponding aspects of Lull's system.

Finally, there is the matter of quantification. If we are not dealing with extension (and Venn diagrams), quantification, which is the instrument for dealing logically with that extension, is no longer indispensable. As an example we can take Euclid's presentation of the Pythagorean theorem:

In right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides containing the right angle. (Euclid 1956, I, 349)

If he had phrased this in the singular, "in a right-angled triangle", it would have made no difference, because he is referring to a mental construction of the figure. But if he had said "in all right-angled triangles" in a logically extensional sense, it not only would have made a difference, but the statement would in fact be false, since it would not be applicable to right-angled triangles drawn in sand or on paper, many of which would be too imprecise to fulfill the equality of the theorem. The reference of such a (Platonic) form can only be changed by modifying its intension, or redefining it. So we can go from a general triangle, to the more specific right-angled triangle, to a particular right-angled triangle printed on the page of a book, etc.

Lull uses an analogous process which he calls "contraction", which he explains in the *AB*:

Each principle, taken by itself, is completely general, as when one says "goodness" or "greatness". However, as soon as one principle is applied (*contrahitur*, literally "is contracted") to another, then it is subaltern, as when we say "great goodness". And when some principle is applied (*contrahitur*) to a singular thing, then it is completely particular (*specialissimum*), as when we say "Peter's goodness is great". And thus the intellect has a ladder for ascending and descending; as, for instance, descending from a completely general principle to one neither completely general nor completely particular, and from a principle neither completely general nor completely particular to one that is completely particular. And in a similar fashion one can discuss the ascent of this ladder. (*SW* I, 582; *DI*, 301)

So "goodness" by itself is completely general, modified by another principle it is neither completely general nor completely particular, and when applied to an individual then it is particular. Since, as we explained above, subject and predicate can be exchanged, making "great goodness" into "good greatness", the mechanism of contraction must also apply to "greatness" in the passage quoted.

As with many other innovations of the Art, this Lullian technique does not preclude classical quantification. In fact in Form 87 of the *AGU* which deals with logic, he *precedes* his presentation of “contraction” by saying that:

The logician predicates higher things of lower ones, and so he predicates “animal” of “man”, “body” of “animal” and “substance” of “body”.⁵⁰

This permits him to use classical syllogisms such as: “Every animal is a substance; every man is an animal; therefore every man is a substance.”⁵¹ Which technique he uses is a question of which sort of entities he is discussing: with (the medieval equivalent of) first-order logic, dealing as it does with extension in the created world, he can use classical quantification; but with (his version of) higher-order logic, which deals intensionally with properties or qualities of the uncreated world, he uses his technique of contraction. The fact that the new process is *also* applicable to the created world, means that, as with most other aspects of Llull’s system (or systems), it is additive with respect to the standard logic of his time.⁵²

This in turn means that we should distinguish three uses of the verb “is” in Llull: the standard one of inherence (“every man is an animal”); that of the mixing or blending of Platonic forms (“goodness is great”); and a third, when this mixing is convertible (“goodness is great” *and* “greatness is good”, a situation which only holds in God), when “is” is the copula of identity.⁵³

⁵⁰ *ROL* XIV, 366. The terms of “higher, lower” (*superior, inferior*) are the usual medieval way of referring to the two terms of the inclusion relation, and were probably derived from the position of the most basic terms on the Tree of Porphyry.

⁵¹ As for instance in the *LN* (*ROL* XXIII, 104; *NEORL* IV, 86), or in the example cited on p. 197 above.

⁵² See Llull’s remarks in the *Compendium seu commentum Artis demonstrativae*, *MOG* III, vi, 156: 446, quoted in Ruiz Simon 1999, 184–5. This is similarly true of the *demonstratio per aequiparantiam*, which does not supplant those of *propter quid* and *quia*, but is used in theological proofs, while the other two can only deal with the created world. When Llull says that the first is more powerful, he means that (1) it can deal with the higher realm of God, and (2) that it is not exclusive to that realm, but is more general and can be used at all levels.

⁵³ For convertibility of the dignities as a basis for the definition of God, see pp. 161 above and 220 below. Llull gives an interesting aside on this question in the *Liber praedicationis contra Judaeos* (*ROL* XII, 29), saying that “Jews and Saracens say that God is not predicable, or in other words that one must not say ‘God is good’, ‘God is great’, etc., and they thus say that God is unproductive (*non est generabilis*). And they say this because they feel there should not be [in God] any composition of subject

The Syllogism

As pointed out near the beginning of this chapter, if the *LN* has the first theoretical discussion of the syllogism, the first work—with the curious exception of one version of the *Apostrophe*—in which it is systematically used is the *Disputatio fidei et intellectus* written five months later. Moreover, the way he does it is both unusual and revealing.

This work begins with a short prologue in which he says he will deal with only four Articles of Faith—the Trinity, the Incarnation, the creation of the world, and the Resurrection—, and in which he gives definitions of “intellect” and “faith”. Here they are still just abstract concepts, but immediately afterwards, the first main section of the work begins with their personification:

Once upon a time it happened that Intellect and Faith, his sister, were on a pilgrimage together, when Intellect said...⁵⁴

After this abrupt change from a theological to a narrative discourse, we find ourselves immersed in a dialogue that is surprisingly vivid and literary. We have *exempla*, such as the one of the Saracen monarch angry at being deprived of his faith by the missionary’s rational arguments and then being told that the latter’s faith cannot be proved by necessary reasons; another about a Saracen scholar with a similar complaint; another of a boy trying to measure the amount of water in a river; and one concerning the ecology of wild boars in Islamic lands.⁵⁵

The arguments of the dialogue itself are closely reasoned, and provide one of Lull’s best counter-arguments to the idea that a faith which

and predicate. This would be true if subject and predicate did not convert, but since in God they do convert...?”

⁵⁴ *ROL* XXIII, 226. This is where the oldest manuscript separates the prologue from the first chapter; the editor has chosen to follow other manuscripts in which the second half of the prologue is placed within the first chapter. Considering the radical change from a metadiscourse, as it were, to the discourse itself, this choice seems debatable. For an excellent analysis of the work see Friedlein 2004, 134–5, 223–242, 273, where he points out how little attention this work has received in the literature on Lull.

⁵⁵ Ed. cit., 226, 231, 233, and 227 respectively. This last one says that because Muslims don’t eat pork, wild boars have proliferated in their lands, to the point of becoming a favorite prey for lions. To counter this, the smaller individuals dig up roots, so the larger ones can eat them without having to blunt the tusks necessary for fighting lions. The analogy, of course, is with Christians having to fight infidels with sharpened intellectual arguments. As Lola Badia has pointed out to me, the same example reappears in the *Llibre de virtuts e de pecats* (*NEORL* I, 196; *ROL* XV, 320).

needs proof is without merit.⁵⁶ At the same time, the argument is quite intense and emotionally charged, with frequent moments of amazement, sorrow, anger, and even insults (“You ox-driver!”) of the one towards the other’s attitude.⁵⁷ In other words, it is a conscious literary exercise in which the two personifications express all too human emotions, one which might seem far from the drier discourse of logic.

Yet in the midst of this discourse, we already find Lull sneaking in three syllogisms, such as:

Faith said: it is impossible for the intellect to reach beyond its own limits without its instrument; but the first principles are its instruments, and they cannot reach beyond sense and imagination; therefore the intellect can not attain the divine Trinity.⁵⁸

For the moment little importance is given to the matter, and the chapter continues with the usual manner of arguing. It ends with Intellect saying that they could go on like this forever getting nowhere; instead why not put the matter to a trial, in which he will try to prove things in such a way that Faith will not be able to deny them.

Now begin the four central chapters of the work, devoted—as promised in the prologue—to the Trinity, the Incarnation, the creation of the world and the Resurrection. After an opening syllogism about which they argue back and forth, the text then says: “Then Intellect said: ‘Faith, I want to prove to you the divine Trinity by syllogizing’”, after which the text starts off in earnest, but still with the technique of arguing pro and con loosely structured.⁵⁹ It is with the next chapter on the Incarnation that it takes on its final formalization, which we give only schematically to show its structuring:

Intellect said: . . . Everything which is more similar to God in unity and trinity is necessary; but Christ is thus; therefore Christ is necessary

I explain the major premise thus: Since everything which is more similar to divine Unity and Trinity is more knowable by divine wisdom, lovable by divine will, bonifiable by divine goodness (*a divina bonitate bonificabile*), and magnifiable by divine greatness (*a divina magnitudine magnificabile*) . . .

⁵⁶ With one of his rare citation of authorities: “No good can be loved unless it is known”. He doesn’t give the name of St. Augustine, but it is from his *De trinitate*, X, 1–2.

⁵⁷ Ed. cit., 229, 230, 235, 237, 238, and 239.

⁵⁸ Ed. cit., 235. There three syllogisms are put in the mouth of Faith and then answered by Intellect.

⁵⁹ Ed. cit., 242ff.

Proof of the minor premise:...

Faith said: Your argument is not valid for the following reason: what divine will could not want is not necessary; but divine will could not want the Incarnation; therefore the Incarnation is not necessary but contingent...

Intellect said: My reasoning is correct and not yours, since...⁶⁰

The first thing to observe about the form of this argument is that the use of the syllogism is standard in contemporary theology, especially in texts wishing to conform to the contemporary model of an Aristotelian science as knowledge derived from principles by way of the syllogism.⁶¹ Moreover, the mechanism of following the original syllogism by a proof of the major premise (*ad maiorem*), one of the minor (*ad minorem*), an objection, and a reply to the objection is also perfectly standard.⁶²

Secondly, Llull imbeds this whole scholastic mechanism within the framework of his dialogue between his two personified concepts, almost as if he were trying to disguise the innovation, or to sugar-coat the pill. Since scholastic syllogistic argumentation was a chain of pros and cons, this imbedding was not difficult; all Llull had to do was ascribe the various arguments to the two disputants. What is unusual, however, is this mixing of literary with scholastic forms. In subsequent works such as the *Liber de Trinitate et Incarnatione* he drops the literary mask altogether:

⁶⁰ Ed. cit. 252. For those unfamiliar with Aristotelian logic, the major premise is the one containing the predicate of the conclusion (in this case “is necessary”), and the minor premise is the one containing the subject of the conclusion (“Christ”). Notice that Faith’s objection is also expressed syllogistically, and that this objection along with Intellect’s reply are similar to the last two lines of the next example, except that here they might have been phrased: “Against the original argument” and “I answer that...”

⁶¹ See *Posterior Analytics*, I, ii. For the medieval trope of a demonstration based on the knowledge-producing syllogism, see p. 194 above. For the premises as the principles from which this knowledge is derived, see p. 213 below.

⁶² One finds it, for instance, in Duns Scotus, but usually in highly extended form with many ramifications. See Purtil 1971, 116–9. Llull himself does not adhere to this scheme strictly; often after the initial syllogism, the arguments for and against are presented in a looser way. Aquinas in his *Summa theologiae* often uses another form: presentation of the question (usually in the form of “whether”, *utrum*); objection 1 (2, 3, etc.) expressed syllogistically; “on the contrary” with reference to the original question; “I answer that” where he reasons his own solution (this was called the *determinatio*); and finally reply to objection 1 (2, 3, etc.).

That subject which converts with its own predicate is perfect; but God is a subject which converts with its own predicate, as I will prove; therefore God is a perfect subject.

Proof of the major premise (*probatio maioris*): Infinite great is infinite greatness, infinite eternal is infinite eternity, and infinite good is infinite goodness, and so on...

Proof of the minor premise (*probatio minoris*):...

Against the major premise (*contra maiorem*):...

I answer that:...

Against the minor premise (*contra minorem*):...

I answer that:...⁶³

Thirdly, since, as pointed out at the beginning of this chapter, these new syllogistic tools were developed for theological discourse such as proving the articles of faith (and particularly those of the Trinity and Incarnation), Lull had to solve the problem of assuring their efficacy in a realm for which such demonstrations were usually considered, at the very least, inappropriate. Now, since contemporary theological discourse usually took it for granted that the form of the syllogisms being used was correct—and hence the argument valid—the acceptability or truth of the conclusion was determined by that of the premises. Merely probable premises led to dialectical syllogisms, true premises to scientific demonstrations.⁶⁴ The first produced, as Lull said, following Aristotle, opinion, whereas the second knowledge (*scientia*), and a science that was divine could not produce something as imperfect as opinion.⁶⁵ So how he dealt with the truth of the premises was of prime importance.

Before going into the premises themselves, however, we must take up an essential component in their construction, which is Lull's new presentation of his:

⁶³ *ROL XII*, 106–7. Note here the theme of the conversion of subject and predicate presented above on p. 161, as well as what might look like a problem of substantized adjectives in the proof of the major premise, but where in fact we have an adjective modifying another adjective and then its corresponding noun.

⁶⁴ The word “probable” in this context had, for medieval thought, a meaning different from the modern modal sense. It derived from *Topics* 100b17–24, and means “generally acceptable, plausible”. Cf. Ruiz Simon 1999, 49 n. 44.

⁶⁵ For the importance of this distinction in Lull, see p. 278 below.

Three Kinds of Demonstration

In the *LN* he presents the same three kinds of demonstration as he had in the *AD*, but in a manner different enough to be worth presenting *in extenso*. After enumerating them, he says:

Propter quid is when the effect is demonstrated by its cause, as for instance: every B is A; every C is B; therefore every C is A. And this demonstration is said to be by cause because A is prior to B and B prior to C. Also, every animal is a substance; every man is an animal; therefore every man is a substance. And this demonstration is said to be by cause since substance is above animal and animal above man.⁶⁶ Moreover, no animal is a stone; every man is an animal; therefore no man is a stone. This demonstration is said to be by cause since animal is the cause of man's not being a stone.

Demonstration *per quia* is when a cause is demonstrated by its effect. This demonstration is of two kinds: one that is completely necessary and one that is not completely necessary. An example of the completely necessary one is: in every essence in which there is understanding (*intelligere*) there is intellect and intelligible; in God's essence there is understanding; therefore in God's essence there is intellect and intelligible. The not completely necessary demonstration is thus: every good effect has a good cause; a castle has a good effect; therefore a castle has a good cause. The reason why this demonstration is not completely necessary is that effect and cause are not conjoint by nature, and he who caused it could have had evil intent to do evil with the castle.

Demonstration *per aequiparantiam* is of three kinds. The first is when several powers are demonstrated by the equality they have in existing; the second is when the demonstration is done by the equality between a power and its act;⁶⁷ the third is when the demonstration is done by the equality between the acts of the powers.

The first kind is thus: Wherever there is infinite goodness and infinite intellect there is infinite equality; but in God there is infinite goodness and infinite intellect; therefore in God there is infinite equality.

The second kind is thus: Every infinite power has an infinite act; God has infinite power; therefore God has infinite empowering (*possificare*).

The third kind is thus: In every essence in which there is infinite goodness (*bonitas*) and infinite intellect, there is bonifying and understanding

⁶⁶ "Above" in the sense of including the lower category, as in the tree of Porphyry; see n. 50 above.

⁶⁷ Many of the Latin manuscripts, as well as the *ROL* edition, instead of "act" have "effect"; the Catalan text has *obra*.

(*intelligere*) equally and infinitely; in God's essence there is infinite goodness and infinite understanding; therefore in God's essence there is bonifying and understanding in an equal and infinite manner. (*ROL* XXIII, 101–2; *NEORL* IV, 84)

If we compare this with the earlier presentation in the *AD* on p. 65 above, we see how much more detailed is this new explanation. In addition, the former simpler examples reminiscent of propositional logic of the Stoics has been replaced by the term logic of the Aristotelian syllogism, including the classic one of “Every animal is...”, and even a purely formal one with letters. His examples also have the advantage of giving us a clearer idea of the various meanings that can be attached to cause, effect and *aequiparantia*.

With this under our belt, we can now go on to discuss the topic which caused our little detour.

Premises

Perhaps the most complete treatment of the subject is to be found, not surprisingly, in the work entitled *Liber de demonstratione per aequiparantiam*. The heart of the work, however, belies this title in a way which is particularly revealing for Lull's program; because, instead of a purely logical work, what we find is one in which logic is applied to a single topic central to Lull's endeavor: proofs of the Trinity. Moreover it is noteworthy that this is the first time the word “demonstration” appears in a title since the quaternary phase,⁶⁸ and this precisely in a work proving the Trinity! But Lull is not innocently courting disaster; he is consciously trying to supplement Aristotelian logic to make it usable for his ends.⁶⁹

To see how he goes about this, we must understand how he faces the challenge of a central dictum of the *Posterior Analytics*. There Aristotle says that for a syllogism to be properly demonstrative, its premises must have four attributes: they must be primary, true, immediate and necessary.⁷⁰ As for this first condition, Aristotle says:

⁶⁸ See p. 187 above.

⁶⁹ In fact, the *Liber de demonstratione per aequiparantiam* was among the ten most copied and studied of Lull's works during the Middle Ages; cf. Bonner 2003b, 91.

⁷⁰ *Posterior Analytics*, 71b22 and 74b16–17. Cf. Kretzmann et al. 1982, 497.

They [the premises] must be causative, better known and prior: causative, because we only have knowledge of a thing when we know its cause; prior, inasmuch as they are causative; and already known, not merely in the one sense that their meaning is understood, but also in the sense that they are known as facts. (71b30–34)

To answer this problem, Lull starts off in the *Liber de demonstratione per aequiparantiam* showing his awareness of this state of affairs, and explaining why his new techniques are necessary:

Whatever was demonstrated by ancient writers was demonstrated either by cause (*propter quid*) or by effect (*propter quia*). Now the subject of this book is to investigate the distinction in the divine persons, and this by demonstration. Such a demonstration, however, cannot be done by cause, since God has nothing above him; and the demonstration by effect is not very strong. For this reason we intend to prove distinction in the divinity by the *aequiparantia* and equivalence between the acts of the divine reasons. (*ROL IX*, 216)

After having thus disposed of the causal problem by placing it in his new framework, he goes on to explain:

And since a demonstration of this sort proceeds from principles which are primary, true, immediate and necessary, therefore it is by such principles that we want to formulate and find that demonstration which we call *aequiparantia*. And just as we will exemplify it in matters divine, similarly it can be used in other sciences each in its own way.

Here we have the four attributes of premises Aristotle said were necessary for a syllogism to be properly demonstrative, but which Lull is recycling to use as foundations for his new demonstration *per aequiparantiam*. As for the little paragraph quoted above in which Aristotle explains what he means by the first one, Lull counters with a parallel proposition:

So as to root out misunderstandings and doubts, we call these principles “primary” simply and absolutely, not because others descend from them but because they themselves descend from no other. In this it can be seen that such primacy retains a more natural primacy than that of a cause of an effect, since it is absolute whereas the other is relative (*respectiva*). For example, in God both the intellect by its understanding and the will by its desiring are primary with respect to all the other dignities, for the reason that the other dignities have understanding by the intellect and desiring by the will. This is not, however, like effects from causes, since the other dignities are equivalent to them in essence and nature. These others are in fact primary to them [i.e., the first two] by their own acts and in their own way, like divine goodness which by its bonifying is primary

to good understanding and loving, and similarly with divine greatness... and similarly with the others circularly according to their acts. And this circularity is primary since it is fixed in God's essence and does not come from outside him. (*ROL IX*, 216–7, cited in Ruiz Simon 1999, 284)

So for Llull the primacy of dignities comes from their unquestionable role as divine attributes, above and prior to anything causal. To this is attached their activity, and their equality with one other, producing their circularity and mutual predicability. And all of this is ultimately based on their definitions, which of course fulfill these requisites.

As for the second of Aristotle's four conditions, the premises of Llull's system are true "because whatever is truly understood has been truly understood through the intellect, and whatever is truly loved has been loved through the will, and whatever is truly bonified has been truly bonified by goodness... and so on for the other dignities." As for the third condition, they are immediate "because there is no mediation (*medium*) between a power or dignity and its act, as between intellect and understanding (*intellectum et intelligere*), between will and willing, goodness and bonifying", etc. As for the last condition, they are necessary "because from the divine intellect there necessarily follows understanding, since in God there is no power without act, and from will willing, from goodness bonifying", etc. (*ROL IX*, 217–8)

He concludes this part of the prologue using three principles from Figure T—concordance, difference, and equality—to prove four crucial conditions for the work's subsequent Trinitarian demonstrations: that each dignity has its act, that from its act follows concordance, that from concordance follows difference, and that from concordance and difference in God there follows equality.

Based on all these mechanisms, in the first distinction of the work he begins with his demonstrations:

Wherever there is concordance there is plurality; but in God's goodness there is concordance; therefore in God's goodness there is plurality.

The major [premise] is evident, since it was proved above in the prologue. The minor [premise] is explained thus: wherever there is an act, there is concordance between the agent and the *agibile* in this act; but in divine goodness there is an act, that is bonifying, as has already been demonstrated; therefore, etc. (*Ibid.*, 222)

Notice how here even the proof of one of the premises is cast in syllogistic form. After giving similar proofs for the dignities of "greatness" and "eternity", he returns to that of "goodness" for a new demonstration:

Wherever there is equality, there is difference between things equalled (*aequalificata*); but in divine goodness there is equality between *bonificativum* and *bonficabile*; therefore in divine goodness there is difference between *bonificativum* and *bonficabile*.

The major [premise] is self-evident. The minor [premise] is explained thus: for if between *bonificativum* and *bonficabile* there were no equality, the act would not proceed equally from both; but it is clear that it does proceed equally; therefore, etc. (Ibid., 223)

The major premise is self-evident as the converse of the last of the four conditions listed above.

So, in this work the premises are primary, true, immediate and necessary because they are based, directly or indirectly—i.e. through a series of simple proofs presented in the prologue—, on the nature of the dignities, of their activity, and of their equality in God.

But this is not the only kind of premises Lull uses. He can also present them directly, as assertions needing no prior justification, in which case Lull sometimes refers to them as “maxims”, the term used in medieval discussions of the *Topics* as a frequent ellipsis for *maxima propositio*.⁷¹ Boethius calls these maxims “principal” since there is nothing which is better known (*notius*) than they.⁷² Accordingly they cannot be proved by anything else, but are usually the principles by which other propositions are proved.⁷³ In Lull these maxims are similarly brief statements acceptable on their own merits, or, because, in the case of God—as in the work we have just studied—they reflect the convertibility, equality, and activity of his dignities. As such they can be used as premises for syllogisms, and sometimes he limits the word “maxim” to that sense.⁷⁴ Other times he uses that word in a sense closer to the modern meaning, as in the *Liber de possibili et impossibili* where he

⁷¹ *OED*, s.v. See Green-Pedersen 1984, 42, 45, 60–63, 148–9, 256–8.

⁷² Lull himself calls them *notabilia* in the *Disputatio Petri clerici et Raimundi phantastici* (*ROL XVI*, 22).

⁷³ Green-Pedersen 1984, 42, citing Boethius, *De differentiis topicis*. Peter of Spain (Peter of Spain 1972, 59) says that “A maxim is a proposition to which no other can be prior, that is to say, better known.” These descriptions, along with the examples given in Boethius and Peter of Spain, are similar or identical to what Aristotle called an “axiom”. For the problems inherent in this term, with the usual Latin translation as *dignitas* and with its Euclidean connotations of an axiomatic method, see the chapter of Conclusions below.

⁷⁴ As for instance in the *Tree of Science* (*ORL XI*, 135, 189, 215, and *XI*, 57; *OE I*, 602–3, 621, 630, and 693; *ROL XXIV*, 157, 212, 240, and *XXV*, 435), where, under the surprisingly Peircian concepts of Firstness, Secondness, and Thirdness (for which see p. 289, n. 91, below), he gives as an example “an argument composed of two maxims and a conclusion”.

proposes a hundred “maxims”.⁷⁵ They are frequently associated with his definitions, as for instance in the *Ars compendiosa Dei*. We will discuss this work in greater detail below in the section on “Contradictory Syllogisms”, but for now we will just use it to touch on his more or less synonymous use there of the concepts of “definition”, “maxim”, “proposition”, and “predication”.

At the beginning of the fifth distinction of the *Ars compendiosa Dei* he says that “since in the third distinction we showed how to prove propositions and maxims, for the rest we do not intend to prove propositions or maxims, since they are self-evident (*per se notae*)” (*ROL XIII*, 50). The idea of wanting to prove a “maxim” seems like a contradiction in terms, but we must remember that with the Godhead, Lull is intent on proving things others accept as matters of faith, *as well as* using them as foundations, which is why here he can use “maxims” in both senses. Lull thus assures the truth of his premises in two ways: by offering acceptable proofs, and by presenting them as *maximae*.

He then offers eighteen predications, definitions, or maxims—he uses all three terms—concerning God’s essence, such as “God’s essence is his goodness”, “God’s essence is his greatness”, “God’s essence is his eternity”, etc. (*Ibid.*, 51 and 53). This is followed by arguments or syllogisms based on them, as for instance from the last of these:

It was said that God’s essence is eternity. I therefore argue thus: Whenever God’s essence is eternity, there one finds *essentiatum* and *aeternatum*, or *essentiabile* and *aeternabile*. But in divine power God’s essence is eternity; therefore in divine power there is *essentiatum* and *aeternatum*, or *essentiabile* and *aeternabile*. (*Ibid.*, 51–52)

Then in a third section “On God’s essence deduced by the Rules”, he begins by asking under the first rule of *utrum*: “Whether in this life we can have a primary, true, and necessary science of God’s essence.” (*Ibid.*, 54) Here we are back with Aristotle’s dictum from the *Posterior Analytics*, without the attribute of “immediate”, which is out of the question between man and God. His answer to this challenge comes in the form

⁷⁵ *ROL VI*, 428ff., on which hundred “maxims” he comments in the *Liber contradictionis* (*ROL VII*, 139ff.). For Boethius’ similar use of the term in a broader sense, see Green-Pedersen 1984, 62. It might be interesting to explore the possible relation between these Lullian “maxims”, the “principles” (or *començaments*) of the *Principia philosophiae* (*ROL XIX*, 91, 160, 265; *NEORL VI*, 7, 58, 141), and his use of proverbs. See Rubió i Balaguer 1985, 311–3, and Pring-Mill 1991, 309–311, for the connection with what Lull calls “examples” and “metaphors”. See also Garcías Palou in Lull 1978, 36ff.

of a metaphor. If a man holds a lily in one hand and a mirror in the other, the whiteness of the lily which he sees when he looks at it directly is primary, true, and necessary, but when he looks at it in the mirror, what he sees is an image (*similitudo*) whose whiteness is not primary, true, and necessary. This image in the mirror, however, *is* primary, true, and necessary as a sign, which sign is secondary (*posterius*) with respect to the reality of the direct sight of the lily's whiteness which is prior. Thus in this life the intellect can make a science of God's essence which is primary, true, and necessary in the secondary sense, which in Heaven we will see as prior. "Otherwise no sign would be primary, true, and necessary with respect to the thing signified."

After this explanation, he can now go on to the second Rule:

By the first species of the second Rule [that of definition] it is asked: What is God's essence? The answer is that it is eternity, which we proved God to be. And his concrete⁷⁶ is his being (*esse*), which being is an eternal thing. Or one can say that God's essence is Godness (*Deitas*), and his concrete being is God (*Deus*), blessed may he be. (Ibid.)

So here we have an argument based on a "maximal proposition", but expanded to include a more involved species of definitions (Ibid., 50).

We cannot end this section without emphasizing the essential role of Lull's definitions as foundations for his demonstrations, as much in his logic as in his earlier ternary Art. In this he is once again following Aristotle's *Posterior Analytics*, which states flatly that "the starting-points of demonstrations are definitions".⁷⁷ This is why he defends them so vehemently, and especially why he defends their peculiar nature making them applicable to both the created and uncreated realms in which his Art and logic had to be equally valid.

New Developments

If the year 1303 saw the appearance of undisguised Aristotelian logic on the scene of Lullian demonstrations, the year 1308 saw an almost equally sudden expansion of Lull's logical techniques. As if plain syllogistics were not enough for his needs, he now adopted not one, but

⁷⁶ "Concrete" is here used as a noun, opposing *esse* to the abstract *essentia*. Compare Ch 3, n. 32 and the passage in the middle of p. 183 above.

⁷⁷ 90b23-24, quoted in Hintikka 1987, 218.

five additional logical techniques: (1) the finding of the ‘middle’, (2) contradictory syllogisms, (3) the systematization of hypotheses and their application to contradictory suppositions, (4) the fallacy of apparent contradiction, and (5) proofs by the superlative degree.⁷⁸ Some of these, such as the first and the fourth, had indeed been discussed in the *LN* of five years before, but no actual use had been made of them in subsequent writings. Now, not only does he adopt them almost explosively, but they become—each one of the five in its own way—general tools central to his theological proofs. Why this sudden interest of Llull’s in expanding his logical tool kit? As Charles Lohr suggested, it was undoubtedly in preparation for his forthcoming visit to Paris, where he would arrive in the autumn of 1309. The first, the finding of the ‘middle’, would be used, as we shall see, to adopt (and adapt) another important component of Aristotle’s *Posterior Analytics*. The next three are techniques which he would use in arguing with the ‘Averroists’, which was the preferred term among their adversaries for the more radical Aristotelians of the Arts Faculty. To a certain extent fomented by rivalry with the more prestigious Faculty of Theology, these were thinkers who tried to set philosophy off on a more or less independent course. If they never actually proposed a theory of double truth—that there might be things philosophically invalid which must all the same be accepted by faith—they developed a style of philosophizing which to a certain extent turned its back on possible theological consequences. Such a separation was, of course, intolerable to a thinker like Llull who had spent his life devising an Art general to all levels of thought, one valid for both philosophy and theology, so that tools developed to argue in one area would be valid in the other, in short one that might refashion the threatened unity of Christian knowledge.⁷⁹

In the midst of these new preoccupations, Llull never lost sight of his apologetic aims. Frequently in the works of these years he points out that non-believers will never be persuaded if one could not argue with them by logical means; it would be seen by them as an abdication of any real ability to prove the truth of Christianity. As if to prove this point, in the principal work presenting the fallacy of apparent contradiction, the *Liber de novis fallaciis*, which we will study below, he devotes

⁷⁸ Contradictory suppositions, as we pointed out on p. 193 above, didn’t in fact make their appearance till two years later, in 1310.

⁷⁹ Particularly good on this subject is Ruiz Simon 1999, 175 and 331ff. (and especially 341 n. 468). See also Imbach 1987 and Domínguez in *ROL XIX*, 15ff.

an entire section applying this new technique to arguments which he had already proved syllogistically in the *Disputatio Raimundi christiani et Homeri saraceni*.⁸⁰

After that brief introduction, we will take up each of these five new developments in turn, beginning with:

The Finding of the Middle

In four works finished in the first months of 1308, the *AGU*, the *AB*, the *Liber de venatione substantiae, accidentis et compositi*, and the *Ars compendiosa Dei*, Llull confronts the reader with the problem of ‘finding’ the ‘middle’.⁸¹ The sudden introduction of the concept is accompanied by an equally sudden declaration of its primary importance, as is clear from his new definition of logic from the *AGU*:

Logic is the art with which the logician finds the natural conjunction between subject and predicate. This [conjunction] is the middle, by means of which he will know how to arrive at necessary conclusions.⁸²

The middle, it will be remembered, is the term or phrase which disappears from the conclusion, after having completed its task of acting as a “conjunction between subject and predicate”, providing the ground of proof or inference, and thus allowing us to “arrive at necessary conclusions”. Because of its explanatory nature, the problem of the middle was central to the *Posterior Analytics*.⁸³ What Llull did was to adapt—or rather expand—it in two ways. The first was to provide a method of searching for (*inveniendi*) the middle, something unknown to scholastic

⁸⁰ This is the last of the four works listed on p. 193 above. The way the second work redoes the proofs of the first is discussed below, pp. 241–2.

⁸¹ Cf. Ruiz Simon 1999, 184–9, 274–293, and Cordeschi 1983. Ruiz Simon provides a more philosophical interpretation of the matter discussed in this section, whereas Cordeschi gives a very detailed map of all the different techniques Llull used to produce these results. Many of the examples cited in this section come from Cordeschi’s study.

⁸² Form 87 of the Hundred Forms in the *AGU* (*ROL* XIV, 365–6), and, without the second sentence, in the *AB* (*SW* I, 623; *DI*, 342). Cf. Ruiz Simon 1999, 186, on the importance of the middle at this point in Llull’s production. We have used the word “middle” rather than “middle term” because in medieval logic it was always just *medium* (and never *terminus medius*), and it often could involve more than a simple term. Cf. Rijk 1990a, 106 and 112; Green-Pedersen 1984, 282–3; and Johnston 1987, 227f.

⁸³ 74b30–75a37, 89b37–90a35, 93a11–b26, as well as many other places in the *Posterior Analytics*.

science, which normally separated the dialectical *inventio* of the *Topics* from the demonstrative science of the *Analytics*.⁸⁴ Suddenly now with the *AGU* and the *AB*, the Third Figure and the Fourth Figure with its corresponding Table become instruments for ‘finding’ the middle.⁸⁵ In the *AGU*, in fact, three pages of the chapter on the Multiplication of the Fourth Figure are taken up in the “Finding of many middles” (*De inventione plurium mediorum*).⁸⁶

Llull’s second mode of adaptation had to do with Aristotle’s statement that any middle would have to be linked to his causal explanations: “The middle is the cause, and this is what we are trying to find in every case.”⁸⁷ Since this meant explanations by *propter quid* and by *quia*, Llull found himself obliged, as he had been with the Art and with his syllogistic premises, to find an additional non-causal middle, one equivalent to his *demonstratio per equiparantiam*, if he wanted to be able to apply his logic to God. His solution had three basic facets, all based on his definitions of God.

As the reader will recall from the previous chapter, he gave many such definitions, three of which are:

God is the being whose reasons [i.e., dignities] convert [with one another]; indeed, the being whose reasons convert is God.

God is the being in which the divine reasons have infinite acts, such as infinite goodness (*bonitas*) which has infinite bonifying, and greatness (*magnitudo*) infinite magnifying. And the being in which there is goodness having an infinite act is God.

God is substance free from all accidents; indeed, substance free from all accidents is God. (p. 161 above)

So in God we must have convertibility, dynamism and substantiality. The first and last imply that any predication applicable to God cannot be between lower and higher (as Llull would put it), or between greater and lesser, but must be on a plane of equality. The same must be true of the correlatives expressing his dynamism *ad intra*; their *-bile*, *-tivum*, and *-are* must exist on a similar plane of equality, as well as between

⁸⁴ Cf. Ruiz Simon 1999, 185–9, and Cordeschi 1983, 259.

⁸⁵ In the *AB* the subject is repeated almost obsessively; see for example *SWI*, 582, 583, 584, 586, 589, 623; *DI*, 301, 303, 305, 309, 342.

⁸⁶ *ROL XIV*, 104–7.

⁸⁷ 90a6–10, where Aristotle asks “Is there a middle?” and “What is the middle”, and gives an example of the eclipse of the moon, where one must ask “Does it suffer eclipse?”, which means ‘Is there or is there not a cause (for its being eclipsed)?’

their inner cause and effect.⁸⁸ As a result, any syllogism applicable to God must be *per aequiparantiam*, but in a broader sense than before, in the sense that equality must now exist not only between terms but also between propositions. But how can Llull do this?

Taking first his dynamic ontology and how it is applied to the problem of convertibility, a simple example of the corresponding middle is found in the *AB*, in the chapter on the Evacuation of the Third Figure, where he says:

Then it is evacuated of twelve middles. And they are called middles because they are placed between subject and predicate, with which they accord in genus or species. . . . Thus, for instance, one can say: "Everything which is magnified by greatness is great; goodness is magnified by greatness; therefore goodness is great", and so on for the others.⁸⁹

This syllogism uses the natural middle from the definition of greatness: "that by reason of which goodness, duration, etc., are great".⁹⁰ In other situations the middle must be 'found', something which Llull explains at length in a passage near the beginning of the *Ars compendiosa Dei*.

The middle which we wish to investigate is made evident by means of this syllogism: Every animal is a substance having senses (*substantia sensata*); but every sentient being (*sentiens*) is an animal; therefore every sentient being is a substance having senses. In this syllogism there is no superior or inferior, since all the terms are equal. By reason of this equality the syllogism can be converted, making the major [premise] into the minor and vice versa, and making the predicate into the subject and vice versa, with the middle remaining as it was. Thus we can say: Every animal is a sentient being; but every substance having senses is an animal; therefore every substance having senses is a sentient being. So, just as in such syllogisms "animal" is the middle for syllogistic reasons, in the same way this act of "sensing" is the middle existing between the things designated

⁸⁸ See the three species of dynamic equality explained on pp. 211–2 above.

⁸⁹ *SW* I, 598; *DI*, 318, the "middle terms" of which I have corrected to "middles". The phrase "with which they accord in genus or species" would tell the medieval reader that he was conforming to Aristotelian strictures (cf. *Posterior Analytics*, I.vii and 84b17). Our translation of "is magnified by greatness is great" necessarily hides the important similarity of Latin roots of *magnificatur a magnitudine est magnum*. The *AB* offers an even simpler, purely Artistic example of "middle" from Figure T. When discussing the Third Figure he says "Each compartment contains two letters, and these represent subject and predicate, between which the artist seeks the middle that will join them, like goodness and greatness that are joined through concordance, and similarly for other terms." (*SW* I, 586; *DI*, 305)

⁹⁰ Cf. p. 134 above.

by the syllogism for reasons of reality, naturality, primitiveness, internality, truth, and necessity. And the reason is that without it, neither animal, substance having senses, nor sentient being could exist. Thus by this investigation we have arrived at the real and natural middle which we intend to use here. And the reason we found such a middle was so that we may conclude necessarily by equality (*per aequiparantiam*), and not by superior or inferior.⁹¹

Llull here distinguishes two kinds of middle: the usual one of “animal” which connects the subject and predicate of the conclusion of standard syllogisms such as “Every animal is a substance; every man is an animal; therefore every man is a substance”, and which he calls the “syllogistic” sort, and his new one which depends on an “act”, in this case on that of “sensing”. It is the introduction of such an “act”, the central and binding component of the correlatives, identical at all levels of being, or to put it another way, at all levels of the tree of Porphyry, that turns a relationship of “superior” and “inferior” into one of equality, thus allowing for convertibility between the two premises, and hence between subject and predicate of the conclusion.⁹² At the same time, this equality turns the copula, “is”, from one of mixing or participation, to one of identity. This new formula is thus able to transform a syllogism that is neither *per aequiparantiam* nor convertible into one that is, and for Llull making it once again conform to Aristotle’s dicta for premises, in addition to giving it a form and force similar to one directly applicable to God, such as:⁹³

Whatever is God is optimum goodness; but maximum greatness is God; therefore maximum greatness is optimum goodness.⁹⁴

Here the equality and convertibility are natural or intrinsic to the subject matter, that is God, whereas in the previous example they had to be “found”.

As for substance, it is perhaps worth stopping a moment to discuss the *Liber de venatione substantiae, accidentis et compositi*, a work mentioned above as one of several written shortly after the *AGU* and the *AB*. The

⁹¹ *ROL XIII*, 17–18. Cf. Gayà 1979, 176–8, and Ruiz Simon 1999, 279.

⁹² “Superior” or “inferior” because on the tree of Porphyry, standard in medieval texts on logic, “substance”, for instance, was higher up than “animal”, which in turn was above “man”.

⁹³ To the subject of conversion Llull devoted an entire work, the *Liber de conversione subjecti et praedicati et mediū*, and it is this work which is principally analyzed in Cordeschi 1983.

⁹⁴ *ROL VI*, 264. It should be remembered that the Latin superlative of *bonus* is *optimus*, and that of *magnus* is *maximus*.

reader might be surprised to find that a book whose title proclaims a quest for substance and accident begins by stating that “the science of logic is difficult, unstable (*labilis*) and prolix”, with three adjectives taken straight from the prologue of the *LN*.⁹⁵ Lull goes on to explain himself by comparing logic with “natural science [which] is delectable, permanent and immutable,” and adding that it is “from the two trees of these sciences that he we will gather this bough, that is to say, this book.” So we are being offered, as it were, a marriage of logic and philosophy. After six distinctions discussing substance and accident in all manner of ways, with only an occasional nod to the logicians, suddenly we come upon a seventh distinction entitled *De venatione medii inter subiectum et praedicatum*, which tackles the application of substance and accident to the “quest” for the middle.⁹⁶ For a syllogism to be ‘necessary’, all three terms must be substantial and not accidental. As an example of one that won’t work, he gives:

No color is a quantity; redness is a color; therefore redness is not a quantity.⁹⁷

He admits that such a syllogism is valid (“dialectical or *probativus*”), but says it is not “necessary”, because “color” is not “the natural middle we were seeking”, but is in fact an accident.

In another, rather more complicated example, he states that, given that “greatness is good”:

All good greatness is the reason for great doing great good; but A is thus; therefore A does great good.⁹⁸

Lull explains that “this syllogism is dialectical and *probativus* but not necessary”, because “only substantial goodness is the reason why good produces good, not accidental goodness”, as is the case here when

⁹⁵ *ROL* XXII, 14, to compare with *ROL* XXIII, 15.

⁹⁶ The first three (of the four) parts of this seventh distinction were printed as a separate work in Lavinheta 1516, and reprinted as such many times thereafter (notably in the Zetzner anthology published in Strasbourg; see Lullus 1996). Because of this separate history, and because no work with that title could be found in Lullian manuscripts, it was long thought to be a pseudo-Lullian work. Finally a librarian from Cologne (Vennebusch 1972) identified it as a portion of the longer work. Consult *ROL* XXII for an edition of the work, and pp. 4–6 for a discussion of the publishing history of this seventh distinction.

⁹⁷ *ROL* XXII, 86. This is, curiously, an almost modern text-book example of second-order logic.

⁹⁸ *Ibid.*, 88.

good is not a substance of its own, but an accident as a modifier of greatness.

Finally, in the same work, he gives an example which throws further light on one we gave before:

Every animal is a substance; every man is an animal; therefore every man is a substance. This syllogism cannot be regarded as necessary, since its middle is not completely natural, for the reason that substance is superior and animal inferior, and animal superior and man inferior. So, in order to remove that by which substance is superior and to raise up that by which animal and man are inferior so that the terms are equal, one needs a syllogism such as the following: Every rational animal is a rational substance; but every man is a rational animal; therefore every man is a rational substance. The resulting syllogism is necessitated (*necessitatus*) because the terms are coequal, since the middle is natural.⁹⁹

In the terminology used before, “animal” would be the purely “syllogistic” middle, whereas “rational” (which is a property, not an accident) would be the natural middle that would put subject and predicate on an equal footing, or as Llull would put it, one of *aequiparantia*.

As one would expect, Llull derives the various middles of his logic from Figure T of the Art. In fact, if the reader will look back to p. 130, he will see how the *AB* divides middle into three species: the conjunctive middle, the middle of mensuration, and that between extremes. In his logical works the middle of mensuration is often called the middle of conversion, and it is the one which permits the conversion of subject and predicate in the cases above where this can take place. As for the other two, their use is best shown by the syllogism;

Every angel is a single species; Gabriel is an angel; therefore Gabriel is his species.

On which Llull comments that:

In this syllogism there is contraction of species to Gabriel. The middle of mensuration does not convert any terms, but by reason of [this] contraction the conjunctive middle joins what is superior with what is inferior. The middle between extremes establishes that these are one undivided essence.¹⁰⁰

⁹⁹ Ibid., 84. Compare the similar syllogisms in *Liber de conversione subjecti et praedicati et medii*, *ROL VI*, 268–9.

¹⁰⁰ From the same *Liber de conversione*, *ROL VI*, 266. Cf. Cordeschi 1983, 261.

As the previous discussion has shown, Lull offers a hierarchy of middles and of the syllogisms in which they appear. The new equalizing middle produces “demonstrative” syllogisms which appeal to the “assertive” intellect, as opposed to the traditional ones which are “opinative” and appeal to the “creditive” or “opinative” intellect,¹⁰¹ therefore smacking more of dialectics than of logic.¹⁰² He makes a further hierarchization based on two kinds “of substance which could be called absolute”, one “which is called absolute because it is simply (*simpliciter*) separate from all accidents” (God and his dignities), and another which is “with accident, and is called absolute because it is ‘predicable’ and any of the nine accidents can be predicated of it, such as quantity, quality...”.¹⁰³ As a result, the last syllogism preceding note 99 (“Every rational animal is a rational substance...”) is less necessary than the one preceding note 94 (“Whatever is God is optimum goodness...”). Even though the terms of the first are coextensive and non-accidental, the last one is more necessary because its terms are “primary, true, and necessary”.¹⁰⁴ The second is also more demonstrative because its terms are convertible simply (*simpliciter*), and not in some qualified way (*secundum quid*) as in the first.¹⁰⁵

Our presentation here of Lull’s hierarchies of middles and syllogisms only gives a small indication of the various possibilities he suggests.¹⁰⁶ What is important to retain is the matter of the ‘natural’ middle which permits Lull’s all-important syllogistic *aequiparantia* and convertibility between subject and predicate. Since these are things which only apply naturally to God, one can see how Lull’s logic is openly dedicated to the problem of developing techniques applicable to theology (and often even more specifically, to proofs of the Trinity and Incarnation), while being at the same time generally applicable to the created world. In this way it is the opposite of his Art, which pretends to a generality which turns out to be *also* applicable to God, as much as that might have been its *raison d’être* from the very beginning.

¹⁰¹ *ROL* VI, 263. See p. 278 below for this much repeated distinction in Lull.

¹⁰² *ROL* VIII, 112.

¹⁰³ *Liber de accidente et substantia*, *ROL* I, 139–140.

¹⁰⁴ *ROL* VI, 264. Cf. Cordeschi 1983, 263. Note once again the same Aristotelian conditions referenced in n. 70 above.

¹⁰⁵ Cf. Ruiz Simon 1999, 277.

¹⁰⁶ Cordeschi 1983 is particularly good on the different hierarchies of middles and of syllogisms which Lull presents, and on their ontological-logical basis.

Contradictory Syllogisms

This new technique presents paired syllogisms, each pair based on the same predication, with a true syllogism followed by its contradiction, where the falsehood of the second proves the truth of the first. It was introduced in a major theological work of 1308, the *Ars compendiosa Dei*, which Lull wrote only two months after finishing the *AGU*. As we did with the *Disputatio fidei et intellectus*, it might be interesting to stop for a moment to describe this work, not only for the way he introduces this new technique, but also as an excellent illustration of the integration of artistic foundations with syllogistics, which will form the basis of much of Lull's production from now on.

After a First Distinction presents the eighteen Principles of the Art (with their definitions) and the ten Rules or Questions, Distinction II carefully explains how this work will function. In the first of its four sections Lull defines twenty-seven other principles, no longer general ones, but those applicable to theology and which will form the backbone of the work: "necessity", "essence", "nature", "unity", "infinity", etc., with definitions of each.¹⁰⁷ Most of these definitions are of the sort with which we're already familiar: "Nature is that being whose proper task is to naturize (*naturare*)" or "Infinity is that being whose proper task is an infinite act, that is to infinitize (*infinire*)". Only the first principle, because of its importance for God, has a slightly more extended definition: "The necessity intrinsic to God is that being which can be no other way, whose act is necessitating (*necessitare*)."

The following section, *De modo praedicandi* (*ROL XIII*, 28), shows how one can predicate the eighteen general principles of these twenty-seven divine principles, as for example "God's necessity is good, God's necessity is great", etc. And then, what was done with God's necessity, can also be done with God's essence, God's nature, etc. Finally Lull explains that the 'artist' can exchange subject and predicate at will, saying "God's goodness is divine necessity, God's goodness is divine essence, and so on".

¹⁰⁷ *ROL XIII*, 27. Lull calls them *principia*; it is the editor of this *ROL* volume who has added the heading *De definitionibus subjectorum*, which would seem to be in none of the sources. The distinction is important: they are things to be used as bases of proofs or premises, not as headings for classifying questions or solutions. Of these twenty-seven, the last three of "comparison", "application", and "question" are more properly devices of the Art than terms applicable to theology.

The third section, *De modo connectendi*, a kind of substitute for the ‘mixing’ of the later Art, shows how “the definitions of the first section are to be connected with those of the second” (Ibid.). He gives two examples: “Divine necessity is divine goodness” and “Divine necessity is divine greatness”, each of which he explains at length (Ibid., 28–29). For the first he starts off saying:

Divine goodness is that by reason of which good does good, and divine necessity is that being which can be no other way. And since divine goodness can be no other way, it is the intrinsic, necessary reason for good to do good. Consequently, intrinsic good is necessitated so that it does good intrinsically.

This permits him to formulate the following syllogism:

Wherever divine necessity is divine goodness one finds necessitating and bonifying; but, in eternity, divine necessity is divine goodness; therefore in eternity one finds necessitating and bonifying.

The seeming jump produced by the introduction of “eternity” is explained:

Proof of the minor term is by the definition of eternity, which says that it causes necessity and goodness to endure;¹⁰⁸ and necessity says that eternity by itself is a necessary thing. We have therefore proved the necessity in eternity, and consequently also goodness (in it). Thus we have discovered eternalizing, for without it necessity could not have eternal necessitating, nor goodness eternal bonifying.

The fourth and last section of the First Distinction treats the finding of the middle. After an explanation which adds to the one already quoted above on p. 221, he gives a series of examples, the first of which is a syllogism based the Principle of “power”:

Wherever the divine intellect is divine necessity one finds divine power; but in the divine will there is divine necessity; therefore in the divine will one finds divine power. (Ibid., 31)

With the technique we have already seen of proving the major and minor premises, he now offers an explanation of the role of the middle, which begins:

¹⁰⁸ The standard definition from the Art (see p. 134 above), with “necessity”, which, as we have just explained, is one of the Principles of this work, inserted under the canopy of “etc.”

Proof of the major premise: Since without divine power the conversion between intellect and necessity cannot be made, one thus finds the middle of power, that is to empower (*posse*), existing between empowering (*potentem*) and empowered (*possitum*), which middle is needed by necessity, intelligible by intellect and lovable by will. And such a middle is primary, true, necessary, and real...

Distinctions IV–XXX take up the twenty-seven new principles one by one, each treated in two parts, one according to the general Principles of the Art and another according to the Rules. So far we're on familiar ground, and especially in the second part with the Rules, which is no different in method from similar sections in the Art. It is in the first part using the Principles that Lull introduces his logical techniques. He subdivides this part into two sections: (1) *De praedicatione*, which predicates the eighteen Principles of whatever new principle is under discussion; and (2) *De argumentatione*, which, as we saw above on p. 216, gives syllogisms based on these predications.

Up to this point we have strayed little from other logical works of the same year; it is with Distinction XII that Lull introduces his new technique of paired, contrary syllogisms. In a first section the eighteen Principles are predicated of God's substance, starting with "God's substance is his goodness."¹⁰⁹ In the second section he gives two pairs of contrasted syllogisms, two of which are "primary, true and necessary, and two not true so that by the ones that are not true we may know the ones that are, and vice versa, since by one of [two] contraries one knows its opposite."¹¹⁰ The first pair states that:

It was said [above] that God's substance is his goodness. And since God's bonifying is the act of his goodness, it follows that God's substance is God's bonifying, and that it has its proper act, that is substantiating (*substantiare*), without which it could not convert with bonifying and with goodness. I therefore argue thus:

Wherever God's substance is his goodness one finds substantiating and bonifying; but in God's greatness God's substance is his goodness; therefore in God's greatness one finds substantiating and bonifying.

¹⁰⁹ Ibid., 86. The complete predication is "God's substance is his goodness without quantity", but since he doesn't use the last two words in the first example I have chosen, I have omitted them.

¹¹⁰ Ibid. 87. On p. 92 he states the situation more categorically: "... so that by the false [syllogisms] we may know the true ones. Since with a syllogism that cannot be true, its opposite must necessarily be true. And this rule is infallible; otherwise real contrariety would be impossible."

Then I make the following syllogism against the first, and I do it supposing that God's substance does not have substantial substantiating nor [his] goodness substantial bonifying. I therefore argue thus:

In that in which God's substance is his goodness there is no substantial substantiating nor substantial bonifying; but in divine greatness the substance of God is his goodness; therefore in divine greatness one does not find substantial substantiating nor substantial bonifying.

This syllogism is correct with respect to its form, but false and erroneous with respect to the matter signified. For divine greatness could not suffer that in the very substance of God there should be his goodness deficient in its proper substantial acts, without which it would be changed into evil and smallness, emptiness and idleness. And the definition of power would be annulled and consequently the definition of end, etc. And since all of these things are false and impossible, it is clear that the second syllogism is false and erroneous, and that the first, which is contrary to it, is primary, true, and necessary.

The last two paragraphs clearly show the difference between formal validity of a syllogism and the truth of its conclusion dependent on that of its premises, a truth which, for Lull, can only be established through the Art, and this especially in matters of theology to which they are, in their generality, principally applicable. Notice too how in the various proofs quoted Lull continues citing the conditions of the *Posterior Analytics* in referring to a syllogism as "primary, true, and necessary."

It might be interesting to see how Lull uses this new tool to solve the apparent contradiction of predestination and free will which he discussed so much in his early works.¹¹¹ Before doing so, however, we must explain a distinction he makes. In the *AGU* (finished at almost the same time) Lull says that predestination, by the second species of Rule C (= what it has in itself), is an idea (in God), and by the third species (= what it has in another thing) it is a created habit by which the predestined man freely is disposed to do good, just as a person with a cape has been dressed in a new cape or a judge is disposed to judging by the habit of justice. So the first is eternal and the second a "habit" (in the Aristotelian sense) with which the first "habituates" or "dresses" the second, which he calls a "new predestination" (*ROL XIV*, 348).

On this matter I argue thus: Whatever is caused by divine predestination is effectively caused without the necessity of the divine, necessary intellect, etc.; but new predestination is effectively caused by divine pre-

¹¹¹ See, pp. 49ff., 70, 86, and 175ff. above.

destination; therefore new predestination is caused without the necessity of the divine, necessary intellect, etc., which is why new predestination is not necessary.

On the contrary I argue thus: Whatever is effectively caused by divine predestination is not caused without the necessity of divine, necessary intellect, etc.; but new predestination is effectively caused by divine predestination; therefore new predestination is not caused without the necessity of the divine, necessary intellect, etc., which is why new predestination is necessary. (*ROL XIII*, 138)

Llull says this second syllogism is false because it condemns the free will of the sinner to take on a habit of vice as opposed to one of virtuous predestination, which falsity makes the first syllogism true and necessary. Notice how now his reasoning is no longer a matter of an apparent contradiction, as in early works of the Art, but of a theological distinction that must be made.¹¹²

Llull's principal interest in this technique, however seems to have been as a method of argumentation against the Parisian 'Averroists', whose doctrines he says imply evident contradictions.¹¹³ He even goes so far as to write a *Liber contradictionis* in which, in a setting reminiscent of the *Book of the Gentile and the Three Wise Men*, a Raymundist and an Averroist decide to go to a lovely field near Paris, where there is a beautiful tree beside a spring, to dispute in a peaceful setting.¹¹⁴ There too they meet a lovely lady, whose name, however, is not Intelligence, as in the earlier work, but Contradiction. She explains who she is by, as it were, defining herself, saying that in her essence lie two species: one intensive governed by the impossible as something which is and is not at the same time, which was and was not, and which is white and while it is white it is black; the other extensive governed by the possible and originating in accident (as opposed to substance), like the

¹¹² A distinction, curiously enough, which I have only been able to find in these two works of 1308, the *AGU* and the *Ars compendiosa Dei*, with the possible exception of the *Liber de praedestinatione et praescientia* of 1310 (*ROL VI*, 162–175), which has a somewhat more complicated gradation of predestination. Aquinas, *Summa theologica* I.1, q. 23, art. 5, makes a vaguely similar distinction between God's predestination as cause, and the effect on a person as a result of his merits.

¹¹³ See for instance *Liber de perversione entis removenda* (*ROL V*, 501), cited in Imbach 1987, 265, which provides a close analysis of Llull's anti-Averroist campaign of 1309–1311.

¹¹⁴ *ROL VII*, 138. Note, as Imbach points out, that the dispute is not between Raymond and Averroes, as in the earlier *Declaratio* where it was between Raymond and Socrates, but between a follower of each.

contrariety between hot and cold, between true and false, etc. The body of the work, curiously, consists of a hundred syllogisms propounded by the Raymundist, to each of which the Averroists has promised he will answer, presumably in a separate work. The following work, the *Liber de syllogismis contradictoriis*, in fact takes up where the first left off, talking about the promised hundred counter-syllogisms. But now the Raymundist says that instead he wants “to do twenty syllogisms, ten for me and ten against me” in order to see “who is on the side of truth, you or I” and he will then use these contrary syllogisms to prove the falsity of forty-four propositions to which the ‘Averroists’ adhere.¹¹⁵ The twenty initial syllogisms come in ten pairs of contradictory syllogisms based on the nine dignities plus “perfection”. We will look at the fourth which uses “power” and is based on an initial premise which he often calls a “maxim”: “divine power is most powerful”, on which he constructs the two following syllogisms:

Whatever is a most powerful power (*potestas potentissima*) can (*potest*) exist and act most powerfully (*potentissime*); God is a most powerful power; therefore God can exist and act most powerfully. As a consequence it follows that there is nothing which can force God not to be most powerfully existing and acting, and that there is nothing which can get around this with infinite empowering (*posse*).

On the contrary it is argued that divine power is not most powerful. Whatever is not a most powerful power cannot exist and act most powerfully; God is not a most powerful power; therefore God cannot exist and act most powerfully. As a consequence it follows that there is something opposing him of equal or greater potency which can impede him and get around this infinite empowering (*posse*), with an empowering (*posse*) equal to his or higher. (*ROL VII*, 174)

In the second distinction he presents forty-four Averroist theses, of which the first was one that seems to have most worried Llull, namely that God is not of infinite vigor.¹¹⁶ The contemporary debate on this

¹¹⁵ *ROL VII*, 170–1. This list also appears in the spurious *De erroribus Averrois et Aristotelis* (printed in *ROL VIII*, 254–7) which gives the source for each error in those two authors. Cf. Imbach 1987, 266ff.

¹¹⁶ See Imbach 1987, 276–280, where Llull’s reaction to this thesis is treated in detail. He begins by saying: “Cette thèse est, selon mes relevés statistiques, la seule dont Lulle fasse mention chaque fois qu’il entreprend d’énumérer ou de décrire la philosophie critiquée et cela dès le début.” I have translated the Latin *vigor* by the English cognate, in spite of its awkwardness, because it was used in this context in a technical sense to describe God’s cosmological action as prime mover.

question concerned how Aristotle's notion of God as a prime mover, immaterial and immobile, could impel the heavens with a movement that was infinite in time but finite in speed. Averroes' answer was to say that the "vigor" of the immaterial prime mover could be neither finite nor infinite, since these are attributes of bodies. In another place Averroes said that the infinity Aristotle had meant to demonstrate was not that of the intensive vigor of the prime mover, but only of its duration.¹¹⁷ But, as Anneliese Maier pointed out, the main problem for Christian writers was to know if one could prove this thesis by reason, i.e. by cosmological arguments.¹¹⁸

Llull's opposition to this thesis went much deeper than mere cosmological concerns,¹¹⁹ since it attacked his conception of God on two absolutely fundamental grounds. Or as Ruedi Imbach puts it, he is much more interested in the ultimate consequences of the Averroist position than in its details.¹²⁰ In the first place it implied a Deity for whom being and quietude were synonymous, which was the opposite of that proposed by Llull, in which being and acting are convertible. And secondly, it limited the absolute perfection of the divine attributes. To put it another way, Llull was much more interested in pointing out Averroist contradictions on theological than on cosmological grounds.

So here he begins the second distinction saying that God's infinite vigor is proved by the ten preceding pairs of syllogisms, positively by the first of each pair, and its negation disproved by the second of the pair. Of these, the fourth pair we have just quoted was clearly the most relevant, and indeed, if one accepts the superlative nature of the divinity therein implied, Llull's argument is cogent. In the second part of his demonstration he just expands on the last consequence expressed above, saying that since everything in God is optimal, maximal, most eternal, and most powerful,¹²¹ anything that impedes his infinite vigor must also be optimal, maximal, most eternal, and most powerful, from

¹¹⁷ The first in *In Phys.* VIII, comm. 79, and the second in *De substantia orbis*, ch. 3, cited by Imbach 1987, 276–7. For an excellent résumé of the different positions on this question of thirteenth- and fourteenth-century thinkers, see Jung-Palczewska 1997.

¹¹⁸ Maier 1955, 235–6.

¹¹⁹ Although he was not unaware of these concerns. In the *Liber de ente quod simpliciter est per se et propter se existens et agens* (ROL VIII, 212–4) he refers the reader to Aristotle's *On the Heavens*, II, and the end of the passages takes up some specific points of Averroes' commentary.

¹²⁰ Imbach 1987, 279.

¹²¹ These are the superlatives of the first four dignities, *optimus* from *bonus*, and *maximus* from *magnus*. See further on for Llull proofs based on superlatives.

which it would follow that necessarily the best (*optimum*) was the worst (*pessimum*), the maximum the minimum, etc., or in other words, a series of impossible contradictions.¹²²

And so he goes, arguing against disbelief or doubts about the provability of the Trinity, the Incarnation, the creation, or against the Averroist theses about the eternity of the world, the unicity of the active intellect, etc., basing his arguments on the previous ten opposing syllogisms, on their agreement with the first or disagreement with the second.

Hypotheses and Contradictory Suppositions

A technique similar to that of the previous section, but now without syllogisms and in general less elaborate, is that of contradictory suppositions. This technique in turn is dependent on what Llull calls the *demonstratio per hypothesim*.

From the very beginning, as we have seen, the Art was based on the testing of hypotheses or suppositions.¹²³ In three works of that pivotal year of 1308, in which Llull leaves one path and sets out on other new ones, he formalizes—or perhaps it would be better to say, systematizes and theoretically justifies—this technique. They are the *Liber de novis fallaciis*, the *Liber de experientia realitatis Artis*, and the *Liber de refugio intellectus*.¹²⁴ In the second of these works he explains that *demonstratio per hypothesim* is one of four kinds of demonstration (along with the familiar three of *propter quid*, *quia*, and *per aequiparantiam* we already know).¹²⁵ Interestingly, he derives it from a passage of Isaiah he quotes often: “If you do not believe, you will not understand.”¹²⁶ It is therefore, as in

¹²² *ROL* VII, 178.

¹²³ For ‘supposition’ as a literal translation of the Greek ‘hypothesis’, see p. 82 above. See Ruiz Simon 1999, 216ff. for the development of demonstrations based on hypotheses ever since the first work of the Art, the *ACIV*.

¹²⁴ The passages referring to or making use of the *demonstratio per hypothesim* from these three works are all in *ROL* XI: (1) 51, 56, 64–93, 111; (2) 187–190, 198, 220–1; (3) 232–246, 260, 294–6, 298–303. It is briefly mentioned in the *Ars compendiosa Dei* (*ROL* XIII, 107).

¹²⁵ *ROL* XI, 220–1.

¹²⁶ *Ibid.* *Nisi credideritis, non intelligetis* (*Isaiah* 7,9), which is the translation from the Greek *Septuagint*; the Vulgate, however, translated from the Hebrew has *Si non credideritis, non permanebitis*, which in the King James Bible became “If ye will not believe, surely

most of his works dealing with theological matters, a method of testing a proposition accepted on the basis of faith. To take two examples from the first work, he says:

The second position is this one: I firmly believe and have true faith that all God's dignities (*rationes*), considered as subject and predicate, convert with one another in identity of number

The fourth position is: I firmly believe and have true faith that all God's dignities are in themselves real beings existing outside the mind (*extra animam*).¹²⁷

And then in the next section Lull proceeds to demonstrate these (sub)positions using the method described in the next section. All this, as I have said, is little but a systematization of one of the basic formulations of the Art. The new twist comes with his use of contradictory suppositions. They first appear in the *Liber de modo naturali intelligendi* of 1310,¹²⁸ but it is with the *Liber facilis scientiae* of a year later that they are used most consistently, the entire work being based on contrary suppositions. As an example we can take his treatment of the question of God's infinite vigor:

I suppose that it is good, great, and true that divine goodness has as much power with respect to itself as divine eternity with respect to itself. And if the contrary supposition were good, great, and true, it would necessarily follow that it would be good, great, and true for divine power to be divided, infinite with respect to eternity in duration but finite with respect to goodness in bonifying, which is false and impossible since it would run counter to the previous definitions. This demonstrates that divine goodness is as infinite in vigor with respect to bonifying as with respect to duration.¹²⁹

In the questions at the end of the work he asks whether God is of infinite vigor, and answers by referring the reader to the above passage,

ye will not be established." Cf. Solà Simon 2003. All of this is, of course, of prime importance for Lull's position on the problem of faith vs. reason.

¹²⁷ *ROL* XI, 65. "Position" here of course refers to "supposition".

¹²⁸ *ROL* VI, 210ff. The same ones are repeated, but with expanded explanations in the *Liber in quo declaratur quod fides sancta catholica est magis probabilis* (see *ROL* VI, 344ff.).

¹²⁹ *ROL* VII, 306–7. The "previous definitions" are given at the beginning of the work by the example: "What is divine goodness, to which one answers that divine goodness is God and it is greatness, eternity, etc." "Run counter to" is a free translation of *laedere* "injure, harm", for which see above, Ch. IV, n. 35 above.

adding that “if not, the contrary suppositions would not imply a contradiction, which is impossible.”¹³⁰

As the reader can see, since the previous contrary syllogisms usually rely on contradictory premises, what we have here is a collapsing of that mechanism into something much simpler where the contradictory supposition is dealt with directly, without any necessary reference to the apparatus of syllogistics. The result is a more methodically presented proof of the kind we analyzed for the *AD*, where the initial hypothesis is proved by disproving its negation.

*The Fallacy of Apparent Contradiction*¹³¹

We have seen how the Art can be seen as a reworking of Aristotle’s *Topics*, and how at a certain point Lull began trying to adapt the *Prior* and *Posterior Analytics* to his needs. What is a bit more surprising is that at the same time he began to see if he could use the fallacies of Aristotle’s *Sophistical Refutations* as a kind of logical testing device.

We have also seen how from the beginning of his career Lull was preoccupied with apparent contradiction,¹³² now, however, he introduces the problem in a specifically logical context and for a different purpose. He does it first in the *LN* of 1303, but still in a somewhat embryonic form and as something to which he seems to attach no particular importance.¹³³ Then suddenly five years later, contemporaneously with

¹³⁰ *ROL* VII, 331. These questions, identical in form to the questions at the end of so many other Lullian works, have in this case often been treated as a separate work entitled *Quaestiones factae supra Librum facilis scientiae*, with no justification in the older sources.

¹³¹ Ruiz Simon 1999, 136–183, is particularly good on situating this Lullian procedure within the general context of his treatment of apparent contradictions. See also Johnston 1987, 263–289, and Bonner 1995b. Platzeck 1962–4, I, 428–445, although somewhat uncentered and going too far in calling it the *lex lulliana*, has some useful information. Hintikka 1987 has interesting suggestions on fallacies as dialectical strategies, or as he puts it (p. 211), as “breaches of rules in the knowledge-seeking questioning games which were practiced in Plato’s Academy and later in Aristotle’s Lyceum”, some of which might be applicable to Lull’s case.

¹³² See in the discussion on predestination vs. free will in Ch. 2, p. 49 above.

¹³³ *ROL* XXIII, 128, 178; *NEORL* IV, 111, 162. He does it in two sections, one following the standard thirteen fallacies (which is why here he calls it the “Fourteenth Fallacy”), and another as a sort of appendix to the work with theological examples, for all of which see the text below at note 155. This, however, was not the only name he gave it. Most often he simply calls it the “New Fallacy”, but in the *Liber de novis fallaciis* (and subsequently) he calls it the “Twentieth Fallacy”, because there he applies

the other new developments we already encountered, he brings it into high prominence and elaborate development in the *Liber de novis fallaciis* and with the *Liber de refugio intellectus*.

To see how this fallacy works, it might be best to start with an example which involves a standard syllogism in Celarent followed by Lull's reworking of it:

No animal is a stone; every man is an animal; therefore no man is a stone.

No animal is a stone; certain animals are stone; therefore certain animals are stone and are not stone.¹³⁴

The first thing to note is how Lull has formalized his new fallacy, with the premises composed of a universal negative (no P is Q) and a particular affirmative (some P is Q), giving an apparently contradictory conclusion (some P is Q and is not Q). This is in fact why it is called the fallacy of apparent contradiction, because, as Lull explains, "it seems to conclude in a contradiction but does not in fact do so."¹³⁵ What is contradictory, however, and this formally according to the traditional square of opposition derived from Aristotle, are the two premises. Since, in Lull's formulation, the minor premise is always true, this not only makes the major premise false, but forces us to find what is making it so. Lull explains it saying:

The major premise is two-fold, since no animal capable of sensation is stone. Certain "fictitious" animals, however, are of stone, as for instance a stone statue of an animal which is so called. The minor premise is simply true, and by taking the second meaning of the major premise, it excludes the first. It is therefore clear how the first syllogism is sophistical and the second declarative. (*ROL XI*, 53)

This explanation is equally formalized, almost always beginning "The major premise is two-fold" followed by an explanation of the case in hand, then by "The minor premise is simply true, and by taking the second meaning of the major premise, it excludes the first," ending

it first to the 19 Rules and Subjects (10 of one and 9 of the other) and then to the 19 syllogistic moods of Aristotle (cf. *ROL XI*, 47, 48, 52, 56, 94, 105, and 113). See Bonner 1995b, 461–2 for these changes.

¹³⁴ *ROL XI*, 53. The first syllogism is a standard medieval example of Celarent; see Peter of Spain 1972, 47, and William of Sherwood 1983, 243. Lull had previously cited it in the *LN* (*ROL XXIII*, 112; *NEORL IV*, 94).

¹³⁵ *ROL XI*, 232. Note also that it forms a syllogism without a middle (term).

with what this implies for the original syllogism (or other question being examined).

As Ruiz Simon has pointed out,¹³⁶ Lull's new fallacy bears a certain similarity to the classical one of *ignorantia elenchi* in two ways. In the first place, there is a similarity of form, in that both have contradictory conclusions, as we can see from a standard example Lull gives of the *ignorantia elenchi*:

Two is twice one and is not twice three; therefore two is twice and not twice.¹³⁷

In this one, however, the major and minor premises are opposed semantically, not formally and semantically as in Lull. And whereas in the classical case, the truth values of premises and conclusion are T–T–F, with Lull, where the contradiction of the conclusion is only apparent, they are (usually) F/T–T–T, where F/T refers to the “two-fold” nature of the major premise discussed below. The second point of similarity is Aristotle's suggestion that the fallacy of *ignorantia elenchi* could be used as kind of general fallacy to resolve all the others, something which Lull claims, as we will see in a moment, for his new fallacy.¹³⁸

What is surprising in the case just presented (and might even seem like gratuitous debunking of the opposition) is Lull's calling the classical example of a syllogism in Celarent “sophistical”. At the beginning of the *Liber de novis fallaciis*, however, he says how “a single thing can be represented by several words, and how a same word can include many things” (*ROL XI*, 12), and it is this distinction between the referents of a word which the original syllogism about animals being and not being stone does not take into account. Looked at this way, the original syllogism is indeed “sophistical”, since its major premise is equally “two-fold”. That this, for Lull, involves much more than merely playing contemporary theoretical games with or scoring off others, should be clear from his explanation at the beginning of the *Liber de novis fallaciis*:

¹³⁶ Ruiz Simon 1999, 171.

¹³⁷ *ROL XI*, 50. Cf. Peter of Spain 1972, 164, and Aristotle, *Sophistical Refutations*, 167a28–31.

¹³⁸ Aristotle, *Sophistical Refutations*, 168a19–21. See Ruiz Simon 1999, 171 n. 149 for the almost identical formulations of this possibility in Aristotle and Lull. Peter of Spain 1972, 180–4, has a long exposé of this possibility.

Since the intellect has two acts, believing and understanding (*intelligere*), and in generating science it believes before it understands so that it may work successively towards its goal. Hence it is in its first act that fallacies takes place, and that, as a result, opinions arise. And thus in the major proposition of this paralogism will appear the cause of the apparentness, while the cause of its defect appears in the minor, concluding in a real truth. (Ibid.)

What is behind this programmatic statement is a matter of prime importance, revealed by a fallacy such as the following:

No being has itself in itself; some being has itself in itself; therefore some being has and does not have itself in itself.

Which Lull explains, saying:

The major premise is two-fold, since in one way it can be understood as applying to created things, in which habit and the thing habituated differ like accident and subject. But in another way, in God, insofar as God the Father has in himself God the Son, he has himself in himself by his essence, in which they don't differ as habit and something habituated, but as the begetter and begotten, existing (thus) through the divine essence. (*ROL XI*, 16)

Here the distinction is crucial, since it applies to that between the created and uncreated realms, and, for Lull, any system of knowledge which is not general enough to take in both, is not worthy of the name of "science". To put it another way, there is a gap between theology and Aristotelian logic which needed to be closed, and this is what he proposed to do with his new fallacy. In the process, as we will explain on p. 244 below, he is tackling a problem of Trinitarian predication that would be a matter of concern for fourteenth-century theologians.

Lull, however, goes further and says that in every area of thought we find the possibility of producing fallacies (and opinions), and he uses this new fallacy as a kind of detective and corrective mechanism, to find them and explain how they can be reworked to make true statements (and science).¹³⁹ To show the generality of the problem, in the *Liber de novis fallaciis* he starts applying it to the ten Questions and nine Subjects of the Art, goes on to the thirteen classical fallacies and the nineteen moods of the three figures of standard logic, and ends applying it to the definitions of the eighteen Principles of the Art and

¹³⁹ See p. 278 below.

a host of other questions, including, as we will see in a moment, some taken literally from the *Disputatio Raimundi christiani et Homeri saraceni*.¹⁴⁰ The *Liber de refugio intellectus* again begins with the nine Subjects of the Art, but then branches out into the seven liberal arts, medicine, nature, theology, and ends solving questions by Thomas Aquinas, Richard of Middleton, Giles of Rome, and a certain Martinus Anglicus, in addition to answering or solving many other questions.¹⁴¹

To get an idea how he carries out this program, in the opening distinction of the *Liber de novis fallaciis* devoted to the Questions of the Art, under that of “What?” (*Quid?*), for its first species, that of definition, Lull give the example of:

No definition is the thing defined; some definition is the thing defined; therefore some definition is and is not the thing defined. (*ROL XI*, 15)

Lull, as usual, explains that the major premise is two-fold. On the one hand it is true in the sense that the reference (*signum*) is not the referent (*signatum*), but false in the sense that both have to do with the same thing. And, as is always the case with Lull’s fallacy of apparent contradiction, the minor premise is simply true, thereby sorting out the problem with the major.¹⁴²

Under the second species of “What a thing has in itself”, we make an abrupt change from semiotics to theology, with the example of “No being has itself in itself . . .” which we studied above.

Another example, along with Lull’s explanation, can be found in the third Question, “Of what?”, whose second species asks “What is a thing composed of?”

No *-bile* has to do with form; some *-bile* has to do with form; therefore some *-bile* has to do with form and nothing to do with form.

The major premise is two-fold. In the first case it is understood as being in creatures, in which matter and form are different essences. In the second

¹⁴⁰ See below p. 241.

¹⁴¹ Both works were edited in *ROL XI*, with interesting introductions by Charles Lohr. As he points out, the second work is described by Lull as *de quolibet* (pp. 225 and 232). A third work using the fallacy of apparent contradiction, the *Excusatio Raimundi*, appears in the same volume, but as Lohr points out (p. 333), “it is made up exclusively of excerpts from” the *Liber de refugio intellectus*.

¹⁴² Notice too what was said above, in the passage preceding n. 34, about this fallacy constituting the greatest exception to Lull’s usual treatment of signification.

case it is understood as being in God, in which the Son is generable from the form of the Father, divested of all matter.¹⁴³

Notice that what he is sorting out with this new technique is not just another exposition of the Rules and Questions, but rather of the problems they could bring on. And it is significant how so many of those problems turn out to be theological, and specifically related to the problem we mentioned before of possible fallacies when predicating something in both the created and uncreated realms.

To see how he applies his new fallacy to resolving the thirteen standard fallacies of medieval textbooks, we can take examples from the first two.¹⁴⁴ The first was called that of “equivocation” (that is, of not properly distinguishing different meanings of a noun), and the second of “amphiboly” (not properly distinguishing different meanings of a phrase, in the example below those of the implications of the possessive). In standard form they are:

Every dog can bark; the celestial star is a dog; therefore the celestial star can bark.

Whatever is Aristotle’s is owned by Aristotle; but this book is Aristotle’s; therefore it is owned by Aristotle.¹⁴⁵

With his ‘new fallacy’, Llull rephrases them as:

No dog can bark; but a certain dog can bark; therefore a certain dog can bark and [a certain dog] cannot bark.

Nothing which is Aristotle’s is owned by Aristotle; but something which is Aristotle’s is owned by Aristotle; therefore something which is Aristotle’s is owned and not owned by Aristotle.

In each case Llull explains that the two “major premises are contrary (to one another), and therefore the classic conclusion is simply (*simpliciter*) false since it applies to one and the same thing. The new conclusion,

¹⁴³ Ibid., 17. The *-bile* is the correlative corresponding to matter, as opposed to the *-tivum* of form. See in Ch. 3 above, the paragraph preceding n. 45.

¹⁴⁴ “Textbooks” in the sense that they appear in Peter of Spain 1972, these two on pp. 100 and 107 (for the first, instead of Sirius, the dog star, he has the dog-fish or shark). Both had previously appeared in the *LN* (*ROL* XXIII, 115 and 116; *NEORL* IV, 97, 99), where Llull explains all these possibilities, saying that “the word. dog has several different meanings, as for instance a dog having a tongue and living on land; a fish lacking a tongue and living in water; a dog in the sky not living.”

¹⁴⁵ *ROL* XI, 48, where we have had to correct Llull’s rephrasing of the second according to the text of the earliest manuscripts.

however, is simply true because it refers to different things.” Again it is a question of referents: do “every dog” or “Aristotle’s book” each refer to a single type, or have we in the first example above confused a flesh and blood dog with an astronomical one, and in the second one something he wrote or to a physical object he owned? Similarly, this fallacy is often expressed as pairs of contrasting syllogisms, but here they are not contradictory with the second false one proving the truth of the first; here it is the second true one which reveals the problems which caused the first to appear false.

But this is not always the case; sometimes the fallacy is used to corroborate the truth of an original syllogism, one in turn based on a hypothesis, as is clear from the following example, consisting of a hypothesis, a syllogism and a paralogism:

I firmly believe and have true faith that all God’s dignities (*rationes*) exist outside the mind and are real entities. To this end I argue thus:

All that by which God can most act with his dignities is necessary; but that is the reason they are real things; therefore the reality of God’s dignities is necessary.

Nothing of that by which God can most act with his dignities is necessary; but something of that by which God can most act with his dignities is necessary; therefore something of that by which God can most act with his dignities is necessary and is not necessary.

The major premise of the paralogism is simply false, since it is directly contrary to the major premise of the syllogism, which was true through a true hypothesis. But the minor premise of the paralogism is true since it directly contradicts its own major premise. For this reason the conclusion of the syllogism is true. (*Ibid.*, 73)

So here it does follow the pattern of contrasting syllogisms, with the falsity of the second proving the truth of the first.

Finally, there is Lull’s use of this new fallacy for religious polemic. As we said before, among the questions of the *Liber de novis fallaciis* there is a section answering some already dealt with in the *Disputatio Raimundi christiani et Homeri saraceni*.¹⁴⁶ In the first part of the *Disputatio* the Saracen expounds his view on a series of problems, defending each

¹⁴⁶ The section in *ROL* XI, 114–121 of the *Liber de novis fallaciis* begins by quoting literally the opening of the other work: “In the book which has as its title: ‘God, with your grace and blessing, here begins *Liber disputationis Raimundi Christiani et Homeri Saraceni*’ . . .”, in a manner reminiscent of the way the opening of the *AB* refers to the *AGU* in the quotation at the beginning of Ch. 4 above.

of them at length. In the second part the Christian answers them one by one, with corresponding defenses of each of his answers.¹⁴⁷ In the *Liber de novis fallaciis* Lull kaleidoscopes this arrangement, giving argument and reply one after the other, as in the following example (*ROL* XI, 115–6):

The Saracen said: Every being existing as one by an infinite and eternal unity is separated from and divested of all plurality; but God is a being existing as one by infinity and eternity of unity; therefore God is separated from and divested of all plurality.¹⁴⁸

Against this the Christian (argued): No unity is infinite without an infinite act; but God's unity is infinite; therefore God's unity has an infinite act.¹⁴⁹

Then, for the separate defenses of the two positions he can substitute a single fallacy of apparent contradiction expressing both:

As a fallacy this is expressed as follows: No being existing as one by an infinite and eternal unity is separated from and divested of all plurality; but a certain being existing as one by an infinite and eternal unity is separated from and divested of all plurality; therefore a certain being existing by an infinite and eternal unity is and is not separated from and divested of all plurality.

Which he goes on, as usual, to explain that the problem is one of two-fold reference: if it refers to plurality of essences it is impossible, if to plurality of persons then it is true.

Since without plurality of persons the divine unity could not have an intrinsically infinite unifying (*unire*), which in turn requires intrinsically infinite unifier (*uniente*) and unified (*unibili*). For without these three cor-relatives, said divine unity could not be infinite, since it would be empty, inactive, and contingent (*extra naturam*).

So with his fallacy of apparent contradiction Lull has managed to conflate opposing arguments into a single syllogism, one which exposes the nexus of the problem, much simplifying the task of clarifying what is true or false and in which situations.

Now that we have seen how Lull uses this new fallacy, and the generality of its possible applications as a kind of system of error

¹⁴⁷ *ROL* XXII, 176ff. and 196ff.

¹⁴⁸ This syllogism is a literal citation of the one from the *Disputatio* in *ROL* XXII, 177.

¹⁴⁹ This one is a paraphrase of the one in *ibid.*, 198–9.

detection, three things about it remain surprising. The first is that one would have thought it also would be directed towards combating the contradictions Llull felt were inherent in the doctrines of the Parisian ‘Averroists’, but for the moment, at least, this does not seem to have been the case. It was not till two years later, when he was already in Paris, that he used this technique to argue with them, but then only in a little three-page appendix to one work: *De fallaciis quas non credunt facere aliqui qui credunt esse philosophantes*.¹⁵⁰ Even there he doesn’t specifically mention the ‘Averroists’, but it’s clear from the context of an opposition between Catholic theologians and philosophers, as well as the subject-matter of the questions discussed, such as God’s infinite vigor or the creation of the world, that they are his target.¹⁵¹

The second mystery is in itself an apparent contradiction: we are presented a method, to which initially Llull seems to have attached considerable importance, presenting it as he does in two long works, the *Liber de novis fallaciis* and the *Liber de refugio intellectus*, in a detailed, general manner, one using his Art, and which he claims is applicable to all fields, but then which he scarcely ever uses again (aside from the other three pages just mentioned).¹⁵² The short life-span of this new fallacy is in strong contrast with that of the other logical techniques described in this chapter, which were used from their beginnings in 1308 (or, in some cases, 1303) right up to the end of his career.

The third mystery is why the second of these two works (along with the *Excusatio Raimundi* extracted from it), should really constitute the only place in Llull’s vast body of works in which he argues by name with contemporary theologians, with figures—as we have already pointed out—like Thomas Aquinas, Giles of Rome, Richard of Middleton, and a certain Martinus Anglicus.¹⁵³

¹⁵⁰ Printed in *ROL VI*, 478–488, with the “Fallacia Raimundi” on pp. 485–488, at the end of which he refers to their sort of thinkers as “sophists”.

¹⁵¹ See Ruiz Simon 1999, 175–6. One might also have expected Llull to have applied the fallacy of apparent contradiction to his old question of predestination vs. free will, but the subject is not mentioned in the *Liber de novis fallaciis*.

¹⁵² That is, except for a few brief references. Four of the new fallacies turn up in a little two-page treatise also of 1308, *Liber de conversione syllogismi opinativi in demonstrativum cum vicesima fallacia* (*ROL XI*, 328–9). In the *AGU* (*ROL XIV*, 116–7) this fallacy is not actually used, but explained in terms of the Multiplication of the Fourth Figure. And finally, in the *Liber de modo naturali intelligendi* of 1310 (*ROL VI*, 198), he lists his new fallacies as one of three types of paralogisms.

¹⁵³ See the passage preceding n. 141 above. Lohr (*ROL XI*, 225) says the last may be Martin of Alnwick; for the other names, see the index of *ROL XI*.

Trinitarian Predication?

As we suggested above, there is the intriguing possibility that Llull was also using the fallacy of apparent contradiction to treat a problem that from Duns Scotus on became a subject of debate in Paris and Oxford: the possibility of paralogsms arising from the application of Aristotelian logic to theology, and especially to the doctrine of the Trinity. As an example of the problems that could arise, the fact that one could say “The essence is the Son” but not “The Father is the Son”, made for a serious dilemma. Since the terms “essence” and “Father” signify the same thing in these propositions, then we are saying that contradictory predicates like “son” and “not-son” could be predicated of one and the same thing.¹⁵⁴ We have seen above on p. 238 how Llull in the *Liber de novis fallaciis* discusses the apparent contradiction of “some being has and does not have itself in itself”.

The same problem was dealt with earlier in the *LN*.

No being is that which it has: some being is that which it has; therefore some being is that which it has and is not that which it has. This is not valid because God the Father has God the Son who is God himself. And this is signified by the first, second, third, and fourth species of Rule C. (*ROL* XXIII, 178; *NEORL* IV, 162)

Now this paralogism arises in a curious little appendix tacked on to the *LN* after the inevitable closing section of questions. After four examples of how to form syllogisms according to the four species of Rule C of the Art, he gives nine examples based on the nine Rules of the Art of his fallacy of apparent contradiction, which here he calls the “Fourteenth Fallacy”.¹⁵⁵ All of the second set of examples are paralogsms arising from predication in the divine realm, some of which are applicable to

¹⁵⁴ The example is from Gelber 1983, 25. Maierù 1981, 484, n. 11, cites a similar contradiction mentioned by the later fourteenth-century thinker, Henry Totting of Oyta: “The divine essence does not generate nor is generated” whereas “The Son is generated from the substance of the Father”. Maierù 1986, 189, explains how the same author offers a solution, explaining that it is incorrect to say “the Son is the Father”, but not that “the Son is the same as the Father”. A century before Totting, Bonaventure (Maierù 1988, 252) was already concerned with statements such as: “the essence is the Father” and “the essence begets.” (I was unable to consult the unpublished doctoral dissertation, Gelber 1974, which initiated much of the subsequent research on the question.)

¹⁵⁵ Nine Rules because he omits the first of *Utrum*. See p. 139 above.

the problem of Trinitarian predication, such as the one just cited, and these two others.

Every son is of himself; you are a son; therefore you are the son of yourself. This deception is due to the first and second species of Rule D, and is an invalid paralogism, because the first species of Rule D and the second do not have the same meaning, since they are different. (Ibid.)

Wherever there exists property there exists accident. In some being there is property and no accident; therefore in some being there is accident and there is no accident. This is not valid because in God there is property and no accident, as is signified by the first and second species of Rule C. (Ibid.)

But to what degree such concerns arise from a desire on Llull's part to enter the fray of academic discourse or are merely a case of spontaneous generation of a similar discussion, would involve a detailed comparison of Llull's writings with those of his contemporaries. In any case, it does not seem to have been a primary concern of his, but more a by-product of his other aims.

Proofs by the Superlative Degree

As opposed to the previous sections of this chapter, all of which have dealt with the forms of Llull's arguments, this new section will deal with a new approach to their foundations. To call it new is in a certain sense inaccurate, since from the beginning he has in fact been using arguments based on the *via eminentiae*, or the idea of the supremacy of God and his attributes.¹⁵⁶ What is new is its systematic use as an absolute, the realization of its implications for theological demonstrations, and its increasing importance as a mode of demonstration.

It becomes systematized in part by the use of the superlative with the divine attributes, so that in the *Ars mystica* Llull can now define God "not by causes, but by his principles, dignities or reasons" by means of the following definitions:¹⁵⁷

¹⁵⁶ For its use in earlier works see Longpré 1926, col. 1116, and Gayà 1979, 25, 29, 34, 37–40. Sala-Molins, in Lulle 1967, 104, refers to the "typically Anselmian" conditions of the first tree of the *Book of the Gentile*.

¹⁵⁷ *ROL* V, 291. I have given the definitions in Latin, to show clearly the superlative forms.

- | | | |
|---|--|--|
| 3. <i>Deus est bonitas optima</i> | 12. <i>Deus est distinctio distinctissima Deus est operatio operatissima</i> | 1. <i>Deus est unitas unissima</i> |
| 4. <i>Deus est magnitudo maxima</i> | 13. <i>Deus est concordantia concordantissima</i> | 2. <i>Deus est operatio operatissima</i> |
| 5. <i>Deus est aeternitas aeternalissima</i> | 14. <i>Deus est principium principalissimum</i> | 15. <i>Deus est perfectio perfectissima</i> |
| 6. <i>Deus est potestas potentialissima</i> | | 16. <i>Deus est substantia substantialissima</i> |
| 7. <i>Deus est intellectus intellectissimus</i> | | |
| 8. <i>Deus est voluntas volissima</i> | | |
| 9. <i>Deus est virtus virtuosissima</i> | | |
| 10. <i>Deus est veritas verissima</i> | | |
| 11. <i>Deus est gloria gloriosissima</i> | | |

Llull's list, presented in the order of the numbers above, has here been arbitrarily divided into three columns to show how in the post-Art phase he feels free to add to the basic components of the Art. In the first column are the usual nine Principles of Figure A; in the second, three components of Figure T; and in the third, four new dignities of "unity", "operation", "perfection", and "substance", to which he even adds "and so on". For the abandonment of the rigid lists of the ternary phase, see below on p. 252. As we will see there, the two dignities most frequently present from now on are "unity" and "perfection", with the second, of course, particularly important for the superlative degree. In the *Ars compendiosa Dei* he defines it as "that which neither in itself nor from outside itself requires anything else,"¹⁵⁸ a definition very similar to one of God from the *AGU*.¹⁵⁹ Since all the dignities *are* God, this means they are all perfections.

The second thing to note is the superlative form added to each definition.¹⁶⁰ In the same *Ars mystica* Llull not only gives the superlative form to each dignity, but gives it a superlative act, saying that:

¹⁵⁸ *ROL XIII*, 27, where most of the other concepts are defined, in the usual Lullian manner, by their infinitive: "Unity is that being whose proper task is (*proprie competit*) to unite."

¹⁵⁹ See p. 161 above.

¹⁶⁰ Usually expressed with the *-issimus* suffix (except for the already mentioned irregular superlatives of *optimus* and *maximus*). Sometimes he expresses it with the word *summus*, "highest"—God is the highest goodness, etc., or he says that each dignity is

Without these principles remaining in the superlative degree and convertible with one another, God could not exist in the superlative degree, for he is identical with them. It is impossible for these principles not to exist in the superlative degree nor without their proper and secondary acts; that is to say, where unity *unissima* would have its proper act of *unissimare*, and *operatio operatissima* that of *operatissimare*, and optimal goodness that of *optimare*, and maximum greatness that of *maximare*... and perfection *perfectissima* that of *perfectissimare*. (*ROL V*, 290)

He justifies the superlative acts as part of the equivalence between existence and agency, for if the principles did not have them, their own “superlative degree would be destroyed and impossible”. Without them, such dignities as the intellect and the will of God would be fictitious principles generated as objects of perception (*in anima genita objective*), without *intelligissimare* and *volissimare*.

... thus these and other principles would be without their abovementioned proper acts, which consequently would be the case for God. But this is impossible, by which it has been proved that his being is true, real and necessary. (*Ibid.*)

In the *De conversione subjecti et praedicati et medii* he puts it more succinctly:

I suppose that God is *bonitas intellectissima*, *voluntas volissima*, *virtus virtuosissima*, *veritas verissima*, *et gloria gloriosissima*, *perfectio perfectissima*, *simplicitas simplicissima*, *et infinitas infinitissima*. If this position is false, it necessarily follows that the human intellect has a higher virtue in holding God and his reasons as its object than God and his reasons would have in existing. This is impossible because [this object] would not be caused by that first cause since it would be objectively above it. Therefore the supposition is primary, true and necessary, since it is made up of primary, true and necessary principals.¹⁶¹

Here we no longer have a kind of Neoplatonic equivalence between positive moral, spiritual, and ontological values, such as the concordance between goodness, virtue, and being (vs. evil, vice, and non-being), nor are we dealing with something vaguely Anselmian, in which the more positive the qualities, the more they should be affirmed. Here we have

“infinite” (see, for instance, *Metaphysica nova et compendiosa*, *ROL VI*, 12ff.) or “absolute” (as in the *Liber de ente simpliciter absoluto* (*ROL XVI*, 190ff.).

¹⁶¹ *ROL VI*, 264. Compare the similar passage in the *Liber lamentationis Philosophiae* (*ROL VII*, 119).

the statement that anything truly superlative in both existence and agency could not be solely the object of our intellection, because then there would be something higher and more superlative: its real existence. This argumentation on the basis of absolute perfection—hence Llull’s new interest in the concept of perfection as a dignity—is now almost identical to the famous Anselmian ontological argument. The main differences are the inclusion of what, for Llull, are the necessary acts of the dignities, as well as a much more rudimentary argumentation. This last point is explainable, I think, by his wish to avoid becoming entangled in the many contemporary discussions of the question,¹⁶² which, as we will explain in the next chapter, he seems to have seen as a kind of smoke-screen obscuring the Christian’s simple but primary task of loving and understanding God.

Because of its foundational nature, arguments based on the superlative degree could be used in combination with the other logical techniques we have just discussed.¹⁶³ Llull also saw it as an important weapon in dealing with Christians, Jews, and Muslims,¹⁶⁴ and indeed he uses it most often for proofs regarding the Trinity and the Incarnation, employing the idea of a highest degree of existence and agency *ad intra* for the first and *ad extra* for the second. To show how he uses it in a syllogistic argument, we give a brief one from the *Ars mystica*, but, as in the above list, without attempting the task of trying to translate the cascade of superlatives. The reader should know that the *-e* ending is adverbial (equivalent to English *-ly*), and the *-a* ending adjectival.

No creator can create a creature *unissime* and *optime* unless he creates one which is *unissima* and *optima*, since one *relatum* cannot be in the superlative degree without the other also being thus; but God is a creator creating a creature *unissime* and *optime*; therefore he creates it *unissima* and *optima*. He could, however, not create such a creature without being joined to it. And that creature is the human nature which God took on when he made himself man. (*ROL* V, 349)

¹⁶² Hartshorne 1965, 154–5, gives a list of fifteen authors between Alexander of Hales and Duns Scotus who discussed Anselm’s ontological argument.

¹⁶³ For instance with contradictory syllogisms in the work of that title (*ROL* VII, 172ff.), with contradictory suppositions in *Liber de novo modo demonstrandi* (*ROL* XVI, 356ff.), and with the fallacy of apparent contradiction in *Liber de refugio intellectus* (*ROL* XI, 298ff.).

¹⁶⁴ In the *Liber de experientia realitatis* (*ROL* XI, 221) he says that all three accept each dignity as a summum, including infinity and eternity, and that once they have conceded this, it is easy to conclude as to the necessity of “production or intensive agency in things divine”. For this use of the term “production”, see Ch. 4, n. 123 above.

Here he justifies the major premise right after presenting it, and after the conclusion he explains the consequence derived from it, which is the Incarnation. The argument is interesting for its typically Lullian insistence on Christ as the bridge between God and man, between the uncreated, infinite realm and that which is created and finite.¹⁶⁵

It would only be natural for Llull to tie in the superlative with the positive and comparative degrees of adjectives. In the *Liber de divina existentia et agentia* he explains that these superlative attributes are causative, whereas as *causabilia* are of two kinds: a positive degree such as the goodness of a stone, and a comparative one such as the goodness of an angel.¹⁶⁶ In the *Ars mystica* he also uses the three degrees of comparison of adjectives to explain the three levels of being, and above all the superlative degree on which the work is based. Here, however, he does it immediately after using the same three degrees of comparison to explain his standard division of demonstration into *quia*, *propter quid*, and *per aequiparantiam*.¹⁶⁷ The connection is not only one of analogy between ontological and logical levels, but also with the fact that the conversion (or “circulation” as Llull puts it here) of the dignities as superlatives is necessarily equivalent to their *aequiparantia*, in that the one implies the other.

In the realm of logic, the three degrees of comparison of adjectives is also used as an analogy for different levels of syllogisms, which again can be classified into how they permit conversion of subject and predicate. In the standard example of a syllogism involving man and animal, these two terms are not convertible and hence the syllogism is positive. One involving man and his risibility or creator and creature, is convertible, although among different levels, and thus comparative. That among the dignities, which involves conversion of terms which operate at an identical level of being is the superlative one.¹⁶⁸ In the earlier *LN* syllogisms can be true, truer or truest according as they deal

¹⁶⁵ See Colomer 1986, 26–26, for a brief, forceful expression of Christ’s role in Lullian thought. For a detailed analysis, see Hughes 2005 and Hughes 2005–6.

¹⁶⁶ *ROL* VIII, 112–3. Cf. Longpré 1926, col. 1116.

¹⁶⁷ *ROL* V, with the three degrees on p. 289, and the three levels of demonstration on pp. 286–7.

¹⁶⁸ *Liber de divina existentia et agentia* (*ROL* VI, 197) and *Ars mystica* (*ROL* V, 342), in this case with an enthymeme.

with sensual-sensual, sensual-spiritual, or spiritual-spiritual, to which he adds a fourth level of those that deal with God's essence.¹⁶⁹

The place where Llull uses the comparison of adjectives most directly is in rhetoric. In the *AGU*, for instance, he says that "the rhetorician adorns his speech in three degrees, namely the positive, comparative and superlative; the comparative adorns it more than the positive, and the superlative more than the comparative. Thus, we can say 'a rose is beautiful', but this expression is better adorned by saying 'a rose is more beautiful than a violet', and even more so by saying 'a rose is the most beautiful flower of all'" (*ROL XIV*, 364–5), a notion developed at greater length in Part II.3 of the *Rhetorica nova*.¹⁷⁰

Sequels

As we have already explained, most of the techniques described above made their appearance in Llull's production in 1308, after he finished the last two works of the Art, the *AGU* and the *AB*, in the first months of that year. As we also said, they seem to have been developed—at least in part—to deal with the Parisian university scene, and especially with the 'Averroist' leanings of parts of the Faculty of Arts. Indeed, it was during his stay there, from the autumn of 1309 to September of 1311, that he developed them most fully. And he continued using these logical techniques in works written up to September 1312, in Vienne

¹⁶⁹ *ROL XXIII*, 105; *NEORL IV*, 87–88. The progression here seems to be a reworking of that of sensual-sensual, sensual-intellectual, or intellectual-intellectual of the green triangle of Figure T (see p. 41 above). The first step is not expressed by Llull as sensual-sensual, which we have inferred from his example and from the other steps. For another version, different again, of the mixture of degrees of comparison with ontology and syllogisms, see the *Ars compendiosa Dei* (*ROL XIII*, 201–3).

¹⁷⁰ The literature on Llull frequently connects the comparison of adjectives with his transcendent points, but strangely enough, in none of the places they cite, nor in any I have been able to find, does Llull do so. The mistake probably came about because the connection seemed plausible. The reason Llull does not make the connection might be because the comparison of adjectives deals with fixed strata, and would thus be an inadequate analogy for the transcendent points which deal with the transition between strata, i.e. how one rises from the sensual to the imaginative, from the imaginative to the intellectual, and from the intellectual to the divine, a process which, in its most developed form in the *AIV*, has many other stages in between. Dealing, as it does, with the steps in the ascent of our knowledge, it is not ontological but epistemological. For Bibliography on the transcendent points, see Ch. 2, n. 96 above.

(south of Lyon, where he attended the Church Council), Montpellier and Majorca.¹⁷¹

With the ten works written in Majorca between then and April of 1313, logical techniques as such are put aside in favor of an almost complete dedication to sermons.¹⁷² It is with the *Liber de compendiosa contemplatione* “begun at sea going from Majorca to Sicily and finished in the city called Messina in the month of May of the year 1313” (*ROL* I, 86) that he begins a series of forty-four works written in Messina and in Tunis during the last two and a half years of his life, almost all of them using the logical techniques we have been discussing.¹⁷³ Because most of them are short works, written when Lull was over eighty (an extraordinarily advanced age for the time), and perhaps because they seem far from the literary and ‘Artistic’ techniques developed in the earlier parts of his career, they have received little attention on the part of specialists. The idea, however, that they constitute minor works, is misleading. By this I don’t mean to imply the opposite—that they are on a level of importance with the *Book of Contemplation*, the *Tree of Science*, or the *AGU*—, but rather that with such an “unquiet spirit”, as Lull has been called,¹⁷⁴ it would be a mistake to imagine that his development stopped at a certain point and that everything written after that was not worth considering. In fact, as we have shown, in the post-Art phase he developed a whole series of new (for him, at least) demonstrational techniques, and it is with these last works that he puts them to the test. In addition to the natural desire of an aged author to concentrate his energies for the little time he has left, their brevity

¹⁷¹ In the catalogue in *SW* II, 1288ff., or on the Lull DB, the works written in Paris are nos. IV.19–47, and those written in the following years using these techniques are IV.49–57 and 59. The Parisian works have been edited in *ROL* V–VIII (see the introduction to *ROL* V for all manner of information on Lull’s two years in Paris), and those of the following year in *ROL* XVI.

¹⁷² Seven works, IV.60–66, containing 182 sermons, edited in *ROL* XV, plus a shorter one (IV.67) on the same subject edited in *ROL* XVIII. The latter volume edits a work on virtues and sins (IV.70) undoubtedly related to Lull’s preaching plans, as well as the only work of this seven-month period which uses syllogistic demonstrations, the *Liber per quem poterit cognosci quae lex sit magis bona* (IV.68).

¹⁷³ IV.72–115, all edited in *ROL* I–II. Of the 44 works, the vast majority, 37 (IV.72–108) were written in his year in Messina (May 1313–May 1314), and 7 in his last six months in Tunis (July–December, 1315). For the mysterious gap in his life (and production) during the year and two months between May 1314 and July 1315, see *SW* I, 50, n. 192.

¹⁷⁴ See the beginning of the next chapter, and n. 1 there.

might be attributable to the fact that these new proofs allow him to pare his demonstrations down to essentials, with little need to present long introductory explanations of the techniques to be used. In any case, they are certainly not without interest.¹⁷⁵

The great majority of these forty-four works are theological, and for the most part dedicated to proving the Trinity and Incarnation, largely by using the logical techniques we have just discussed, rather than by appealing to the generalities of the Art. As a result, the rigid lists so necessary for the combinatory mechanisms of the ternary Art no longer hold sway. In fact, in subsequent works we find different lists, sometimes without the three concepts from Figure T we saw on p. 246 above from the *Ars mystica*, and frequently with the addition of other concepts such as those already mentioned of “unity” and “perfection”, plus “simplicity”, “singularity”, “necessity”, “infinity”, “sanctity”, “action”, etc., with “unity” now by far the most common newcomer.¹⁷⁶ The structures of the Art have given way to theological works in which Llull can tailor his needs to the specific kinds of argument he wishes to generate. Moreover, since Llull’s Principles now refer almost exclusively to the divine realm, they can once again be called Dignities, Reasons (*rationes*), or Principles, as they were in the quaternary phase.

As we have also seen above, it is precisely for theological subjects that Llull uses his logical techniques. A quick survey shows that fifteen of these works use plain syllogisms (some with a defense of the major and minor premises), two others use the technique of contradictory syllogisms, and eight that of contradictory suppositions.¹⁷⁷ Sometimes along with one of these techniques, and sometimes separately, twenty-five are based on the superlative degree (or infinity, or other terms indicating superlativeness). This means that the immense majority of these works use at least one of the techniques we have discussed in this chapter.

To give an example of how Llull does this, and of the variety he can introduce in a theme he has treated so often, we will look at the

¹⁷⁵ They were of interest, for example, to Nicholas of Cusa, who copied, or made excerpts from, nine of them in the manuscript preserved at Bernkastel-Kues, no. 83, which one can consult on the Llull DB.

¹⁷⁶ See the lists in *ROL* II, 468–472, for these added dignities. In the *Liber de civitate mundi* the civil/political theme elicits, in addition to “perfection”, a completely different list of added qualities: “domination”, “justice”, “mercy”, etc.

¹⁷⁷ As we said above in the paragraph referenced in n. 152 above, the fallacy of apparent contradiction appears nowhere here.

Liber de potestate pura finished in January of 1314 in Messina, in which the superlative component is the purity of God's power. He begins the work with a summary of the relation between faith and reason.

It is believed that God is pure act, and that he has pure power. Believing this, we intend to investigate how to understand it, so that we may know that God is pure act and has pure power. And by such an ascent from believing to understanding the human intellect will be distanced from ignorance, from errors and opinions, and will be lofty and profound, and be invested (*habituatus*) with great and useful science. (*ROL* I, 407)

Notice that the step from believing to understanding is that of an ascent.¹⁷⁸ Two paragraphs later, Lull explains what he wants to do:

We have written many books proving the divine Trinity, and each book has its own way of doing so, different from that of any other. In this book we will prove the Trinity by means of pure power, deducing it from pure goodness, greatness, eternity, intellect, will, virtue, truth, glory, sanctity, and perfection. With these ten principles¹⁷⁹ we intend to make a science of the divine Trinity, arguing in new and old ways, and first by belief or supposition, and afterwards by understanding (*Ibid.*, 407–8)

We have, as before, the equation between belief and supposition (or hypothesis), the testing of which has been a constant since the beginning of the Art. But now he uses the technique we have already seen of two contrasting suppositions:

I suppose that it is a great good and a great truth for divine power to be pure, and that to understand and love this is a great good and a great truth. And if the contrary supposition is true, it follows that it is a great good and a great truth for divine power not to be pure, and that to understand and love this is a great good and a great truth, which is false and impossible.

We have proved that divine power is pure, and I therefore argue thus: All pure power is pure empowering (*potens*); God is pure power; therefore God is pure empowering. He would not, however, be pure empowering if he did not produce a pure empowered (*possitum*), and from both a pure act of empowering (*posse*) which would be the procession

¹⁷⁸ A sixteenth-century manuscript has a gloss in the margin: *Ascensus utilissimus de fide ad intellectionem*.

¹⁷⁹ The Principles are ten because "power" has been omitted from the usual list of nine dignities, since it is now what is going to be proved, and two have been added, "sanctity" and "perfection".

(*processum*),¹⁸⁰ for this would make him idle as well as implying a contradiction, namely, that he would be pure empowering and not pure empowering. And since a contradiction cannot be, it is demonstrated that there is pure empowering existing in its own pure number, and that there is pure empowered existing in its own pure number, and similarly for the act of empowering. It is these three pure correlatives that we call the divine Trinity. (Ibid., 408)

Llull's contradictory supposition leads him into a syllogism, which in turn introduces the correlatives of "power" as the basis of his Trinitarian proof. So if the foundations of his arguments are familiar, they continue to be structured in the more strictly logical manner characteristic of the post-Art phase.

If these last works are predominantly theological, and centered on proofs of the Trinity and Incarnation, other themes are not abandoned. We find there two works on logic, *Liber de medio naturali* and the *Liber de quinque praedicabilibus et decem praedicamentis*, with the first using the middle and syllogisms, and the second the superlative degree and contradictory suppositions. At the beginning of his stay in Messina he wrote the *Liber de compendiosa contemplatione*, a manual on contemplation using superlatives and a modified version of contradictory suppositions, and the *Llibre de consolació d'ermità*, which uses superlatives and contradictory syllogisms. Listed coldly this way, one might think that Llull had suffered a kind of category confusion, but read attentively these can be quite moving. Especially the second—one of the few works of this period written in Catalan—, set in a *locus amoenus* and presenting a kind of Saint Anthony in tears because he finds himself unable to disentangle his thoughts from the sinful life he left.¹⁸¹

At the end of his stay in Messina, Llull wrote the *Liber de civitate mundi*, using the superlative degree with a list of sixteen dignities, the nine usual ones plus "perfection", "domination", "justice", "mercy", "grace", "humility", and "piety". Here the *locus amoenus* is a hall made of precious stones, gold, and silver set in the middle of green fields, and inhabited by the sixteen Dignities and their heralds, the seven Virtues. These last deplore the sorry state of the world, but then the

¹⁸⁰ The *possitum* is, of course, the Son, and the *processum* the Holy Ghost.

¹⁸¹ See Spöner 1935 for the Catalan original, and *ROL* I, 94–120, for the two medieval Latin translations.

Dignities (each expressed in superlative degree) offer their counsel.¹⁸² The following work, also on counseling, the *Ars consilii*, the first written in Tunis, surprisingly uses none of these logical techniques, but rather is a return to the Art, albeit in a much modified form, but still with its Alphabet, combinatorics, definitions, Hundred Forms, etc.

¹⁸² Latin text in *ROL* II, 173–201; modern translation into Spanish and Catalan, with extended introduction in Ramis 1992. The rest of the works cited in these two paragraphs are printed in *ROL* I–II.

CHAPTER SIX

OVERVIEW

An Unquiet Spirit

With any thinker one can expect a certain development over time, corrections of original positions which no longer seem valid, or refinements of earlier considerations in the light of new information, but Llull's case is altogether exceptional. Not only did his system go through various quite different stages, but within each stage he was continually trying out new tactics. One could reply that he was proposing methods rather than a body of thought, but this would be begging the question on two counts. In the first place, there is no objective reason why a method should be less stable than a body of thought; in the second place, form and content are never that separable, and, as we have seen in the course of this book and as will be explained further in these conclusions, the content also shifted, perhaps not so frequently and certainly not as to its foundation, but noticeably all the same. So Flasch's characterization of Llull as "ein unruhiger Geist" is not out of place.¹

To attempt an overview of the various stages of Llull's system, however, we must simplify to a certain extent. Perhaps the best way to do so is to chart the course of his development in terms of the titles of four key works, and what those titles would have implied—and what Llull surely intended them to imply—to his contemporaries. They are:

Ars compendiosa inveniendi veritatem (ACIV)
Ars demonstrativa (AD)
Ars generalis ultima (AGU)
*Logica nova (LN)*²

For a contemporary, the words *invenire* or *inventio*, "to find" or "finding",³ in the first of these titles referred to the science of dialectic, as

¹ Flasch 1986, 382.

² Corresponding to the two cycles of the quaternary phase, the ternary phase, and the last roughly to the post-Art phase, but with a five-year overlap with the end of the ternary.

³ The verb is always translated as *trobar* or *atrobar* in Catalan.

expounded in Aristotle's *Topics* and Boethius' *De differentiis topicis*, which involved the attempt to find argumentative strategies, called *loci*, usable in debating, to arrive at probable (as opposed to scientific) conclusions.⁴ For Lull finding a *locus* involved discovering the relation of the thing sought (which he calls the "particular") to certain components of the figures and their combinatory "compartments" (the "universals" of the Art). The difference between this and classical dialectics was that Lull tried to provide a system that went beyond probable conclusions and that could "find the truth", as the title proclaimed. As we pointed out before, however, the *ACIV* gives few theoretical considerations as to how this aim was to be achieved.⁵

In that same place we pointed out that the word "demonstration" did not appear in the *ACIV*, whereas "signify" or "signification" appeared over and over again. We will see in a moment what this represents in terms of Lull's methods, and how at this stage of his career he could even use these concepts synonymously.

What changes with the second work is not so much the addition of demonstrative techniques, which in fact were ever-present in the *ACIV*,⁶ as that of a new theoretical formulation. Contemporary readers' assumption from the title that we were entering the domain of Aristotle's *Posterior Analytics*,⁷ would be confirmed by the prologue, in which, to the Greek author's well known demonstrations of *propter quid* and *quia* is added a third, *per aequiparantiam*.⁸ So to the original *pars inveniendi* of the *ACIV* Lull has now declaredly adjoined one aspect of a *pars iudicandi*, that which has to do with demonstration. Thus the reader would not only know where to find the solution to a problem, but would be able to arrive at a conclusion that was true, according to a systematic method of reasoning.⁹

The third title in our list brings us to the Art's pretensions to generality during the ternary phase. The word "general" not only begins to

⁴ The classic work on the subject is Green-Pedersen 1984.

⁵ See p. 102 above.

⁶ Shortly after the *ACIV* Lull did in fact write a *Llibre de demostracions*, but it is more a tract justifying the possibilities of demonstrations in theology than a work of the Art as a technique of demonstration.

⁷ Aristotle's *Posterior Analytics* was known in Arabic as *Kitāb al-Burhān* (*Book of Demonstration*), and in Gerard of Cremona's Latin translation as, *De demonstratione* (Dronke 1988, 156).

⁸ See, p. 65 above.

⁹ See Ruiz Simon 1999, 87–88.

appear then in the title of works, but suddenly from the *TG* on, he for the most part ceases to recommend specific works of the Art in favor of just directing the reader to the system itself—“as can be seen in the *Ars generalis*”—, with the implication that he can find the information almost interchangeably in any one of the more recent expositions of his system. And as we saw above, the word “general” is repeated almost obsessively in three of the most important works of this period, the *TG*, the *AGU*, and the *AB*.¹⁰ At every turn he is telling the reader that the Principles, the Third and Fourth Figures, the Table, the Rules and Questions, the Nine Subjects, etc. are general, or that everything that exists is implied by or can be found in them. Of special relevance in this domain is the univocality of the definitions, generally applicable to all realms from God down to the four elements. Finally there is the pretension of the Art as a science of sciences, as a challenge to both the *Posterior Analytics* and contemporary views of the place of theology (which last subject will be discussed in the section below on the “Relation of the Art to science and theology”).

As for the roles of ‘invention’ and demonstration in the ternary Art, the first is now dealt with in two ways: one is carried out by means of the combinatory mechanisms of the last two figures and the Table, and the other is assigned to the questions at the end of the work, where the reader is told in what place of the main text the answer can be found.¹¹ As for demonstration, it is significant that in the entire *TG* the word only appears in the explanation of Rule B of *Utrum?* or “Whether?”.¹² The reason it has been so largely relegated to this one Question is because of the new generality of the Art, which can now not only demonstrate something like the Incarnation, for instance, but it can also explain its what, its why, its how, etc.

With the *LN*, the fourth title of our list at the beginning of this section, Lull enters into the field of syllogistics corresponding to the *Prior Analytics*. He thus completes his journey from a *pars inveniendi* into a *pars judicandi*, into the medieval model of demonstrative science based on

¹⁰ See Ch. 4, n. 10 above.

¹¹ See above, p. 171, where the relation to dialectic and the *Topics* is explained. Nor should one forget the importance of the ‘finding’ of the middle in the *AGU*, for which see p. 219 above.

¹² *ORL* XVI, 339; *ROL* XXVII, 54. It also appears under that Rule in the *Lectura Artis quae intitulatur Brevis practica Tabulae generalis* (*ROL* XX, 363), and under the trio of “Affirmation, negation, and doubt” in the *Principia philosophiae* (*ROL* XIX, 288; *NEORL* VI, 160).

the “knowledge-producing syllogism” (*syllogismus faciens scire*), taken from Aristotle’s assertion in the *Posterior Analytics* that “by demonstration I mean a syllogism which produces scientific knowledge.”¹³ We will see below the larger implications of this move.

It should also be pointed out that in this last phase, Lull never gives up using key components of the previous ternary Art (Principles or Dignities, virtues and vices, etc.); what he abandons is its totalizing argumentative structures. Moreover, the former position of generality is progressively relinquished in favor of a more exclusively theological approach (once again the Principles are restricted to their role of Dignities), and progressively too the Art’s rigid combinatorial mechanisms are abandoned (allowing him to vary the number of dignities, virtues and vices, etc.).

Parallel with the changes of method implied by the four titles we have chosen, we find important changes in the content of the Art. We find ourselves taken from the explicitly theological foundations and Christian aims of the *ACIV*, to the more neutral theological ground of the *AD*, to the generality of both basis and aims of the *AGU*, finally to what one would have expected to be the totally neutral terrain of contemporary logic—or rather Lull’s reworking thereof—which would, paradoxically, be used almost more exclusively than with the *ACIV* to argue once again from an openly theological basis to accomplish declaredly Christian aims, with a clear predominance of proofs of the Trinity and the Incarnation.

The stages of the Art we have just outlined had important consequences for the sections of questions with which works of the Art usually ended. On p. 77 we saw the prominence given to them in works of the Art, a prominence which remains unchanged between the quaternary and ternary phases. What does change is how they are answered; in the quaternary they are done so by chains of compartments using concepts and techniques explained in the body of the work, whereas in the ternary the reader is usually referred to the place (*locus*) in the work where they have already been answered.¹⁴ In the former the reader has to do the work, in the best of cases—i.e., for questions presented in

¹³ See p. 194 above.

¹⁴ The ternary Art also presents an intermediary method, in which, as we have seen, questions are answered by the ‘compartments’ of the Table. But notice that these are usually differentiated, as they are in the *AGU*, from the questions at the end of the work.

the text—of interpreting the symbols of the chains of compartments, or in the worst of cases—for questions exterior to the Art—of setting up *and* interpreting them. In the ternary Art he must just find his way back along the paths of the labyrinth of the work indicated in the text where he will find the answer. The first is, of course, a natural consequence of the ‘upside-down’ system of proof of the quaternary phase, in which, as with natural deduction, every proof starts from the result. The second is an equally natural consequence of the ‘right-side-up’ system of the ternary phase, where the result is the consequence of pre-established premises, i.e. the definitions. In the post-Art phase, as one might expect, the structuring of questions at the end of works is much more varied and *ad hoc*. They can be answered either by the referential method of the ternary Art, or by purely syllogistic means which need no previous explanatory mechanism. They also at this stage are of diminishing importance, and one now begins to find many works without them.

Another factor in the evolution from one system to another might have had to do with contemporary readers’ reactions to what we would now call the learning curve. If works such as the *AD* are indeed geared to be able to answer questions, the reader is asked to struggle through 121 pages explaining the techniques (all extremely unfamiliar) he should learn in order to be able to answer those questions, which are then presented in a section occupying 129 pages. In other words, half of the book is preparatory material, and moreover of a nature so unusual that the reader could not just leaf through it as if to brush up on techniques with which he was already familiar. This was in sharp contrast to standard treatises of the time which usually after the briefest of prologues explaining what the work was about would plunge immediately into their questions asking “Whether...?” It should be remarked, in this connection, that Le Myésier, perhaps all too aware of such problems with Parisian audiences, in the vast anthology of the *Electorium* included no works of the quaternary Art.¹⁵ With the ternary Art, the amount of initial material the reader was asked to absorb and memorize was greatly reduced, the techniques proposed would probably have struck contemporary readers as less strange, and he would

¹⁵ Of the 88 works there anthologized, there are only four from the quaternary period: the *Liber chaos*, two parts from the *Disputatio fidelis et infidelis*, the *Book of the Gentile*, and the *Llibre contra Anticrist*, none of which are of the Art of that period.

start encountering and answering questions much earlier in the work. Finally with the post-Art phase, the reader would find works with little or no preparatory material, and whose techniques differed little from those he had learned in his university education.¹⁶

All these changes had obvious effects on what sort of persuasive strategies Llull used, going from one based on signification and metaphor, to one based on the testing of hypotheses, one based on definitions and “mixture”, to a full-blown incursion into logic proper. We will take these up in turn.

Signification, Metaphor, and Demonstration

Signification and metaphor

Llull defines “signification” as “the revelation of secrets which are demonstrated by a sign”.¹⁷ To this we should add what we said before about how for Llull “signification” refers not to how a word refers to a thing, but to how one thing can be a sign of another. In the *Book of the Gentile* he says; “It is clear that God created creatures and their properties to signify his great power and charity.”¹⁸ In the *Book of the Lover and the Beloved*, the lover, when asked who was his teacher, “answered that it was the signs which created beings give of his beloved.”¹⁹ We have remarked above the role of “signification” in the first work of the Art, the *ACIV*, as well as the fact that Figure T is there called the “the figure of principles or significations”, with the explanation that the elemental part can be used “metaphorically”.²⁰ With this use of “signification” we are very close to the dictionary definition of “metaphor” as “One thing conceived as representing another; a symbol”,²¹ close enough so that Llull could use them side by side—as he does in the two quotations

¹⁶ Le Myésier included some 30 works of the post-Art phase in the *Electorium*. Curiously, however, he did not include the *Logica nova*; instead, at the beginning of the anthology he placed an anonymous *Summula sive introductio in logicibus*, of which he was perhaps the author (see Hillgarth 1971, 351).

¹⁷ In the *AB*, Hundred Forms No. 36 (*SW I*, 619; *DI* 338). We have translated it more literally here. Note that “secrets” here has the older meaning of something hidden, unknown, and has no esoteric or magical connotations.

¹⁸ *SW I*, 220; *DI* 132.

¹⁹ *DI* 196.

²⁰ See above pp. 102 and 96–97.

²¹ *The American Heritage Dictionary*, s.v. 2.

we have just given—in the first period of the Art. To see how they are woven together, we can look at the *Principles of Medicine*, one of the four *Libri principiorum* derived from the *ACIV*. In a chapter entitled, “The reason why in this Art we deal with fever, urine, pulse, and metaphor”, one finds the following passage.²²

...Now this metaphor signifies to you that, in the science of medicine, the greater the mixture of simple medicines in a drink you make, the less the assurance with which you can work. Now this medicinal metaphor signifies another in the science of law, for the more different laws and cases you want to make accord towards a single end, the more difficult it is to attain that end. Now this metaphor from the science of law signifies another from natural science, that is, that the more A B C D are diversified in different species in revealing the colors they have engendered, the more they conceal and deny the existence of color in the simple elements. (*SW II*, 1129–1130)

From degree to degree, from example to example, and from one principle to another, I could go on at great length discussing metaphor with you, but we will reserve this for the distinction on that subject. The greatest virtue of this Art, however, lies in its metaphorical nature, and this is due to the fact that the elements and the universal sciences conceal and reveal their secrets and their operations to the intellect in very subtle ways. Because of this obscurity, the intellect must be uplifted to understand metaphorically so that these secrets be revealed to it, and so that a person, through this uplifted understanding, may know how to pose and solve questions. (*Ibid.*, 1130)

Then the work ends with a “Tenth distinction which treats of metaphor”, entirely devoted to the subject, which begins with the explanation:

Metaphor is one thing signifying another, like the sick person who is near death and is cold, yet desires cold and searches around the bed to see if he can find it. This signifies that the feeling of cold is destroyed by large amounts of heat...²³

Metaphor or analogy, which plays such a major role in the quaternary Art, is thus based on signification; how one process can “signify” another;

²² I have modified the translation to show the insistence on the word “signify” of the original, for which see *NEORL V*, 49, which presents a new critical edition of the Catalan text of the work.

²³ *SW II*, 1199. For the reappearance of this same metaphor, see n. 26 below. See also the other important quotations from the *Principles of Medicine* and from the *AD*, preceding and following Ch. 2, n. 60 above.

and often how various heterogeneous bits of the Art can be related to one another and to the outside world. Not only did he exploit the structural similarities between Figure S and the Elemental Figure, but the latter was extensively used for what Frances Yates called Llull's "elemental exemplarism". All this, in the quaternary phase, functioned as an immense web of signification.²⁴

A splendid example of Llull's literary use of these techniques can be found in *Felix or the Book of Wonders*, a kind of tapestry woven out of a vast chain of exemplary stories, usually accompanied by explanations of what they "signify".²⁵ Once again, however, Llull takes something that was a commonplace of the time and puts it to a different use. In the Middle Ages *exempla*, as such stories were called, were the material out of which sermons were constructed, usually offering models or countermodels of conduct from stories of saintly, wise, upright, sinful, ignorant, or slothful lives. In Llull, however, as Lola Badia has pointed out, these *exempla* are hardly ever exhortatory; they are mostly doctrinal, that is theological, philosophical, or scientific. Book I of *Felix*, for instance, has chapters asking "Whether God exists?", "What is God?", "The unity of God", "The trinity of God", and Book IV discusses "The simplicity and composition of the elements", "The generation and corruption of the elements", "The movement of the elements", and so on, in which the *exempla* are used to explain, to "reveal the secrets", of the things the protagonist doesn't understand.²⁶

²⁴ Johnston 1987, 162–175, has an excellent section on "Signification", where he points out the definition from the *LN*: "Signification is the being through which something secret [i.e., hidden, concealed] is revealed" (*ROL* XXIII, 93; *NEORL* IV, 75), which corresponds closely to the passage in the second quotation above where Llull explains how "the elements and the universal sciences conceal and reveal their secrets".

²⁵ As well as chains of simpler significations between things, such as: "Light signifies wisdom and wisdom signifies light; and light signifies glory, whereas darkness signifies suffering and ignorance" (*SW* II, 729).

²⁶ Lola Badia's remark is from a review in *SL* 39 (1999), 137. See too her study of the opening *exemplum* of *Felix* in Bonner-Badia 1988, 87–90 (or Badia-Bonner 1993, 109–113). The best recent studies of Llull's *exempla*—with bibliographies that will lead one to earlier studies—are Badia 2004, Gisbert 2004, Bonillo 2004, and Bonillo 2005. The second of these, Gisbert 2004, gives a good survey of Llull's use of metaphor, of its roots in classical (principally Aristotelian) and medieval thought, and on the reasons for his using the term "metaphor" rather than "analogy". The same study (p. 37ff.) discusses the more explanatory, less exhortatory nature of Llull's *exempla*, while mentioning (p. 39, n. 30) the curious fact that the first two metaphors of the *Principles of Medicine* reappear in almost identical form in Book VIII of *Felix*; cf. *SW* II, 873–4 and n. 74. On the intentional obscurity of some *exempla* as instrument for stimulating

The culmination of this approach took place in the “Arbre exemplifical” of the *Tree of Science*, where Lull performs what Pring-Mill called “the transmutation of science into literature”.²⁷ Here we have *exempla*, for example, in which four elements are personified: “Water wept, saying to Air that he didn’t know the villainous thing that Fire had done to Earth...” (*OE I*, 799).

In that same work, however, things soon start branching out. In the brief prologue to the “Arbre exemplifical” he had already said, “The examples we propose to give are divided into two parts, that is to say into little stories (*recontaments*) and proverbs” (*Ibid.*) First he gives little stories, like the one quoted above about water weeping, which is a page or so in length, and then when he gets to what—rather surprisingly—he calls proverbs, we continue to have the same four elements personified, but now in the form of single sentences: “Air begged Fire not to heat it too much, for if it did, Water would not want to takes its humidity from him” (*Ibid.*, 805). Then come personifications of Autumn, Winter, the Carob tree, the Lettuce (*Ibid.*), followed by ones with animals such as, “The donkey said to his master that he knew how to sing, and the master replied that he knew how to weep” (*Ibid.* 806). Then we come to something nearer to real proverbs: “Simple circular motion has no middle,” “No angel has spatial extension,” “Eviternity has an end without limit,” or, “Eviternity is the mirror of eternity”.²⁸ These last two are summarized in one from the *Proverbs of Ramon* which says, “Eviternity is the mirror of eternity at its end”.²⁹ Then if we look in this latter work under “goodness”, we find the familiar definition of “Goodness is that by which good produces good...”.³⁰ In turn, for another proverb of this last work, “No being comes naturally from non-being”, we find an almost identical formulation in the *Principia philosophia*, where it is a “principle” on which one of the consequences of the following section

active participation and thought on the reader’s part, see *SW II*, 709, 722, and Gisbert 2004, 40 and 43. For Felix’s progression in his understanding of the *exempla* he encounters, see p. 294 below.

²⁷ Pring-Mill 1976. See also Cabré, Ortín, and Pujol 1988, and Hauf 2002.

²⁸ *Ibid.* 808 for the last three examples. Part IV of the *Rhetorica nova*, combines proverbs with *exempla*, the latter used to explain or illustrate the former.

²⁹ *ORL XIV*, 26. Eviternity was the usual scholastic term for an eternity with a beginning and without an end, such as that of angels.

³⁰ *Ibid.* 105. This is not an isolated example; there are hundreds of entries in the *Proverbs of Ramon* that are in fact nothing but definitions, and part of of the work even offers another list of a Hundred Forms (see Bonner and Ripoll 2002, 89–91).

will be based.³¹ All of this represents an extraordinary gradation from exemplary stories, to proverbs, to the definitions of the Art, to simple propositions used as premises, and finally to maxims used for similar purposes, as we discussed earlier.³² Lull defines “proverb” as “an instrument which briefly certifies the truth about many things” a definition which would fit all these categories.³³ If we substituted “signifies” for “certifies the truth about” it would cover their explanatory role.

The one element of this gradation that will diminish in importance in the ternary phase and finally almost disappear in the post-Art phase is that of metaphors or *exempla*. This could be related to the fact that one of the traditional arguments for reasoning by analogy was the impossibility of speaking about God directly in ordinary language.³⁴ Once Lull, however, had developed a unified world-picture through the Trinitarian structure induced by the correlatives, as well as a unified language achieved through the univocality of his definitions, he no longer needed the recourse to analogy or metaphor as basic tools of his system.

This new orientation coincided more or less with his abandonment of literary forms, based as they were so much on these tools, and their replacement with sermonizing. Lull’s ventures into the latter field, interestingly, coincided chronologically with those into logic. They began with rhetoric, which against its use in classical times as a tool for public speaking and politics, in the later Middle Ages was studied almost exclusively as the foundation for preaching. Aside from brief discussions of the subject in earlier works, Lull’s first more extended treatments of rhetoric are found in the *Aplicació de l’Art general* and *Rhetorica nova*, both of 1301. The former work also contained his first incursion into logic, and the latter work was translated from Catalan into Latin (the only version preserved) in 1303, the same year as the composition of the *LN*. In 1304 came his first work on preaching, the

³¹ Ibid. 103 for the first work, and *ROL* XIX, 160; *NEORL* VI, 58, for the second.

³² See p. 215ff. above, and Rubió i Balaguer 1985, 289–293. Notice that Lull refers to the verses of the *Book of the Lover and the Beloved* as “moral metaphors” (See *DI* 190, where in n. 5 it is pointed out that, in addition to its usual meaning, “moral” in the Middle Ages could also mean “figurative, symbolic, requiring interpretation”). See Lull 2006, 73–74, 82, 85, for a particularly perspicacious discussion of these different categories.

³³ In the *Mil proverbis* (*ORL* XIV, 327). For other definitions see Bonner and Ripoll 2002, s.v.

³⁴ See Gisbert 2004, 34.

Liber de praedicatione, and, one year after that, the *Liber praedicationis contra judaeos*, both contemporary with the first works to use logic listed on p. 193 above. Then already in the post-Art phase we get the group of sermons which Fernando Domínguez has called the *Summa sermonum* of 1312–1313, centered on the *Llibre de virtuts e de pecats* and ending with the *Ars abbreviata praedicandi*.

In his books on sermonizing, he basically discusses and/or uses three techniques. The first is the *exemplum*, as in the *Liber de praedicatione*; but even here he uses it sparingly, and less and less so in subsequent works. In that same book he also uses the classic technique of basing his sermons on a Biblical *thema*, but here by the extraordinary means we will discuss in a moment of ‘reducing’ it to necessary reasons.³⁵ Finally, near the end of his life, he tends to base his sermons more on combinations of components of the Art (mainly the virtues and vices).³⁶

Demonstration and metaphor

In the *Lectura Artis quae intitulatur Brevis practica Tabulae generalis* Llull explains that there are four kinds of proof: syllogism, enthymeme, induction, and the *exemplum*.³⁷ This is a division he took straight from the standard textbook of the time, Peter of Spain’s *Summulae logicales*,³⁸ who in turn took it from the opening paragraph of Aristotle’s *Posterior Analytics*. Aristotle, however, rather than just listing them, subdivides them into two categories, by referring to “examples, which are a kind of induction, [and] enthymemes, which are a kind of syllogism”

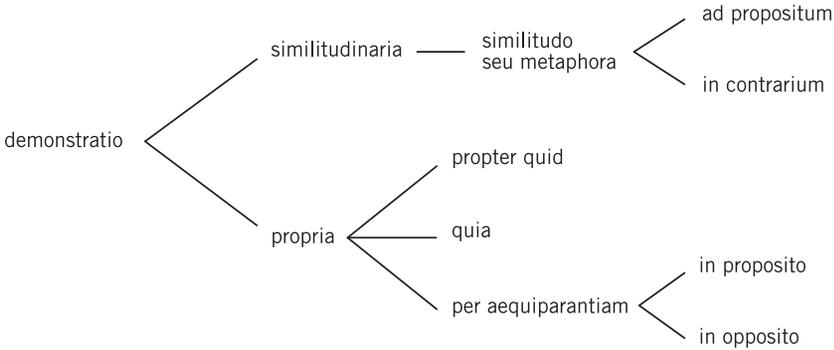
³⁵ See pp. 280–281 below.

³⁶ In the sermons of 1312–1313 he mixes these techniques. To take the two most important of these works, in the *Llibre de virtuts e de pecats* he mainly uses combinatory methods of the Art, but he bases it on a single *thema*: “Thou shalt love the Lord thy God with all thine heart, and with all thy soul, and with all thy might” (Deut 6,5). In the *Art abreuçada de preicació* he likewise mixes the two, but with a more varied use of both *themata* and artistic devices. These sermons are available in *ROL* III–IV, XV, XVIII, with the Catalan version of the *Llibre de virtuts e de pecats* edited in *NEORL* I. See the introductions to these editions, as well as Cabré 2000 and Bonner 2005 for more information.

³⁷ *ROL* XX, 423 and 428, a division Llull used in the work written immediately afterwards, the *Liber ad probandum aliquos articulos fidei catholicae*, which is like a testing ground for the previous *Lectura*. He had originally presented this classification in the *Compendium logicae Algazelis* (Lohr 1967, 102), as well as in the Catalan translation, the *Lògica del Gatzell* (*ORL* XIX, 23).

³⁸ Peter of Spain 1972, 56–58. For Llull’s early dependence on Peter of Spain for this division, see Lohr 1967, 22–23.

(71a9–11). Since the difference between the *exemplum* and induction is purely numerical (whether you base your argument on one or many cases), and since enthymemes only differ from syllogisms formally, by lacking an explicit middle, Llull could discard one of each set and focus his attention on examples and syllogisms.³⁹ In the *Ars inventiva veritatis* he expands this two-fold schema in the following way:⁴⁰



First of all, here he is treating demonstration more broadly, including any tactic that might serve to convince. As synonyms of “demonstrate” he often uses not only, as we have just seen, “signify”, but also “prove”, “show” (in Catalan *mostrar*, which is the root of *demonstrare*), “persuade”, “solve” (when the proof is formulated as the answer to a question), and, often at the end of a demonstration, he simply says “it is therefore clear” (*patet ergo*). It is this broad meaning which permits him to include under *demonstratio* the exemplarism (*similitudinaria*) discussed in the previous section.

³⁹ I have only found enthymemes used in the *Ars mystica*, *ROL* V, 338–342, and as far as I have been able to discover, induction is only mentioned in the three works cited in n. 37 above.

⁴⁰ *MOG* V, 45–47. This chart has “proper demonstration” rather than the “syllogism” of the previously cited division, because the *Ars inventiva veritatis*, finished in 1290, predates his ventures into syllogistics by more than a decade, whereas the *Lectura Artis quae intitulatur Brevis practica Tabulae generalis* dates from 1304, after the *LN*. This substitution does not mean, of course, that the terms are synonymous; just that after a certain point the second becomes the chief instrument for the first.

On the same page of the *Ars inventiva veritatis*, he gives two illustrations of this exemplarism, both significantly from elemental theory. First he gives the positive one:

If someone cannot easily understand how the four elements form a single mixture, one can adduce the following example. Gingerbread is made from sugar, ginger, honey, and saffron, in such a way that the smallest amount of each blends with the smallest amount of each of the others, and from their four forms there results a gingerbreaded form, and from their four matters its matter. This is similar to how a given mixture can be made from the four elements.

Then the negative one:

If someone does not easily understand how one element can exist continuously throughout another when they are mixed, one can adduce the following contrary example. If wine and water are separate, it is impossible for one to exist continuously throughout the other. From a contrary simile [i.e., realizing that when they are together there is no problem with their forming a continuous mixture, one can see how] with the simple elements, since they are subtler than wine and water when they are joined, each of them can exist continuously throughout the other.⁴¹

He then explains “that metaphors do not infer necessity, but acceptability (*convenientiam*), and if they do infer necessity, then they cross over into the category of a ‘proper’ demonstration, under the species of ‘equivalence’.” This means that the two techniques are very close to one another, and in fact only differ in the degree of conviction they produce. This combined with what we said above about his *exempla* being more theologically and philosophically explanatory than exhortatory, plus the fact, as with the Art, that they can also answer more general questions, such as asking what a thing is, of what it is composed, etc., shows how narrow is the gap between the literary and the philosophical Lull.

Another reason for the progressive abandonment of exemplarism and metaphor, aside from that mentioned in the previous section, might be that in the Parisian circles which Lull wanted so much to influence it was not to everybody’s taste. Albert the Great, for example, had said that “In rejecting Plato’s opinions, Aristotle is not rejecting their substance

⁴¹ *MOG* V, 45. Lull adds an interesting observation as to why metaphors must sometimes be obscure: “One should take care not to make the metaphors clearer to the understanding than those things which have induced them,” reminiscent of Niels Bohr’s pledge to try never to speak more clearly than he thought. For the different matter of the intentional obscurity of metaphors, see n. 26 above.

but their form. Plato had, in fact, a bad method of exposition. Everything with him is figurative and his teaching is metaphorical and he attaches to words other things that they signify.⁴² Such considerations might have played a role in his slow changeover from Platonic methods of persuasion to Aristotelian methods of proof.

As for demonstrations proper, we have already met the three-fold division of *propter quid*, *quia*, and *per aequiparantiam* (loosely, by cause, effect, or equality), of which the first two are Aristotelian and standard in the Middle Ages, and the last an invention of Lull's. We have also seen how the last, which allowed him to prove the Articles of Faith—and above all the Trinity and Incarnation—, is absolutely central to his program, but the distinction with the first two is equally crucial for justifying his proofs in matters of faith as not being demonstrations in the usual Aristotelian, causal sense. This is why this ternary division, after its first appearance in the *AD*, is discussed almost obsessively throughout his career, and why he could even indulge in one of his rare outbursts of anger against those who refused to understand this distinction.⁴³

His new method of demonstration *per aequiparantiam* is also central in another sense, that of being crucial to his understanding of the nature and intelligibility of the Divinity, or, to put it another way, how Lull's epistemology is grafted onto his ontology. In the *AD* he explains that each dignity is "of the same essence" as another, and in fact "the divine dignities are all equal in essence and nature",⁴⁴ in addition to being jointly concordant. It is in the ternary phase that Lull begins talking about their convertibility. In the *AGU*, as we have seen, the principal definition of God is:

⁴² See Le Goff 1957, 122. The Franciscan, St. Bonaventure, was however much more favorable to analogical methods and thus someone to whom Lull was much closer. Gilson 1965, 185, says "Where the reader expects syllogisms and formal demonstrations, St. Bonaventure usually offers him only correspondences, analogies, and conformities, which seem to us hardly satisfactory but which seem to satisfy him entirely. Images cluster together in his thought and follow one another indefinitely..." A bit later on (p. 207) Gilson says "The metaphysic of analogy must therefore be completed by a logic of analogy, and it remains for us to consider its laws." These passages come from a most illuminating chapter called "Universal analogy".

⁴³ See p. 187 above.

⁴⁴ See the passage referenced in Ch. 2, n. 81 above; this is what permits the *demonstratio per aequiparantiam*.

God is the being whose reasons [i.e., dignities] convert [with one another]; indeed, the being whose reasons convert is God.⁴⁵

This in turn is tied in with the circular nature of the First Figure, the predicability of the Principles of the ternary Art, and with their mutually referential definitions, all of which, when applied to God, makes them convertible with one another. Finally the chief mechanism of Lull's finding of the middle explained in the chapter on Logic is an attempt to find a mechanism to achieve "aequiparance" where it doesn't occur naturally between propositions.⁴⁶

Techniques of Demonstration

The Art

In the chart presented three pages ago, we saw how the last column had positive/negative pairs both for *exempla* and for demonstrations in the stricter sense. What this implies in the latter case is explained by Lull in the *Art of Proposing and Solving Questions* written some four or five years after the *Ars inventiva veritatis* from which that chart was taken. But before giving that explanation, it might be best to clarify the two crucial terms of "universal" and "particular" used in that explanation:

In this science we call "universal" each Principle and its definition, along with each mode and Rule of the Questions, since all of these are universal principles and general to all sciences, and this is why this science can be called "general". Moreover, we also call "universal" the mixture of the Principles one with another, as it appears in the Figures, which is why we call each compartment of the Figures "universal".

On the other hand, we call "particular" the question which one proposes, such as when one asks whether God exists or not, whether the world is eternal or not, whether angels exist, or what is an angel, of what is an angel, or why is it, and thus with other things similar to this.⁴⁷

⁴⁵ Cf. pp. 161 and 220 above. This convertibility also, of course, includes the cor-relatives; cf. *TG*, in the description of the First Figure.

⁴⁶ See pp. 220–221 above.

⁴⁷ *MOG* V, v, 10: 368, cited in Ruiz Simon 1999, 192, n. 187, in a whole section devoted to this terminology. For the "species" of the first paragraph, the text has "mode", but in the presentation of the Rules at the beginning of the work the terms are interchangeable.

Notice how the “universals” are not just the Principles, but also the other basic components of the Art, and the “particulars” are not only things to be demonstrated, but any kind of question. Moreover, under the first category, when Lull refers to definitions, he does not just mean those of the Principles, but any definition “which shows what the thing is really, and that what is said of it applies to no other thing”, giving the example of when “God is defined by the unity of goodness, greatness and eternity, since in no other being are goodness, greatness and eternity one in number except in God alone.” This, of course, means definitions according to Lullian methods, such as those found under the Nine Subjects or the Hundred Forms.

All this is equally applicable to the quaternary Art, except for the fact that there under “universal” there is no question of definitions or Questions and Rules. In that earlier presentation “universal” refers to the terms of the Art, the compartments formed from them, and the propositions based on them, which usually consist of comparisons based on understood—that is to say, generally acceptable—things such as the concordance of good, perfection, or virtue with being (and hence with one another) and their contrariety to evil, imperfection, or sin, which were similarly concordant with non-being (or privation), forming a set of ontological/moral Neo-Platonic positive and negative values equivalent to modern truth-tables.⁴⁸ From all this it is clear that for Lull “universal” and “particular” do not usually refer just to simple terms or concepts but also to propositions, or even to complex comparisons of terms or compartments.

These considerations should help us understand the dual positive/negative presentation of the basic demonstrative technique of the Art from the *Art of Proposing and Solving Questions*. After presenting the Alphabet of the Art, Lull says:

Thus each of these letters signifies that for which it stands, along with its definition and explanation, so that the particular sought can be found in its universal by affirmation or negation. If the particular is concordant with its universal, the affirmative should be maintained in the conclusion of the question; if the particular is contrary to its universal through an affirmative, the negative should be maintained; and similarly if the particular is contrary to its universal with a negative, the affirmative

⁴⁸ For which see p. 83 above.

should be maintained in the conclusion. And this rule is general in this science.⁴⁹

Here Llull, by breaking down the negative alternative into two subcategories, gives us three possibilities. Letting P and U stand for “particular” and “universal” respectively, \rightarrow for the conditional “if . . . then”, \vdash for “gives” or “leads to the conclusion that”, and \neg for “not” we would get: (1) $(P \rightarrow U) \vdash P$; (2a) $(P \rightarrow \neg U) \vdash \neg P$; (2b) $(\neg P \rightarrow \neg U) \vdash P$.⁵⁰ Two things should be emphasized with these formulae. The first is that Llull’s universals function as criteria by which the affirmation or negation of the particulars can be established. Secondly, whereas (2) is clearly *modus tollens*, (1) is not *modus ponens*, which would establish the truth of the consequent from that of the antecedent, something, as we will see in a moment, Llull never does with this type of argument. (1) just shows that the original particular *can* lead to a correct conclusion, which in logical terms means that the deduction is not necessarily inconsistent, or in other words that it is at least satisfiable, and that the particular in question is a valid possibility; this is undoubtedly why Llull usually does not use (1) alone, but rather in conjunction with (2).

Llull’s standard mode of demonstration was thus to rephrase what he wanted to prove (i.e., the particular) as an “if” clause, and then study the conclusions which followed from the “then”. To take the question “Whether God exists?” from the *AD*,⁵¹ after a little preamble about F and G, he begins his demonstration by saying “if A exists”, then we find ourselves with a “universal” consisting of concordances among the “positive” terms of the Art, thereby affirming the validity of the initial “if” clause, all of which gives us case (1) above. He then tries out the negative, “if it [A] does not exist”, which leads to a series of contraries between those same “positive” terms, showing that God cannot not exist, or in other words that he does exist, which gives us case (2b) above. Case (2a) would be the tool for disproving an opponents negative assertions about Christianity, like those of the Muslim, Homer, in the *Disputatio Raimundi christiani et Homeri saraceni*.⁵²

⁴⁹ *MOG* V, v, 10: 368. Cf. Ruiz Simon 1999, 220ff., whose analysis of this passage we have followed in great measure.

⁵⁰ Note that U is a positive universal, such as “being” concordant with “perfection”, and $\neg U$ a negative one, such as “being” concordant with “imperfection” or contrary to “perfection”.

⁵¹ See p. 80 above.

⁵² See, for example, Llull’s refutation of Homer’s assertions that “God has greatness, but he himself is not greatness” (*ROL* XXII, 219), showing that his negative proposition leads to a contradiction, thus proving the positive of the initial proposition.

Notice also how (1) and (2) correspond respectively to the positive and negative parts of the chart on p. 267 above, in both its metaphorical and logical branches.

For Llull, the negative conclusion could either be because he had arrived at a logical contradiction (the simultaneous assertion and negation of a proposition), producing a *reductio ad impossibile*, or because he had arrived at a contrariety (as in the proof of the *AD* just cited), an onto-theological impossibility (an assertion that God was imperfect, lesser, etc.),⁵³ or, in the ternary phase, because the conclusion contravened one of the premises of the Art (the definitions, the mixing of same, the Rules, etc.), thus producing a *reductio ad absurdum*.

This fundamental technique of demonstration is found throughout Llull's works, from the *Book of the Gentile* right up to his last works written in Tunis. One of the reasons for which he gives it such priority is that he is not proving a conclusion, but testing a hypothesis or supposition.⁵⁴ Generalizing the patterns of demonstration given above (i.e., ignoring negatives), we have $A \rightarrow B \vdash A$, and never $A \rightarrow B \vdash B$, or in other words, he is testing an antecedent, not arriving at the truth of a consequent. Thus to prove, for instance an Article of Faith, he did not have to say that it was thus and so because of some prior reason or cause, but simply because, taking it as a hypothesis, the conclusions one could derive from it were acceptable, or those derived from its negation, unacceptable. It is what the logician Melvin Fitting has called an "upside-down" system, which begins with the desired result.⁵⁵ In this it is similar to a modern "tableau" proof, which is a refutation system, and even more to natural deduction, both of which can prove such basic laws of thought as, for instance, that of non-contradiction, $\neg (A \wedge \neg A)$. In the sense that such laws can be considered axioms, it is precisely the nature of an 'upside-down' system to be able to prove them. Similarly, Llull's system is 'upside down' in that it can prove what, for thinkers like St. Thomas, were the axioms of theology as a science, that is the Articles of Faith. Or rather to test them; because—as Llull was at pains to point out—he is not proving them in the normal Aristotelian causal sense, but testing them to see if their affirmation or negation does or does not lead to a contradiction. Such disputation for or against a hypothesis has been the standard technique of dialectic

⁵³ Based on conditions such as those of the trees of the *Book of the Gentile*, to which, as we know, Llull attached such importance.

⁵⁴ See p. 82 above.

⁵⁵ Fitting 1983, 4–5.

since Plato, and it permits Llull to confirm or refute any proposition whatsoever, be it a proverb, a legal maxim, or a citation from the Bible. So, in a technical sense, he is not proving the proposition in question, but merely showing—as he says again and again—that it conforms to ‘necessary reasons’.

From simple concepts to propositions

Llull’s disciple, Thomas Le Myésier, in the *Epitome* which had originally preceded the *Electorium*, makes an important distinction between those of the master’s works based on “incomplex” and those based on “complex” principles.⁵⁶ The first were the components of the figures of the Art, which, up to the end of the quaternary phase, were the *only* principles of the Art. But as Le Myésier says in the same passage, some found this unsatisfactory, since they “preferred to base arguments on propositions, which, if they are necessary and true, are the principles on which demonstrations can be based”. And the propositions on which Llull could base his demonstrations were the definitions which in the ternary Art took on such importance. Not only are the eighteen principles of the Art defined, but all manner of other foundational material as well, giving way to proverbs, maxims, etc.

One could in fact say that this passage from incomplex to complex principles, from simple concepts to propositions, was perhaps the most fundamental change between the quaternary and ternary Arts. It permitted Llull to go from proofs based on comparisons tested against Neo-Platonic equivalents of modern truth tables, to proofs based on his definitions. It permitted him to operate a complete reversal in his demonstrative methods, to go from an ‘upside-down’ system, which tried to sustain or refute an original hypothesis, to a ‘right-side-up’ system which began with the definitions and worked it way through to the desired conclusion.

This change also involved a different kind of relation between the principles of the Art. Instead of the above-mentioned comparisons, we now have the mixing to which Llull attaches so much importance in the ternary period. The difference is crucial. With the first method,

⁵⁶ From a lost portion of the *Electorium* preserved in a copy made for the Florentine humanist, Pier Leoni (see Hillgarth 1971, 200). The quotations in this paragraph are transcribed in Hillgarth 1971, 403, and cited by Domínguez in *ROL* XIX, 43.

one principle is concordant with, contrary to, greater than, etc. another, and it is the implications of this comparison that are studied. With the second, it is one principle which ‘enters into’ another, usually by means of their definitions, to produce something new—the mixture itself—,⁵⁷ something on which the next step of the demonstration or explanation can be based in the outwardly expanding process of the ternary Art discussed in chapter 4.

Finally, this is what is behind the changed role of Figure T. It is no longer so much an instrument of comparison as a second set of principles which can enter into a mixture, along with those of Figure A and the Rules.

As is usually the case with Llull, one system did not exclude or negate the other. As we saw in the *AGU*, in what for him was an unusual procedure, he formulated questions in two parts of the work: in the first half of the text (where he included the proofs that the world could not be eternal), as well as in the familiar end section. We also saw how, before the chapter on Mixing, he still follows earlier methods of testing hypotheses, that is, continues to use his former ‘upside-down’ system; from then on he begins a build-up of material based on the original definitions, in a ‘right-side-up’ system that allowed him to formulate questions that did little more than refer the reader to the place in the Art where the answer could be found or was implied.

The syllogism

It was surely this use of propositions as a basis for the Art that suggested to Llull that he could use them as the premises for standard Aristotelian syllogisms. For such syllogisms to produce correct demonstrations or proofs, however, they required premises that were primary, true, immediate and necessary; but, as we explained in the previous chapter, this is precisely what in the *Liber de demonstratione per aequiparantiam* Llull said was supplied by his dignities, their definitions, and definitions derived from them. Notice how these definitions can more easily be shown to be primary, true, immediate and necessary when applied to the Divinity, wherein their circularity and their correlative action have been shown by

⁵⁷ Like the newness of the gingerbread created out of sugar, ginger, honey, and saffron on p. 268 above.

the Art to be irrefutable, than in the created world, where propositions such as “man is a manifesting animal” caused more notable resistance and might be felt as being more difficult to accept as “primary” and “immediate”. This might be one of the reasons why Llull, at the end of the ternary and in the post-Art phases, more and more used logical methods for matters theological. This produced the apparent anomaly of a technique considered by his contemporaries least apt for any direct application to theology, being used by him chiefly to that end.

This is another change of first importance. Suddenly Llull is confronting Parisian university audiences with a battery of weapons from their own arsenal, as opposed to the Art, which might have struck them as a rather home-grown artifact. To be sure, many achievements of the Art are used in these syllogisms, but the end product is the result of a studied adaptation of Aristotle’s then prevailing theory of science. Llull is now meeting his contemporaries on their own ground. And the very adequacy of these new methods might have played a role in the progressive abandonment of the Art.

The one thing that remained constant, was that Llull’s methods of argumentation, whether the comparative techniques of the quaternary Art, the Table of the ternary Art, the build-up from the mixing of the same stage, or the final syllogistics, are collectively what he called his “necessary reasons”. And at every point he tried to formulate them in such a way as to avoid the strictures of the Church; or in other words, instead of having to decide on a disjunction between faith and reason, his idea was to develop systems in which the two went hand in hand, either by the first providing the hypotheses which the second could confirm, or by the second offering definitions general enough so that they could be applicable to the Divinity yet not reasonably refutable.

Relation of the Art to Other Fields

An axiomatic system?

It has been suggested that there might be parallels between Llull’s Art and medieval attempts to formulate theology or philosophy in an axiomatic form. Before discussing this possibility, however, some initial points must be clarified. The first is a possible element of confusion in Llull’s choice of the word “dignities” for the divine attributes. The fact that *dignitas* was the usual medieval Latin translation of the Greek

axioma, should not obscure the fact that classically an axiom is a proposition, not a single term. In the Art, however, which is what has been suggested might be modeled on such ventures, it was not a connection Lull ever made—however much he might have liked the resonances it created in the reader’s mind.

In the second place the quaternary Art obviously does not proceed from axioms to conclusions, but functions in a reverse direction of testing conclusions in a manner similar to that of dialectic, so it can at once be ruled out as a candidate. And the logic of the post-Art phase is not systematized in any way which might lead one to describe it as axiomatic. This leaves us with the ternary Art as the only possible candidate for an axiomatic system.

But before discussing that, however, we must sort out what was involved in a medieval axiomatic method. Charles Lohr has distinguished two kinds, one which is properly axiomatic, and another which he has called “deductive”.⁵⁸ The second, used in the *Liber de causis*, starts from one initial proposition so patently true as to be unquestionable, and deduces everything else from there, using a method obviously unconnected to Lull’s Art. This leaves us with the more strictly axiomatic method—based on varying combinations of axioms, definitions, and postulates—which made its way into the Middle Ages through two works: Boethius’ *De hebdomadibus*,⁵⁹ and Euclid’s *Elements*, and which acted as models for the twelfth-century Nicholas of Amiens’ *Ars Catholicae fidei*, Alan of Lille’s *Regulae caelestis juris* (also known as *Regulae de sacra theologia*), and other works. Now Lull’s ternary Art—based as it is on definitions of eighteen basic principles—could be compared to this second method, although the connection seems tenuous. Lull never accompanies his definitions by anything like axioms or postulates,⁶⁰ and the Art is not structured as a chain of theorems based on said

⁵⁸ Lohr 1986b for a good, succinct overview. See also Charles Burnet in Dronke 1988, 162–5; and Aquinas 2002, xvii. I have not been able to consult Dreyer 1996.

⁵⁹ Or, to give its more authentic title: *Quomodo substantiae in eo quod sint bonae sint cum non sint substantialia bona*.

⁶⁰ What I mean is that Lull presents no theoretical justification, as he does when he is recycling, for instance, Aristotle’s *Topics* or *Posterior Analytics*. Nor should we be misled by Lull’s ventures into geometry, which are much more Pythagorean in the sense of being symbolic and numerological. These involve the *Liber de geometria nova* and the *De quadratura et triangulatura circuli*, which give the numerological categories used in the *Principia philosophiae*. From a purely logical point of view, as we pointed out before and as Fernando Domínguez has suggested, these works could be more easily tied in with the medieval theory of consequences.

definitions, axioms, or postulates, nor on previously proved theorems. So until more research is done on the taxonomy of the many similar axiomatic systems of his time,⁶¹ it is difficult to see anything more than a vague resemblance, or to say how much this resemblance is useful or historically justified.

Relation of the Art to science and theology

The most important aspect of the Art's generality which we discussed above is its pretension to being a science of sciences. At the beginning of the *AGU* Lull says:

Since the human intellect is more given to opinion than to science, and since each science has its own principles different from those of other sciences, the intellect requires and desires one science which is general to all sciences, one with its own general principles in which the principles of the other individual (*particularium*) sciences are implicit and contained, like any particular in a universal. The reason is so that by means of these principles other principles would be subalternate (*subalternata*), ordered, and regulated, so that the intellect might come to rest in these sciences through true understanding, far removed from erroneous opinions. (*ROL* XIV, 5)

Such a statement would have aroused in his contemporaries very strong associations. First of all it is a clear reworking of Aristotle's theory of science in the *Posterior Analytics*. The distinction between opinion and science (or knowledge),⁶² on which Lull insisted frequently,⁶³ is the main subject of section I, xxxiii, of Aristotle's work. In addition, the body

⁶¹ Libera 1994, 119, for instance, mentions the desire to axiomatize theological knowledge as characteristic of Rhineland thinkers of the thirteenth and fourteenth centuries, specifically mentioning Ulrich of Strasbourg and Master Eckhart. Since Ruiz Simon 2005 has suggested other connections between Lull and these thinkers with their roots in Proclus, it might be interesting to explore what methodological links might be found.

⁶² It must be kept in mind that in the Middle Ages "science" didn't have its modern meaning of drawing conclusions from empirical observations. It was closer to its original meaning of "knowledge", derived as it was from the verb *scire*, "to know", and could refer to fields such as theology and philosophy, as well as to mathematics and astronomy. See Lohr 2004. Since St. Augustine it usually referred to knowledge of a "highly intellectualist quality" (see Chenu 1969, 67), or "scientific knowledge".

⁶³ See for instance the *Liber de novis fallaciis* (*ROL* XI, 12–13), where he explains that opinion has to do with belief, and knowledge (*scientia*) with understanding (*intelligere*). Cf. *De conversione subjecti et praedicati et medii* (*ROL* VI, 262), and the *Liber de ente reali et rationis* (*ROL* XVI, 50, 79, 103).

of the passage would inevitably remind readers of Aristotle's theory of science as explained earlier in the same work, where he says that each science has its own principles, but that:

The special principles of each genus cannot be demonstrated; for the principles from which they would be demonstrable would be principles of all existing things, and the science of those principles would be supreme over all... However, demonstration is not applicable to a different genus, except as we have explained that geometrical proofs apply to the propositions of mechanics or optics, and arithmetical proofs to those of harmonics. (76a17–25)

Llull is thus opposing a basic tenet of Aristotelian science by saying there *is* a science whose “principles are supreme over all”.⁶⁴

When Llull says in the same quotation that “by means of these principles other principles are subalternate”, he is stepping quite consciously—at this point in his life it could be no other way—into major contemporary discussions about the hierarchy among the various medieval sciences.⁶⁵ Thomas Aquinas, answering the question at the beginning of his *Summa theologiae*, “Whether sacred doctrine is a science?”, had said that:

There are two kinds of sciences. There are some which proceed from a principle known by the natural light of the intellect, such as arithmetic and geometry and the like. There are some which proceed from principles known by the light of a higher science. Thus the science of perspective proceeds from principles established by geometry, and music from principles established by arithmetic. And in this way sacred doctrine is a science, because it proceeds from principles established by the light of a higher science, namely the science of God and the blessed. Hence, just as the musician accepts on authority the principles taught him by the mathematician, so sacred science believes the principles revealed to it by God. (I, Q. 1, Art. 2)

⁶⁴ This is not the same as Aristotle's *Metaphysics*, in which he studies general being, as opposed to that of special sciences which just “cut off a part of being” (1003a20–26). In the *Posterior analytics* he is talking about the principles of demonstration, an area in which he says there can be no “crossing over” from one discipline to another. These parallels with the *Posterior Analytics*, it should be added, were first pointed out to me by Ruiz Simon.

⁶⁵ Gayà 2002, 105, remarks that the term *subalternata* which Llull used in passages quoted here was one with profound resonances in medieval theology. Note the similar formulation concerning the subordination of theology—along with philosophy, law and medicine—to the Art at the beginning of his career discussed on p. 53 above.

Aquinas is here reworking Aristotle as to the hierarchy of sciences (with the same examples of arithmetic and geometry) to make a place for theology, which, as a science, was *subalternata* to a “science of God”, or revelation as expressed, for instance, in the Articles of Faith “which are the self-evident (*per se nota*) principles in the knowledge (*scientia*) that God has of himself.”⁶⁶

This, as we will see, contrasts notably with the place Llull accords to theology, and especially in its relationship with the Art. That one was intended for the other was something which Llull stated flatly:

One must know that, even though this Art is necessary in every subject, it has principally been developed for the sake of theology, for theology is the goal of all other sciences. (*Introductoria Artis demonstrativae* [MOG III, ii, 3: 57])

But all the same, the place he accorded to theology was remarkable. In the *Lectura Artis quae intitulum Brevi practica Tabulae generalis* he says:

God is the subject of theology. We would like to show, moreover, how this science [i.e. the Art] is applicable to theology, that is to say, how to apply the principles and rules of this science, which are general, to the principles of theology, which are subalternate to these general principles and [at the same time] proper to theology itself. Thus, when one says that goodness is that by reason of which good does good, etc., this can be applied to God, angel, man, fire, etc. (*ROL XX*, 388)

So, after contravening Aristotle about the possibility of a general science, he is now, with such an open statement of theology as a *scientia subalternata* to his Art, opposing the most generally accepted contemporary formulations on the place of theology. And he does so by explaining the generality, or univocality, of his definitions, equally applicable as they are to all levels of being, as are the Rules, which is what he shows in the section following the one just quoted. Then, after using the Art to answer questions about theology (among them, again, that of “Whether theology is a science?”), he adds a little colophon:

We have explained the application of this Art to theology by solving some questions by means of reasons. Now we would like to apply it by solving questions based on authorities, by reducing authorities to the necessity of

⁶⁶ *In Boetium de Trinitate*, Q. 2, art. 2, ad 5, cited in Chenu 1969, 72, in a whole section devoted to “Théorie de la subalternation”.

reasons or arguments. And this we do to give a doctrine how by means of this Art authorities can be reduced to demonstrations.⁶⁷

This is followed by examples of four Biblical passages from Genesis, Isaiah, and two from the Gospel of John. So theology is not only sub-alternate with respect to the Art, but the latter can be used to show how the Bible can be “reduced” to necessary reasons, that is to Lullian demonstrations.⁶⁸ This was an extraordinary pretension for a medieval thinker, but as with many other surprising aspects of Llull’s system, it was not gratuitous; it was not only consistent with his views on the question of faith vs. reason—i.e. that if faith is true, that truth must be demonstrable—, but also seems to have been carefully considered with respect to its possible impact on his contemporaries. Both of these points can be seen in two works written in Paris in 1298, with an eye to the Faculty of Theology.⁶⁹

The first is the *Disputatio eremitae et Raymundi super aliquibus dubiis quaestionibus Sententiarum Magistri Petri Lombardi*, written to prove that the Art was capable of answering questions deriving from the famous *Sentences*, which opens with the question “Whether theology is properly speaking a science?” The question is almost identical with that of Aquinas we have just quoted, as are the first words of the answer about the existence of two kinds of science, but almost immediately it veers off in a very different direction:

Theology is a science in two ways, appropriated and proper. Appropriated according to faith given by God, as a Catholic through faith believes the Articles of Faith and Holy Scripture which he does not understand by [rational] arguments. It is properly speaking a science since it is not the proper function of the intellect to believe but to understand (*intelligere*). (*MOG IV*, iv, 2: 226)

He goes on to explain that faith permits the believer to attain those things which he might otherwise be unable to reach, either because they are too lofty and subtle, or because ordinary, intellectually untrained

⁶⁷ *ROL XX*, 395. See Bonner 2005, as well as p. 154 above. Hames 2000, 235–8, 255, suggests a possible impact of this approach on leading contemporary Jewish thinkers such as Solomon ben Adret.

⁶⁸ In the *Ars compendiosa Dei* (*ROL XIII*, 327) he even reduces to necessary reasons the quotation of Gregory the Great which he knew others would oppose to his Art: “Faith has no merit if it needs to be proved by human reason,” for which see Ch. 1, n. 44 above. Two pages earlier he did the same for “If you do not believe, you will not understand” from Isaiah, for which see Ch. 5, n. 126 above.

⁶⁹ See p. 178 above for these two works and the bibliography on them.

people are incapable of understanding them. He then explains the second point:

That theology is properly speaking a science, I show thus: it is the proper nature of the intellect to understand, as I said before, just as—and much more so—it is the proper nature of fire to heat, and of the eyes to see. Now since God is the subject of theology, since he is intelligible with his greatness of intelligibility, it follows that... (Ibid.)

We won't go into his arguments, because our concern here is more with the question of how Llull reformulates the *ordo scientiarum*. Notice the central place given Rule G of the ternary Art, which treats of proper and appropriated qualities, as well as the phrase which Llull repeats frequently, that it is not the proper function of the intellect to believe but to understand.

The second work is the *Declaratio Raimundi per modum dialogi edita contra aliquorum philosophorum* written to prove the correctness of the condemnation twenty years earlier of 219 articles circulating in the Faculty of Arts and which the Church found to contain matters of dubious orthodoxy. It is presented as a dialogue between personages called Raymond and Socrates, which begins with a discussion as to what foundations might be acceptable for their dispute. Raymond suggests the Articles of Faith which he has recently proved in the *Book of the Articles of Faith*,⁷⁰ but Socrates will have none of it, saying they were already theological, which was a “positive” science, not something proved by necessary reasons. Raymond insists that in the aforementioned book he has done just that, i.e. proved the Articles by necessary reasons, but Socrates is unmoved. So Raymond finally gives in and suggests that instead they argue according to the principles of the “newly formulated (*inventa*) *Tabula generalis*”, a proposition as acceptable to Socrates as the previous one was unacceptable.⁷¹

The word “positive” in the previous passage refers to a dichotomy between positive theology based on faith, on the one hand, and demonstrative or argumentative theology, on the other. The first was so called because it was based on generally accepted “positions” (“theses” in Greek). Llull's formulation of the dichotomy is summed up in the *Proverbs of Ramon* in terms of two powers of the soul: “Positive theology is based

⁷⁰ This is the work usually known by the title of its introductory poem, *Apostrophe*.

⁷¹ *ROL* XVII, 255–6; Lulle 2006, 4–5.

on the will, and demonstrative on the understanding.”⁷² And of course the whole point of his system was to be able to approach theology—and above all the Articles of Faith—from an acceptably rational, demonstrative point of view. This was essential for the conversion of unbelievers, as he explained in one of the instances when he tells the story of the missionary who succeeded in persuading the sultan of Tunisia of the falsity of Islam. When the sultan asked for the corresponding proof of Christianity, the missionary replied: “The Christian faith cannot be proved, but here is the Apostles’ creed (*symbolum*) expounded in Arabic. Believe it!” On this statement, which obviously angered the sultan, Lull comments: “This is what the brother replied, since, although he was a man versed in letters and moral philosophy, he was only *positivus*, and not *probativus* by means of reasons.”⁷³

It is this same dichotomy which justifies his “reducing” authorities—even the Bible—to necessary reasons. As Lull expresses the matter in the *Liber de refugio intellectus* Lull says that “authorities can be explained either by positive or by argumentative sciences, positive through belief and argumentative through understanding, by which we can avoid false opinions.” (*ROL XI*, 232)

The mention of “opinion” here connects this dichotomy to that between opinion and scientific knowledge, which we found in the quotation above from the beginning of the *AGU*. As for reducing authorities to necessary reasons, since most medieval sermons were based on Biblical themes, this was what would permit him to apply his Art to the construction of sermons, a topic to which he would devote considerable attention from 1304 on.⁷⁴

⁷² *ORL XIV*, 301. Garcías Palou 1958 suggested that Lull might have been the first Latin author to use the term “positive theology”, and Lohr 1973 pointed to a possible Islamic origin of the term. See also Chenu 1969, 73, n. 1. For the *modus argumentativus* at the heart of the scholastic method, see *ibid.*, 67.

⁷³ *Liber de acquisitione Terrae Sanctae*, ed. Longpré 1927, 276, and Kamar 1961, 126. See p. 13 above for this oft repeated parable.

⁷⁴ See p. 266 above.

*Key Aspects of Llull's Art**Dignities, Principles*

The importance accorded to the Principles of the Art, as well as their role as dignities when referred to God, can scarcely be over-emphasized. Already in connection with the *Book of Contemplation*, Jordi Gayà speaks of the decision, transcendental to all of Llull's production, of placing the divine dignities in the center of his scientific discourse.⁷⁵ Whether they had their 'origins' in the divine names of (Pseudo-)Dionysius the Areopagite, the primordial causes of John Scotus Eriugena, the Islamic *ḥadrās*, or the Judaic *sefirot*, and discussing how similar or different is their role in these other areas, is perhaps less fruitful than thinking of them as something that would set up resonances with the three religions.⁷⁶ They were a starting point familiar to everybody and on which everybody could agree; from there Llull could argue that, given that God had the attributes of goodness, greatness, etc., according to his *rationes necessariae* they *must* be of a certain nature. The trick was to ensnare his interlocutors with unobjectionable foundations, whose consequences they would then be unable to refute.

Very soon, the dignities become inseparable from their activity. Jordi Gayà has remarked, however, that in the *Book of Contemplation* there is as yet no mention of their activity.⁷⁷ But by the time we get to the *Book of the Gentile*, Llull speaks of an activity in God, whereby "from infinite good in greatness, eternity, etc. [there] would be engendered infinite good in greatness, eternity, etc.", a mechanism which would describe the self-production of the three persons of the Trinity.⁷⁸ And on the following page he gives a kind of precursor of the correlatives (I give the original Catalan as well, so the reader can see the verb forms):

By this impossibility is signified the fact that God's power must empower, his wisdom make wise, and his love love. (*Per la qual impossibilitat és significat que al poder de Déu se cové poderejar, e a la saviea saviejar, e a la amor amar.*)⁷⁹

⁷⁵ Gayà 1979, 14.

⁷⁶ See Yates 1960; Yates 1966, 178ff.; Gayà 1989; Cruz Hernández 1977, 75–78; and Hames 2000, 119–120, 124–131.

⁷⁷ Gayà 1979, 17.

⁷⁸ *SW I*, 196.

⁷⁹ *SW I*, 197 (see n. 16 there, as well as *SW I*, 233, n. 47). For the Catalan text see *NEORL II*, 94.

Then, as we saw, with the *Lectura super figuras Artis demonstrativae* appear the correlatives, not only in God, but in all levels of being, each component of which has its *-tium*, *-bile*, and *-are*. This is a trio which permits Llull to articulate a remarkably totalizing pattern, which Pring-Mill has called a “Trinitarian world picture”.⁸⁰

At the same time, it permits him to generalize the dignities and their semblances (*similitudines*, *semblances*) to all levels being, with divine goodness mirrored in the goodness of the created world. Then Llull shifts gear, as it were, and they become, along with the concepts of Figure T, the Principles which form the corner-stone of the ternary Art. There, however, he hardly mentions their role as semblances, possibly to present a more general point of view, one no longer to be seen as working exclusively from the top down.⁸¹ But, as we have seen, when applied to God they, along with most of the concepts of Figure T, reappear as Dignities. Yet whether applied to the Principles or to the Dignities, Llull’s definitions of “goodness” or “difference”, which begin with the ternary phase, are equally applicable to all levels of being, or as Llull himself put it in the passage we quoted several pages back: “Thus, when one says that goodness is that by reason of which good does good, etc., this can be applied to God, angel, man, fire, etc.”⁸² Finally, in the post-Art phase, since Llull is now concentrating almost exclusively on theological works with no new formulations of the Art, they reappear as Dignities, while their role as Principles practically disappears.

During the ternary phase, however, these Principles with their correlative articulation form the backbone not only of the Art, but also of the structure of being, as Llull so remarkably showed in the *Tree of Science*.⁸³ The gap between ontology and epistemology has been closed; the necessary, substantial and accidental principles of the mode of being of a thing are now identical with the Principles of the Art, permitting Llull to present a unified *modus essendi et intelligendi*.⁸⁴

⁸⁰ Pring-Mill 1955–6.

⁸¹ Certainly in no work of the Art of the ternary period are the Principles referred to as “semblances” of the dignities. Of the few occasions in which they are referred to that way in other works of the same period, most of them seem to be concentrated in the *Liber de possibili et impossibili* (see *ROL* VI, 410–1, 413, 419, etc.).

⁸² See p. 280 above.

⁸³ See the various papers in Domínguez Reboiras et al. 2002.

⁸⁴ *Ars inventiva veritatis* (*MOG* V, 38). See Colomer 1975, 57, and Colomer 1997, 25, where he comments on its Platonic and ‘onto-theo-logical’ nature. See also Ruiz Simon 2005, 192, where it is associated with the turn towards Pseudo-Dionysius and Proclus taken by Llull at the end of the quaternary phase.

As with Platonic forms, an essential characteristic of the Principles/Dignities is their intelligibility. In this sense, they play a primary role in allowing man to fulfill his obligation to love, understand (*intelligere*), and remember God. And they even allow one to define God:

[God] is knowable through his most holy reasons or properties. For if it is asked “What is God?”, one should answer that God is that being in which his goodness and greatness, eternity, power, wisdom, will, etc., convert and are one in number. (*Disputatio fidei et intellectus* [ROL XXIII, 232])

The presence and structure of the dignities permits Lull to make all kinds of other assertions about the Divinity, and above all to articulate his all-important proofs of the Trinity and the Incarnation, as expressions of God’s activity internally and externally.

In this connection one should point out the central place accorded to the Trinity, not only doctrinally and ontologically (as the pinnacle of the “Trinitarian world picture”), but even tactically and methodologically. Methodologically, because its ontological primacy permits Lull to use the Trinity even to prove the Incarnation (*ibid.*, 252). Tactically, because Lull knew full well that, along with the Incarnation, it was for Muslims and Jews the most controversial aspect of Christianity, and that, if he were incapable of proving it, his entire project would be destined for the scrap-heap. Thus we find him insisting on it in places that might seem surprising. In the *Liber praedicationis contra Judaeos*, for example, where one might expect the usual emphasis on the Jews’ error in believing that the Messiah had not yet appeared, Trinitarian arguments are more prominent and frequent.⁸⁵ Harvey Hames has shown how it was precisely these arguments that the Jewish community of Barcelona might have found most threatening, and how it seems to have forced a serious theological debate on a topic with which the Jews had had to come to terms.⁸⁶ He adds:

If, however, the Trinity could be proven in a conclusive manner it would be far more difficult to avoid acceptance of the necessary Christian doctrines that arise from it. It was for this reason that much of the Lullian literary creativity was focused around the issue of the Trinity. Lull developed

⁸⁵ The opening sermons of the book are all on the Trinity. Next in importance is the Incarnation; and far behind the Messiah. Word counts done with the CETEDOC show that (with the asterisk indicating alternate following possible letters) “trinita*” appears 101 times, “incarna*” 64 times, and “messia*” 10 times.

⁸⁶ Hames 2000, in two chapters entitled “The Lullian Trinity: A Means to an End?” and “Unity and Trinity: A Jewish Response”.

a theory of the Trinity which he truly believed would be acceptable to both Jews and Muslims and would lead them to realize the truth of the Christian religion. Moreover, Llull specifically targeted his exposition towards the Jewish and Muslim elites, knowing that they themselves were wrestling with the problem of the nature of the Godhead on the one hand, and the relationship between the Divine and His Creation on the other. Converting these elites would be tantamount to a mass conversion of Jews and Muslims, and it was this that Llull ultimately hoped to achieve. (Hames 2000, 192–3)

A combinatorial, relational system

The most striking and unusual aspect of the Art has always been its combinatorial mechanisms, and especially the wheels and charts with which its various presentations begin. And since it is an aspect of the Art which has elicited such a variety of historical interpretations (Agrippa von Nettesheim, Giordano Bruno, Leibniz, etc.), it might be best to try to get an overview of its nature and purpose.

In the first place I would like to emphasize some of its mathematical aspects. We saw how in the quaternary stage—*avant la lettre* but quite correctly—he used graphs and adjacency matrices to display his binary combinations.⁸⁷ We have also seen how in the next stage, realizing these structures would not work for ternary relations, he had to resort to revolving disks (the Fourth Figure) and the Table to display them. Another difference between the two stages is that in the *AD* Llull allowed combinations with repetitions, whereas in the ternary period not. The reason was that with relations he could study those within a concept or between one expression of a concept and another; but with the predication which made its appearance in the ternary Art, repetition (“goodness is good”) made little sense.

One aspect of his binary combinations which has attracted mathematical attention lately is how he used it—and especially its adjacency matrices—to formulate theories of voting that people had until recently thought had been first proposed in the eighteenth century by Borda

⁸⁷ This systematization is the work of the *AD*; as explained on p. 96 above, in the earlier *ACIV* he had not yet worked a consistent set of first and second figures, i.e. of graphs and their adjacency matrices.

and Condorcet.⁸⁸ This conjunction of graph theory with the theory of voting seems to me quite remarkable.

For Llull, all this combinatorial apparatus was important because it permitted him to systematize the essentially relational basis of his system.⁸⁹ He rarely presents even its foundations in an isolated manner. Figures S and the Elemental Figure of the quaternary Art form typically complicated webs of relations, in which the important thing is not so much any single component as its place within that structure. Figure T is, of course, the instrument for studying relations. Even in Figures A, V, and X the cross-lines show relations of concordance between all or some of their components. Nor should one forget that metaphor, so important in the quaternary phase, is a relational device. In the ternary phase the foundational definitions are relational: “Greatness is that by reason of which goodness, duration, etc. are great”, “Difference is that by reason of which goodness, etc. are clearly distinguishable from one another”, and so on. With the introduction of the correlatives it is all of being whose essence has become relational, and this at every step of the ladder. In both phases Llull’s methods of demonstration are relational, comparatively in the quaternary and by his system of mixture in the ternary. And Llull even uses these techniques in mystical works, such as the *Flowers of Love and Flowers of Intelligence* and the *Tree of the Philosophy of Love*. Similarly, in the novel *Blaquerna* the chapter of the *Book of the Lover and the Beloved* is followed by a kind of how-to-do-it manual, called the *Art of Contemplation*, with one section, for instance, entitled “How Blaquerna contemplated God’s virtues” in groups of three (*de tres en tres*).⁹⁰

In the *Logica nova*, Llull in studying “relation” under Question F of Quantity says:

⁸⁸ See McLean and London 1990 and 1992; Hägele and Pukelsheim 2001 and 2004; Pukelsheim 2002, and Drton et al. 2004.

⁸⁹ See Bonner 1994, of which much of this exposition is a summary.

⁹⁰ All three of these works have been translated into English, in *SW* II, Llull 1926, and Llull 1925 respectively. In another field, in the *Principia philosophiae*, Llull carries out his investigations on concepts in groups of one, two, and, three, ending the works saying that he hasn’t had the time to carry out higher groupings as he had hoped (see the editions in *ROL* XIX and *NEORL* VI). Furthermore, these groupings are based on geometric figures taken from the *De quadratura et triangulatura circuli* and *Liber de geometria nova* written a few months before, geometric figures conceived in part as a visualization of different relational possibilities.

“Relation” has quantity in two ways: dual and ternary. Dual as in father and son, action and passion, abstract and concrete, and so on. Ternary as in *intellectus, intelligibile, intelligere; possificativum, possificabile, possificare; calefactivum, calefactibile and calefacere*.⁹¹

The novelty of this formulation can best be explained by quoting a modern text on computer logic:

What Aristotle missed was the basic building-block character of binary relations $R(x, y)$ such as “ x is less than y ” and of ternary relations $S(x, y, z)$ such as “ z is the sum of x and y ”, etc. He used only unary relations or predicates $P(x)$ such as “ x is red”. He generally coded relations $S(x, y)$ such as “ x is the grandfather of y ” as the property $S_y(x)$, x has the property of being the grandfather of y .

There was no real defect in Aristotle’s theory of quantifiers. He rather lacked explicit propositional connectives and relations of multiple arguments in his logical formulas. This lacuna was really only remedied by authors of the late nineteenth century such as C.S. Peirce (1839–1914), E. Schröder (1841–1902) and G. Frege (1848–1925).⁹²

I have quoted this in extenso not only because once again we find Llull in his own way anticipating later developments, but because it helps reveal the nature of a very essential innovation of his Art, one of which he seemed conscious enough to feel the need to define and explain it.

The exterior signs of this are, of course, all the graphic devices we have just mentioned, which are Llull’s way of organizing and displaying all possible binary and ternary relations. They have, moreover, another

⁹¹ *ROL* XXIII, 67; *NEORL*, 49–50. It was the late David Rosenblatt who first pointed out to me the historical importance of this definition. One can find a similar definition in the *Tree of Science*, “Elemental Tree”, V “Leaves”, 3. For other definitions, see Bonner and Ripoll 2002, s.v. See also Llull’s use of the somewhat related concepts of *prioritas, secundioritas*, and *tertioritas* (ibid., s.v.), so remarkably similar to Charles Peirce’s “firstness”, “secondness”, and “thirdness”, explained in his significantly entitled “The Logic of Relatives” reprinted in Peirce 1992. Fidora (in the forthcoming *SL* 47) explores the possibility that Peirce took this from Llull, through a copy of the 1617 printing of Lullus 1996 which he owned and where the terms appear in Valerio’s commentary on the *Tree of Science*.

⁹² Nerode and Shore 1993, 299. One can understand their omission of Llull, but they are perhaps a bit unfair to Leibniz who, in his *Dissertatio de arte combinatoria*, referred to the two sorts as *com2nationes* and *con3nationes*. Ruiz Simon has pointed out to me that this kind of relation, although absent from the *Prior Analytics*, is discussed in the *Topics*. Since the foundations of classical logic are to be found in the first work, and those of Llull’s Art (in a more indirect way) in the second, this division is not without significance.

important function. In modern logic, a system is called sound if all its acceptable (or as logicians call them, well-formed) formulas are true, and complete if all well-formed formulas are theorems of the system. Since in the Middle Ages—in fact at any time before the twentieth century—demonstrating either of these concepts was out of the question, the only way Lull could do it was to offer a *closed* system within which one could *display* all its possibilities. The soundness of the system could thus be shown—at least in theory—by testing any of its components, and the positive result of such a test within the closed system ensured its completeness. Because of his desire to show the validity of so novel a system, Lull repeatedly explained its soundness by stating how any of the other combinations displayed in such-and-such a figure or Table could be used to achieve a similar result, and how at the same time they were general to any result desired, thereby trying to insure its completeness. Here the graphic devices were essential, since only by *displaying* all the possibilities were such assertions possible.

Moreover, the combinatory mechanisms with their accompanying graphic devices made possible what was perhaps the most innovative of Lull's accomplishments: to create an Art that was generative, which upon a base of a strictly limited number of concepts could build a whole constellation of demonstrations and explanations. And it was this generative nature of the Art which held such a powerful fascination for later thinkers, such as Agrippa von Nettesheim who presented it as an alternative to the rhetorical-mnemonic-logical system of Ramus, Giordano Bruno who saw it as a way to explore the connections among his infinity of worlds, to Leibniz whose youthful interest in the Art planted the seed that led to his later *Mathesis universalis*. Not only were their aims different, but the use they made of Lull's techniques varied from something vaguely similar to something entirely different, but it was Ramon Lull who had first shown how one could construct a generative system. The fact that the aims and results in later centuries were so different is perhaps the final irony of something generative: its very nature suggests other possibilities, of new material to generate or of other ways to generate it.

Lastly, the combinatorial Art with its graphic devices, all used as a generative system, has made the Art into a distant precursor of computer science. As Eusebi Colomer has said, "We are all aware that information processing rests upon a dual basis: the idea of a logical calculus and its subsequent automation. Both items form part, though in a slightly rudimentary manner, of the combinatorial project which

formed a basic element of Lull's Art."⁹³ Not only that, but I would not be surprised if some day large portions of the Art could be worked out in a computer language such as PROLOG.⁹⁴

A closed system

The fact that the Art worked with a strictly limited set of concepts also meant that it was, to coin a word, endo-referential: all the material it needed to construct its arguments was found within its own domains, with no need of any exterior input. And if outside material was introduced, it was merely to show how this closed system could deal with it. Now Lull was proposing this method in a world where, for theology and philosophy, intertextuality was the order of the day, a world whose basic technique was to answer a question by presenting the arguments for or against it on the part of a series of authorities, to reply to each of them, and to end saying "I declare that..." In a world where these authorities held an importance which for us is perhaps difficult to imagine, Lull put aside any reference to them as foundations for his arguments, and offered instead a closed circuit, one which looked inward to its own mechanisms, and if it made references, it was to other works of the Art in which such and such a question had been treated in greater detail.⁹⁵ This is particularly noticeable when we think that Lull was trying to demonstrate the tenets of a revealed religion, whose revelation was expressed in a book. But the Bible—which was, after all, *the* authority—is hardly ever mentioned in Lull's vast work, and when it is, again it is to use the Art to show how a particular passage *must* be true, or, as we have seen that he puts it, "to reduce it to necessary reasons".⁹⁶

⁹³ Colomer 1979, 114, reprinted Colomer 1997, 86.

⁹⁴ In fact one programmer, Cameron Buckner, told me he did not think it at all out of the question, but his intentions in this direction were waylaid—unfortunately but understandably—by his need to find a more practical outlet for his abilities.

⁹⁵ See Minnis 1984, where (p. 13) he says that "Every discipline, every area of study, had its *auctores*. In grammar they were Priscian and Donatus together with the ancient poets; in rhetoric, Cicero; in dialectic, Aristotle, Porphyry and Boethius; ... in medicine, Galen and Constantine the African; ... in Canon Law, Gratian; in theology, the Bible, and subsequently, Peter Lombard's *Sentences* as well." See Bonner 1993 for more information on this subject.

⁹⁶ Somebody from a culture totally unfamiliar with Christianity could read Lull's vast opus—dedicated principally to that religion—without ever realizing that the Bible

The need to keep arguments within such strictly limited bounds undoubtedly began with his desire in religious polemic to steer clear of the use of authorities, which inevitably turned such disputations into futile exercises in hermeneutics, in which nobody convinced anybody because each could take refuge in the idea that their own interpretation was the only correct one. It also permitted him to address himself to “the beliefs and practices” of his opponents, as opposed to his Dominican contemporaries, who “were still arguing with books, rather than with real and living people.”⁹⁷

By limiting the premises of arguments to a set he had chosen, but which might seem acceptable to his opponents, he also kept the discussion on his own chosen ground, preventing its being side-tracked into the usual business of “yes, but so-and-so said that...” Nowhere within his closed system could such objections be introduced. His opponents—Muslim, Jewish, or others—, once they had accepted Llull’s bases, had to try to undo the arguments he had forged from them, or use them to forge their own counter-arguments. But since they had been removed from their own familiar ground, this made things much more difficult for them.

Like much else with the Art, however much this endo-referentiality may have begun as a polemical tool, very soon it took on a life of its own, becoming one of the most characteristic aspects of Llull’s endeavor. I would also say that Llull quickly found another use for it, more within his own religion. If his principal aim was to persuade people to remember, understand, and love God, these were things which could not be done in the usual rote fashion in which most people deal with their religion. True remembrance, understanding, and loving cannot be done without the necessary effort, principally to understand and love. In *Felix* he introduces *exempla* which are purposely difficult, so as to oblige the reader/listener to exercise his mind and soul. Llull feels that the high goals of the faith require a commensurate effort on the part of the faithful. The usual comforting approach with Bible stories, well-known

is essentially a narrative work, peopled with myriad prophets and saints; Llull only cites it for an occasional sentence or precept, and very seldom at that.

⁹⁷ For this formulation of Harvey Hames, see the passage corresponding to Ch. 1, n. 40 above. Llull says that his *Disputatio Raimundi christiani et Homeri saraceni* “was written so they could dispute by reasons and not by authorities, for authorities can be rejected because of differing interpretations (*diversarum expositionum*).” (*ROL XXII*, 261), the *Proverbs of Ramon* where he flatly states “in disputation by authorities one finds no rest” (“*disputar per auctoritatis no ha repòs*”. *ORL XIV*, 271).

exempla, ecclesiastical ceremony, etc. breeds a familiarity which banishes deeper understanding and contemplation.⁹⁸ One could in fact say that much of Lull's endeavor is a conscious attempt to uproot both non-Christians and Christians from familiar ground, a ground based on the automatic acceptance of their authorities.⁹⁹

An open system

If Lull's system was closed as far as input was concerned, it was open with respect to its output, something which ties in very closely with what we just said about his expecting the reader's cooperation and effort. To the medieval reader the word "art"—the normal translation of the Greek *technē*—referred to a practical science, like medicine, one intended for the production of concrete results, one that offered a system of precepts and rules for attaining a given end.¹⁰⁰ Such a technique was not so much directed to telling people what to think as to presenting a method by which they could on their own arrive at the correct conclusion. One can see this in so many works of the Art that have sections on applying, teaching, or accustoming oneself (or "habituating oneself" as Lull frequently puts it) to the use of the Art.¹⁰¹ An author does not suggest such things for a philosophical or theological treatise, only for a 'practical' art.¹⁰²

⁹⁸ His own *exempla*, with few exceptions, are not taken from the medieval anthologies of same so abundantly used by medieval preachers.

⁹⁹ In the *Book of Contemplation*, Lull expresses the reasons for his insistence on novelty: "A person take greater pleasure in words that are new and strange, because by them he can better arrive at his desired goal, than with words he is accustomed to hear, in which and by which his soul does not find satisfaction." As a further justification for "novel reasons and demonstrations", he says: "Any new, strange word is closer to the soul's desire for the things it doesn't have, than old words which the soul has used for those things in which it has not found satisfaction." (*ORL VIII*, 541–2)

¹⁰⁰ Lohr 2001, 166–8; Lohr 2004, 268–272; and Teeuwen 2003, 358–9. The term was applied in the Middle Ages, for example, to *artes liberales*, *artes mechanicae*, *ars dictaminis*, etc. As well as to music, as in *Blaquerna* (*ENC II*, 158), where Lull recommends that readers try the *ACIV*, "which is an art by means of which the understanding rises up to understand, just as the voice does in singing by means of the art of music."

¹⁰¹ In the *AGU* and *AB*, Distinctions X, XII, and XIII are respectively on Application, Habituation, and Teaching of the Art. Of these, that on Application is by far the longest, and contains, for instance, the Hundred Forms.

¹⁰² In the *AD*, in one paragraph of *SW I*, 433, Lull twice mentions the need for "frequent practice" in using the methods of the Art. Can one imagine Aquinas, Ockham or Peter of Spain suggesting such a thing? One does not practice a treatise; but an art, yes.

But Llull takes matters further. As we have seen, in the *AD* he solves each of the 1,080 Questions at the end of the work by means of a chain of compartments containing letters or terms drawn from the figures. For the first 39 he gives explanations of how to interpret these compartments, but then for the remaining 1041 the reader is left to his own devices. As Llull puts it, “the second group is dealt with in a more subtle fashion”, which is a delicate way of asking the reader to make the effort to do it on his own. As if this weren’t enough, Llull adds yet a third group, in which the ‘artist’, with the first two groups as models, is asked to make up *his own* questions and solutions. This new group is not only, he explains, “subtler” than the other two, but it is “the general goal of the entire Art”.¹⁰³

This progress has a fairly remarkable parallel in the *Book of Wonders*, in which the protagonist, Felix, during a long trajectory of listening to *exempla* propounded by hermits, philosopher, and the like, goes from an apprentice who needs to have the meaning of the stories explained, to a more or less expert listener who can interpret them on his own.¹⁰⁴ These stages would correspond to the first two groups above. That corresponding to the third comes at the end, when Felix, on his deathbed, bequeaths to someone else his task of going “throughout the world... to recount the *Book of Wonders* to one and all, and to add to the book as he added to his stock of wonders during his wanderings”. Indeed, the monks presiding at his funeral already find a “wonder” to add to the stock, and the monk who takes the name of the Second Felix then “went throughout the world recounting the *Book of Wonders* and adding to it, according to the wonders he encountered.”¹⁰⁵ So the techniques learnt in the course of his wanderings can now be applied by followers.

¹⁰³ See above pp. 63, 90–92. An almost identical three-stage process is recommended for the *Art of Proposing and Solving Questions* (see *MOG* V, v, 310: 668). At the end of the chapter in the *AB* “Which Treats of Questions”, Llull similarly suggests the reader could formulate and answer other questions concerning the Hundred Forms, and how this could be a springboard for formulating all sorts of other questions and solutions (*SW* I, 643; *DI*, 361).

¹⁰⁴ Bonillo 2005, 225 says that Felix’s long voyage has permitted him at the end to achieve his intellectual emancipation, a voyage that is a metaphor of *homo viator* in his attempt to arrive at the glory of God.

¹⁰⁵ *SW* II, 1104–5. Similarly the end of the *Principia philosophia* Llull explains that he has had to break off the work to study Arabic, but that it could be completed by “some devout and wise man to the glory and praise of our Lord God” (*ROL* XIX, 323; *NEORL* VI, 184). Lola Badia has pointed out to me that the little prologue of the *Arbre exemplifical* in the *Tree of Science* explains that since the topic is vast, since he has others matters to attend to, and since people dislike prolixity, he has had to abbreviate things,

Another work in which the open ending has caused much comment is the *Book of the Gentile*, where, after listening to the extended arguments of the Jew, the Christian and the Muslim, the culmination of the work arrives when the protagonist offers to tell which religion he has chosen. Not only the gentile but the reader himself is taken aback when the three wise men reply that they would prefer not to know, so that they can continue their discussion “according to the manner the Lady of Intelligence showed us”. Part of the reason for this surprise is surely tactical, to avoid making the work seem weighted in a particular direction,¹⁰⁶ but part of it is also doubtless to suggest that the “manner” of the Art which the Lady of Intelligence had showed them and which they have already developed so successfully is something they (as well as the readers) should all continue practicing.

The idea of trying other possibilities is something we have seen frequently in the course of this book. There was, for instance, the Demonstrative Figure of the *AD*, whose alphabet could be adapted to “many other principles of science”, or the Table of the ternary Art where one can “multiply twenty arguments towards a single conclusion” or use other columns each with twenty more. And these are only a few of the many ways in which Lull presents his system as providing models or examples for further elaboration. One could even say that it is an essential feature of the Art, one that makes the Art itself an exemplary structure. It thus can generate not only answers or demonstrations, but even the questions themselves!¹⁰⁷ It is therefore “inventive” in a much broader sense than the normal medieval meaning derived

but that “in accordance with what we will say, we will be indoctrinating the reader on how to make up (find, *atrobar*) new proverbs and new stories (*recontaments*), thus stretching his understanding according to the great matter of this tree.” *OE I*, 799.

¹⁰⁶ Should there be any doubt, it is clear from subsequent works which religion Lull thought had won the day, as for instance when he says in the *Book of the Lover and the Beloved*: “Tell us, fool, how do you know that the Catholic faith is the true one and the beliefs of the Jews or Saracens are in falsehood and error?” He answered, ‘From the ten conditions of the *Book of the Gentile and the Three Wise Men*’” (*DI*, 226). And indeed, that’s where the trap for the unwary lies, and especially in the first condition of the first tree: “One must always attribute to and recognize in God the greatest nobility in essence, in virtues, and in action” (*SWI*, 114; *DI*, 88). As his adversaries will discover, only a divinity with an inward and outward activity capable of producing the Trinity and Incarnation can fulfill the condition of this superlative.

¹⁰⁷ Note the title of the work we have so often cited: the *Art of proposing* (literally *making*) *and solving questions*, where Lull quite literally shows us how to do this. Cf. Ch. 4, n. 120 above.

from the *Topics*, of “finding” strategies for dialectical arguments; Llull’s suggestions for its use as an exemplary, self-generative structure, makes its possibilities almost limitless. The ability to apply the Art beyond its own borders make it what Umberto Eco has called an *opera aperta*, one in which the user is invited to continue the work begun on the pages he has before him.¹⁰⁸

The Novelty of the Art

It has been rightly pointed out that:

Llull was fully aware of the novelty of his epistemological project and frequently insisted upon this aspect. But... it is a mistake to concentrate solely upon this novelty and to forget that the nature and significance of this Art and the meaning of what it offered in the way of innovations cannot be interpreted without taking into account the ‘old’ epistemology with respect to which it defined itself.¹⁰⁹

Llull was quite conscious of the relation of the Art and his logical techniques to contemporary and earlier formulations, and to forget this connection would indeed lead to a serious misunderstanding of their place and their functioning. To be more precise, it would seem that after a beginning in which he confided overmuch in what one might call the stand-alone efficacy of his God-given system, he became increasingly aware of the contemporary climate of thought, and adapted his techniques to conform to what he felt was needed to persuade his contemporaries, until finally, as we have seen in the section on logic, he addressed himself to very specific problems of the *Posterior analytics*.

One of the principal innovations of the Art was the desire to fuse two fields that had until then been treated separately: the *pars inveniendi* of dialectic and the *pars judicandi* of logic (a desire shared by later thinkers such as Ramus or Francis Bacon), and to create a system that could deal with both. The Art could also be ‘additive’ with respect to previous and contemporary thought. We have seen how Llull’s definitions don’t supplant, but can be used alongside the standard type, how he adds a new kind of syllogism and a new fallacy to the classical ones, how the *demonstratio per aequiparantiam* is presented as added to the Aristotelian

¹⁰⁸ The term is from Eco 1989.

¹⁰⁹ Ruiz Simon 1999, 87; also *ibid.*, 16–17.

ones of *propter quid* and *quia*, and how his new ‘middle’ is an extension of the standard logical use, etc.¹¹⁰ We have also remarked during the course of this book on how Llull consciously recycled contemporary material in logic, metaphysics, rhetoric, literature, sermonizing, etc. Or to put it another way, the Art allowed him to present a kind of alternative logic, alternative metaphysics, and so on. At a certain point in his life he even decided that in order to get people to accept his message, and especially its novelty, he must try to present *himself* as an alternative ‘authority’.¹¹¹

On the other hand, just as we must not doubt Llull’s dependence on contemporary formulations, we should not doubt the novelty of his system. One only has to compare the figures and alphabets at the beginning of any work of the Art with the opening pages of contemporary treatises to see that we are looking into different worlds, and in fact many of the things discussed in this book are quite unlike anything found in contemporary thought. Llull was quite aware of this novelty, and announced it not only with reference to the Art itself (“I bring you a General Art newly granted by a spiritual gift”), but also to a series of works in which he applied it to other fields and which have the word “new” in their title.¹¹² All of this was fairly unusual at a time when people were much more comfortable with tradition and with already established ‘authorities’, and when innovation was preferably disguised as tinkering with bits and pieces of the past.¹¹³ To put it another way, many of the trees of Llull’s grove are those of the surrounding forest,

¹¹⁰ See Ruiz Simon 1999, 184ff. The *Compendium seu commentum Artis demonstrativae* (MOG III, vi, 154: 444) asks “Whether this science adds something over and above other sciences”, followed by a lengthy answer which includes not only the *demonstratio per aequiparantiam*, but also shows how the generality of the Art makes it “additive” with respect to theology, logic and the other sciences.

¹¹¹ See Bonner 1998.

¹¹² The quotation is from the *Desconhort*, stanza VIII. For the works with “new” in the title, see p. 18 above.

¹¹³ Umberto Eco (Eco 1987, 5) said that “Medieval culture has a sense of innovation, but people were taught to hide it under the guise of repetition (as opposed to modern culture which pretends to innovate even when it repeats)”, cited by Ruiz Simon 1999, 87, n. 111. Lewis 1964, 211, said: “If you had asked Layamon or Chaucer ‘Why do you not make up a brand-new story of your own?’ I think they might have replied (in effect) ‘Surely we are not yet reduced to that?’... The originality which we regard as a sign of wealth might have seemed to them a confession of poverty.” Llull in his *exempla* is more often than not making up brand-new stories of his own. Even his attitude towards novelty is unusual; see n. 99 above. Ruiz Simon, loc. cit., adds that this insistence on the novelty of the Art, which permits innovation in all the sciences dependent on it, is very modern.

but he has also introduced some quite startlingly new species, and above all the ecology of his grove is fundamentally different from that of the woods around it.

Perhaps the greatest difference between the Art and contemporary treatises, however, is that whereas the latter are primarily directed at offering solutions to problems or clarifications on points of doctrine, Llull's Art offers a method for arriving there, and if it gives solutions and clarifications—which it of course does—they are presented as inevitable by-products of that method. The same could be said of his works of religious polemic, where, instead of the usual tactics of defending one's religion or attacking that of one's adversary, Llull offers a method based on apparently neutral principles, which can end up providing demonstrations (*per aequiparantiam* and hence non-causal) for such things as the Trinity and the Incarnation. So what Llull did was to take the idea of an 'art', which his contemporaries understood as a technique or craft, a practical science like medicine, but to make into one not limited to a single field. Within this broad framework, the reader, rather than being the passive recipient of information, was more like an apprentice smith or mason learning how to use the tools of his trade.¹¹⁴ Thus the possibility we have just seen with the *AD* or the *Book of Wonders* of the reader going from an apprentice, to an assistant working *with* the master—or, as Lola Badia has put it, to a system that is 'interactive'—, finally to the full status of a new master who can 'invent' his own questions or *exempla*.

This in turn is fully concordant with one of the prime goals of Llull's project: to make his readers capable of thinking for themselves, and to show them how do to so correctly in both an intellectual and spiritual sense. This in turn is why he wants to make everything new: not to *épater* but to root out ingrained habits, to rid people of the sloth of routine, to cut them loose from the familiar, so that their understanding and love of God can be more than superficial. Unlike modern paid guides who carefully take people to the top of mountains, he wants to teach them all the techniques they'll need, point the way, and then let them achieve the goal on their own.

Perhaps the most unusual aspect of the Art for a medieval polemicist is the fact that it can be used impartially by everybody, as in the *Book*

¹¹⁴ See the example of the smith used in connection with Figure T on p. 41 above.

of the Gentile it is used identically by the Jew, the Christian, and the Muslim to prove their respective articles of faith. One cannot imagine a contemporary polemical work handing out a perfectly viable argumentative strategy to the author's religious opponents, allowing all to dispute on an equal footing.¹¹⁵ If the framework of the paired concepts in the 'flowers' of the trees is a 'light' version of the Art, one only has to examine the arguments, for instance, of the Jew in Book II, to see that their technique is the same as that of the *AD* analyzed in Ch. 2 above. And this identity of polemical opportunity among the disputants is carried out to the end, where, as we just said, the reader is left hanging as to which religion the gentile has chosen, partly because the three wise men want to keep on using the technique of the Art they have by now mastered to dispute among themselves, to see if one can convince the other two so that all three can be of a single faith.¹¹⁶ So the chief protagonist is the Art, a technique—or a set of techniques—created to take the place of sterile doctrinal confrontations or hermeneutical discussions. Similarly, the elaborate mechanisms of the *Ars demonstrativa* and the *Ars generalis ultima*, so carefully made to appear theologically neutral and non-doctrinal, are in fact nothing but the complete working out and justification of Lull's method for religious persuasion, and at the same time the foundations for a science of sciences. This was why Lull's Art and logic were at the very center of his entire enterprise.

¹¹⁵ Arnaldez 1993, 340, makes the interesting suggestion that "the Art would seem to be capable of allowing not only the discovery of the truth, but also the rules for its own use, which would make it 'autocritical', a necessary condition for arriving at a universal agreement." That the *Book of the Gentile* concerns an ideal, not a real, discussion makes no difference to Lull's intention, which was to present a model of how such a method might function. Moreover, the fact that in later works such as the *Disputatio Raimundi christiani et Homeri saraceni* the tone is harsher is not really relevant here, since that work purports to be the account of a real dialogue which took place during Lull's imprisonment in Bougie, not a school-book model like the *Book of the Gentile*.

¹¹⁶ Till now they have only been trying to convince a fourth party, the gentile, and not each other.

APPENDICES

THE MARTIN GARDNER PROBLEM

I said in the preface that I would omit criticisms of predecessors. If I make an exception in the case of Martin Gardner it is because of the negative impact his essay on “The Ars Magna of Ramon Lull” has had in the scientific community. This is not the case with medievalists, who hardly know this essay; but for every non-medievalist who tackles Lull from the angle of the history of logic, mathematics, or combinatorics it would seem to constitute their first introduction to the subject, one which colors most of their subsequent thinking thereon. This is hardly surprising in the light of Martin Gardner’s justifiably high reputation, and considering that this essay is the opening gun of his popular and otherwise excellent *Logic Machines and Diagrams*.

In this essay, however, he admits to having “relied almost entirely on E. Allison Peers’ magnificent biography *Ramon Lull*, London, 1929”, a work which, by the time Gardner was writing in 1958, was already out of date. In addition, Peers was a literary scholar, not a philosopher. His chief interests were in Spanish mysticism and in the romantic movement in Spain, all of which fit in perfectly with the vision he—in consonance with earlier Catalan nationalists—had of Lull. The medieval Majorcan’s fervor, which permitted such marvelous flights of mystical and literary fancy, fitted these people’s needs like a glove. That he was perhaps a bit unbalanced mentally, and certainly out of his depth in the more serious fields of theology or philosophy, only made the romantic image more convincing and lovable.

It is by having “relied almost entirely on” Peers’ biography that Gardner trots out the stories about Lull’s riding into church on horseback in pursuit of a woman who turns out to have a breast eaten by cancer, about having had a verbal duel with Duns Scotus in Paris, about his martyrdom at the hands of a Muslim mob, etc., all of which, by 1958, were beginning to be seriously questioned. Curiously enough, Gardner

not only copies, but embroiders on some of Peers' Victorian sexual obsessions. In *Felix* a hermit tell an exemplary story of a burgher who suddenly worries about being more disturbed by the idea of urinating in public in the main square of the town than that of secretly sinning with a prostitute. The anecdote is told simply, without any descriptive or lurid details. Peers found this story "moral in principle, but in detail, repulsive to a degree"; Gardner embellishes Peers by saying that in *Felix* we find Llull "describing profane love in scenes of such repulsive realism that they would shock even an admirer of Henry Miller's fiction."¹

Gardner's not bothering to read Llull's texts themselves might be anecdotal with *Felix*, but with the Art, since that was the subject of his essay, the consequences were more serious. A quick glance at some of its works would have saved him from presenting the Art as a practically endless series of "spinning wheels" (he seems to assume that all of the circular figures rotate; he explicitly has Figures S and T rotating!), wheels capable of producing all sorts of combinations of concepts which will solve almost any problem. We might even, he explains, be able to find some uses for these circles, such as finding the first and middle name for a newborn baby, making a children's toy where one could combine a giraffe's head with the body of a hippopotamus, or combining plot elements to produce a novel. Gardner's only addition to the reissue of his essay in 1982 was a final endnote concerning, among other things, a device called the Think Tank, widely advertised in 1975, for combining 13,000 plastic chips each with words on them to come up with sets of seemingly unrelated words with which to make mental associations.

He cites, but clearly does not use, Longpré 1927 and Carreras 1939–43, who could have set him on a surer path. By the time of the second edition of 1982, the English writings of Yates, Pring-Mill, and Hillgarth had come on the scene, with a renewed vision of Llull that made Peers seem to be describing someone out of the Spain of Washington Irving's *Tales of the Alhambra* or Mérimée's and Bizet's *Carmen*.² Gardner's source for the Art itself was Erdman 1910, which, for its time

¹ See *SW* II, 939; Peers 1929, 217; Gardner 1982, 7.

² Hillgarth 1971, vi, was already saying that Peers 1929 was "not only out of print but out of date", and that while "Peers fully appreciated Llull's gifts as a poet and novelist, he showed no understanding of or interest in Llull's philosophy and especially the Art."

was excellent, but, by improving on that source the way he did with Peers on *Felix*, he came up with his chains of “spinning wheels”.

The problem is that Gardner’s description of a “bizarre, now forgotten Art”, forged by someone with “unmistakable hints of paranoid self-esteem” very like those “eccentrics who become the founders of cults”, has projected its long shadow over people writing about Lull from the scientific/logical side of the fence. This “troubadour who sang his passionate love songs to his Beloved and twirled his colored circles as a juggler twirls his colored plates” still finds people assuming Lull’s combinatorics are all due to twirling disks. Given Gardner’s well-earned reputation for scientific solvency (I myself for years read with great relish his articles in the *Scientific American*), it is only natural that people should follow his lead, but in this case he had simply not done his homework.

CHRONOLOGICAL LIST OF LLULL'S WORKS

We give this list so that the reader, using the catalog number in the Index of Works, can orient himself among Llull's vast production. It only includes works mentioned in this book. Indented works form part of the one they follow. For further details the reader can consult the Llull DB at <http://orbita.bib.ub.es/ramon/>, going to the section on Works.

- I.1—*Compendium logicae Algazelis, Lògica del Gatzell* [1271–2 (?), Montpellier]
- I.2—*Book of Contemplation* [1273–4 (?)]
- II.A.1—*Ars compendiosa inveniendi veritatem (ACIV)* [ca. 1274, Majorca]
- II.A.2—*Lectura compendiosa super Artem inveniendi veritatem* [1274–6 (?), Majorca]
- II.A.3—*Ars notatoria* [1274–6 (?)]
- II.A.4—*Llibre de demostracions* [1274–6]
- II.A.5—*Book of the Order of Chivalry* [1274–6]
- II.A.6—*Doctrina pueril* [1274–6]
- II.A.8—*Llibre contra Anticrist* [1274–6 (?)]
- II.A.9—*Book of the Gentile and the Three Wise Men* [1274–6 (?)]
- II.A.10–13—*Quattuor libri principiorum* [1274–83]
- II.A.10—*Principles of Medicine, Liber principiorum medicinae* [1274–83]
- II.A.11—*Principles of Philosophy, Liber principiorum philosophiae* [1274–83]
(Not to be confused with III.43 below)
- II.A.12—*Principles of Theology, Liber principiorum theologiae* [1274–83]
- II.A.13—*Principles of Law, Liber principiorum juris* [1274–83]
- II.A.14—*Ars universalis* [1274–83]
- II.A.17—*Llibre d'intenció, Liber de primera et secunda intentione* [1276–83]
- II.A.19—*Blaquerna* [1276–83, Montpellier]
 - II.A.19e—*Book of the Lover and the Beloved*
 - II.A.19f—*Art of contemplation*
- II.B.1—*Ars demonstrativa (AD)* [ca. 1283, Montpellier]
- II.B.3—*Ars inveniendi particularia in universalibus (AIPU)* [1283–7 (?)]
- II.B.4—*Liber propositionum secundum Artem demonstrativam compilatus (PropAD)* [1283–7 (?)]
- II.B.5—*Introductoria Artis demonstrativae* [1283–5 (?)]

- II.B.6—*Liber de quaestionibus per quem modus Artis demonstrativae patefit* [1283–7 (?)]
- II.B.9—*Lectura super figuras Artis demonstrativae (LFAD)* [1285–7 (?), Montpellier]
 II.B.9a—*Liber chaos*
- II.B.10—*Regles introductòries a la pràctica de l'Art demostrativa* [1283–5 (?)]
- II.B.11—*Liber exponens figuram elementalem Artis demonstrativae* [1285–7 (?)]
- II.B.13—*Disputatio fidelis et infidelis* [1287–9, Paris]
- II.B.15—*Felix or the Book of Wonders* [1287–9, Paris]
 II.B.15a—*Book of the Beasts*
- II.B.17—*Compendium seu commentum Artis demonstrativae* [1289, Paris]
- II.B.18—*Quaestiones per Artem demonstrativam seu inventivam solubiles* [1289]
- III.1—*Ars inventiva veritatis (AIV)* [1290, Montpellier]
- III.2—*Art amativa, Ars amativa boni (AA)* [1290]
- III.2.bis—*Taula d'esta Art* [1290]
- III.6—*Quaestiones quas quaesivit quidam frater minor* [1290 (?)]
- III.11—*Tabula generalis, Taula general (TG)* [1293–4, Tunis-Naples]
- III.12—*Liber de affatus* [1294, Naples]
- III.13—*Flowers of Love and Flowers of Intelligence* [1294, Naples]
- III.14—*Disputació de cinc savis* [1294, Naples]
- III.16—*Arbre de filosofia desiderat* [1294, Naples-Barcelona-Majorca]
- III.18—*Liber de levitate et ponderositate elementorum* [1294, Naples]
- III.19—*Art of Proposing and Solving Questions* [1294–5, Naples-Rome]
- III.20—*Lectura compendiosa Tabulae generalis* [1295, Rome]
- III.22—*Desconhort* [1295, Rome]
- III.23—*Tree of Science* [1295–6, Rome]
- III.24—*Book of the Articles of Faith, or Apostrophe* [1296, Rome]
- III.26—*Proverbs of Ramon* [1296 (?), Rome]
- III.29—*Tractatus novus de astronomia* [1297, Paris]
- III.30—*Declaratio Raimundi per modum dialogi edita contra aliquorum philosophorum et eorum sequacium opiniones* [1298—Paris]
- III.30.bis—*Investigatio generalium mixtionum secundum Artem generalem* [1298, Paris]
- III.31—*Disputatio eremitaie et Raimundi super aliquibus dubiis quaestionibus Sententiarum Magistri Lombardi* [1298—Paris]
- III.32—*Arbre de filosofia d'amor, Tree of the Philosophy of Love* [1298, Paris]
- III.36—*Ars compendiosa (Brevis practica Tabulae generalis)* [1299, Paris]
- III.37—*De quadratura et triangulatura circuli (Principia theologiae)* [1299, Paris]

- III.39—*Liber de geometria nova* [1299, Paris]
 III.40—*Quaestiones Attrebatenses (Quaestiones Magistri Thomae Attrebatensis)* [1299, Paris]
 III.41—*Dictat de Ramon, Tractatus compendiosus de articulis fidei catholicae* [1299, Barcelona-Majorca]
 III.43—*Principia philosophiae* [1299–1300, Paris-Majorca] (Not to be confused with II.A.11 above)
 III.44—*Medicina de peccat* [1300, Majorca]
 III.47—*Liber de homine* [1300, Majorca]
 III.49—*Aplicació de l'Art general* [1301, Majorca]
 III.50—*Rhetorica nova* [1301, Cyprus]
 III.51—*Liber de natura* [1301, Cyprus]
 III.53—*Mil proverbis* [1302, on shipboard]
 III.56—*Logica nova (LN)* [1303, Genova]
 III.58—*Disputatio fidei et intellectus* [1303, Montpellier]
 III.59—*Liber de lumine* [1303, Montpellier]
 III.60—*Liber de regionibus sanitatis et infirmitatis* [1303, Montpellier]
 III.62—*Liber de intellectu* [1304, Montpellier]
 III.63—*Liber de voluntate* [1304, Montpellier]
 III.64—*Liber de memoria* [1304, Montpellier]
 III.64.bis—*Lectura Artis quae intitulatur Brevis practica Tabulae generalis* [1304, Genova]
 III.64.ter—*Liber ad probandum aliquos articulos fidei catholicae per syllogisticas rationes* [1304, Genova]
 III.65—*Liber de significatione* [1304, Montpellier]
 III.69—*Liber de praedicatione* [1304, Montpellier]
 III.70—*Liber de ascensu et descensu intellectus* [1305, Montpellier]
 III.71—*Liber de demonstratione per aequiparantiam* [1305, Montpellier]
 III.73—*Liber praedicationis contra Judaeos* [1305, Barcelona]
 III.74—*Liber de Trinitate et Incarnatione* [1305, Barcelona]
 III.77—*Ars brevis (AB)* [1308, Pisa]
 III.79—*Liber de venatione substantiae, accidentis et compositi* [1308, Montpellier]
 III.79a—*De venatione mediū inter subjectum et praedicatum*
 III.80—*Ars generalis ultima (AGU)* [1305–8, Lyon-Pisa]
 III.81—*Disputatio Raimundi christiani et Homeri saraceni* [1308, Pisa]
 III.84—*Ars compendiosa Dei* [1308, Montpellier]
 IV.1—*Liber de novis fallaciis* [1308, Montpellier]
 IV.4—*Liber de experientia realitatis Artis ipsius generalis* [1308, Montpellier]

- IV.5—*Liber de refugio intellectus* [1308, Montpellier]
 IV.5a—*Liber de conversione syllogismi opinativi in demonstrativum cum vicesima fallacia*
- IV.6—*Excusatio Raimundi* [1309, Montpellier?]
 IV.12—*Liber de acquisitione Terrae Sanctae* [1309, Montpellier]
 IV.19—*Ars mystica* [1309, Paris]
 IV.20—*Liber de perversione entis removenda* [1309, Paris]
 IV.21—*Metaphysica nova et compendiosa* [1310, Paris]
 IV.22—*Liber novus physicorum* [1310, Paris]
 IV.25—*Liber de praedestinatione et praescientia* [1310, Paris]
 IV.26—*Liber de modo naturali intelligendi* [1310, Paris]
 IV.28—*Liber de conversione subjecti et praedicati et medi* [1310, Paris]
 IV.30—*Liber in quo declaratur quod fides sancta catholica est magis probabilis quam improbabilis* [1310—Paris]
 IV.31—*Liber de possibili et impossibili* [1310, Paris]
 IV.32—*De fallaciis quas non credunt facere aliqui qui credunt esse philosophantes* [1310 (?), Paris]
 IV.35—*Liber lamentationis Philosophiae* [1311, Paris]
 IV.36—*Liber contradictionis* [1311, Paris]
 IV.37—*Liber de syllogismis contradictoriis* [1311, Paris]
 IV.38—*Liber de divina unitate et pluralitate* [1311, Paris]
 IV.41—*Liber facilis scientiae* [1311, Paris]
 IV.44—*Liber de divina existentia et agentia* [1311, Paris]
 IV.46—*Liber de ente quod simpliciter est per se et propter se existens et agens* [1311, Paris]
 IV.47—*Vita coetania* [1311, Paris]
 IV.49—*Disputatio Petri clerici et Raimundi phantastici* [1311, Paris-Vienne]
 IV.50—*Liber de ente reali et rationis* [1311, Vienne]
 IV.52—*De ente simpliciter absoluto* [1312, Vienne]
 IV.57—*Liber de novo modo demonstrandi* [1312, Majorca]
 IV.60–7—*Summa sermonum*
 IV.65—*Llibre de virtuts e de pecats* [1313, Majorca]
 IV.67—*Ars abbreviata praedicandi* [1313, Majorca]
 IV.68—*Liber per quem poterit cognosci quae lex sit magis bona* [1313, Majorca]
 IV.72—*Liber de compendiosa contemplatione* [1313, Messina]
 IV.73—*Llibre de consolació d'ermità* [1313, Messina]
 IV.75—*Liber de accidente et substantia* [1313, Messina]
 IV.76—*Liber de ente absoluto* [1313, Messina]

- IV.80—*Liber de medio naturali* [1313, Messina]
IV.89—*Liber de quinque praedicabilibus et decem praedicamentis* [1313, Messina]
IV.94—*Liber de potestate pura* [1314, Messina]
IV.107—*De ostensione per quam fides catholica est probabilis atque demonstrabilis* [1313–14, Messina]
IV.108—*Liber de civitate mundi* [1314, Messina]
IV.109—*Ars consilii* [1315, Tunis]

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